Approximation of singular Sturm-Liouville problems at regular singularity in indefinite metric framework

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We consider the singular expressions

$$\ell = -\frac{d^2}{dx^2} + v(x), \quad v(x) = \frac{\nu^2 - 1/4}{x^2} + \frac{v_{-1}}{x} + v_{reg}(x)$$

on $(0, b), b \leq \infty$ and assume $\nu \geq 1$. In this case the 'limit point case' prevails at $x = 0$ in L^2-setting of the boundary problem.However, in this case, there is a Pontryagin space realization S of the expression ℓ in a Pontryagin space Π_κ with $\kappa = [\frac{\nu+1}{2}]$, which is symmetric and has defect indices $(1, 1)$, and the 'limit circle' case at $x = 0$ is reproduced in Π_κ.

We disscuss the approximation of self-adjoint extensions of S by realizations of the following regular boundary problems

$$\ell y = zy, \quad y'(\varepsilon) = \beta(\varepsilon, z)y(\varepsilon),$$

on extending intervals (ε, b), when $\varepsilon \to 0$. Here $\beta(\varepsilon, z)$, for each $\varepsilon > 0$ is a polynomial of z of degree $[\nu]$. The coefficients of these polynomials are determined by asymptotic expansion in ε of the Titchmarsh-Weyl coefficients $m(\varepsilon, z)$ associated with family of the regular boundary problems for ℓ on (ε, b).

The talk is based on joint papers with A. Dijksma and A. Luger in [1].

References

[1] A. Dijksma, A. Luger, Yu. Shondin, *Approximation of $\mathcal{N}_\kappa^\infty$–functions I: models and regularization*, Operator Theory: Adv. and Appl., v. 188 (2008) 95-120; *Approximation of $\mathcal{N}_\kappa^\infty$–functions II: Convergence of Models*, Operator Theory: Advances and Applications, V. 198 (2009) 125-169 .