

Micro-localization from Clifford Analysis

F. Sommen

The theory of formal boundary values of holomorphic functions in the upper and lower half plane leads to the definition of the spaces of hyperfunctions and microfunctions on the real line. This can be thought of as singular analytic signals. In the higher-dimensional case there are two approaches to the theory of hyperfunctions and of microfunctions: the classical one using functions of several complex variables and an approach from Clifford analysis, which is related to the Riesz transform and makes use of the monogenic Cauchy kernel, so that it is related to monogenic signals. In the process of micro-localization one starts from the Radon transform which gives the delta function as an integral over the sphere of one-dimensional delta functions on hyperplanes. The microlocal decomposition of the delta function is obtained as a deformation of the Radon inversion formula, where the kernel is singular in one point and in one direction. In our presentation we derive this formula from the Radon decomposition of the Cauchy kernel restricted on a parabolic surface. In this way complicated techniques from several complex variables can be bypassed.