Progress in wedge diffraction

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Erhard Meister developed with me in the 1980's an operator theoretical approach for so-called canonical problems in diffraction theory. Starting from a weak formulation of Sommerfeld type (boundary value or transmission) problems we defined the associated operator (mapping the solution space into the data space) and constructed relations with singular operators, e.g., with Wiener-Hopf operators in Sobolev spaces, for which (generalized) inverses could be explicitly obtained, in closed analytical form, by matrix factorization methods. This research gave an impact to an intensive study of (1) the factorization of non-rational matrix functions (such as Daniele-Khrapkov matrices), (2) operator relations in general (such as equivalent after extension relations and its generalizations) and (3) new techniques for the normalization of singular operators (associated with ill-posed problems). In 1990-94 our cooperation with Francisco Teixeira and Frank Penzel resulted in a series of papers on rectangular wedge diffraction problems, in which Wiener-Hopf-Hankel operators had to be inverted. So far, this was possible only in particular cases with rather sophisticated methods. In 2002-05 I came back to this class of problems when preparing the Erhard Meister Memorial Volume (in the Birkhäuser OT series, Vol. 147). Together with Lus Castro, Francisco Teixeira and (finally) Roland Duduchava, we developed a rigorous approach to solve boundary value problems for the Helmholtz equation in a quadrant with rather arbitrary boundary conditions (including impedance and oblique derivative data). The results were based upon new techniques for an asymmetric factorization of scalar and matrix functions due to the Wiener-Hopf plus Hankel operators in view. The present method consists of a combination of our knowledge about the analytical solution of Sommerfeld and rectangular wedge diffraction problems with new symmetry arguments that relate the present to previously solved problems and yield the explicit analytical solution in a great number of cases. For this purpose we introduce here so-called "Sommerfeld potentials" (explicit solutions to special Sommerfeld problems) whose use turns out to be most efficient. It is surprising that the case where the angle is an integer part of 2π can be solved completely whilst the case of "rational" angles $\alpha = 2\pi m/n$ for $m > 1$ appears much harder and remains, in general, unsolved at present. As an interesting and very direct conclusion we obtain the result that, for the angles under consideration, the behavior of the field shows the same singularity in the corner as the singular behavior in the corresponding half-plane or Sommerfeld potential cases. This work is based on joint research with Torsten Ehrhardt and Ana Paula Nolasco. Some key references are given below.

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