AAK approximants to functions with branch points

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We are concerned with the asymptotic behavior of AAK (Adamjan-Arov-Krein) approximants to functions f that are holomorphic outside the unit disk $\mathbb{D}$ and a little bit beyond, and have all their singularities in a compact set of capacity zero in $\mathbb{D}$. Among these singularities there should be also branch points. Our main object is the investigation of the asymptotic distribution of the convergence behavior of the AAK approximants.

AAK approximants are meromorphic functions in $\mathbb{D}$ with a controlled, finite number of poles, and they have a minimal deviation in the uniform norm on $\mathbb{T}$ from the function f to be approximated. There is an important and very interesting connection with Hankel operators and their singular values, which is central for their understanding. In our talk this topic will play a minor role, instead we shall concentrate on questions that are important for understanding of the analytic background of the arcs in $\mathbb{D}$ that form the support of the asymptotic distribution. Methodologically, the main tools of the investigation belong to potential theory and geometric theory of functions.

The talk is based on a joint work with L. Baratchart and M. Yattselev.