When is a non-self-adjoint Hill operator a spectral operator of scalar type?

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We consider a Hill operator

$$H = -\frac{d^2}{dx^2} + V(x), \qquad x \in \mathbb{R},$$

with a complex-valued π-periodic potential $V(x)$ such that $V \in \mathcal{L}^2([0,\pi])$ and prove a criterion for it to be a spectral operator of scalar type in the sense of Dunford [1]. This criterion is stated in two versions.

The first version is given in terms of three entire functions which are independent parameters uniquely determining the potential V, *cf.,* [2], and the second one is formulated in terms of algebraic and geometric properties of spectra of periodic/antiperiodic and Dirichlet boundary problems generated by H in the space $\mathcal{L}^2([0,\pi])$.

The problem of deciding which Hill operators are spectral operators of scalar type appears to have been open for about 50 years.

This is a joint work with F. Gesztesy published in [3].

References

1. N. Dunford, *A survey of the theory of spectral operators*, Bull. Amer. Math. Soc. **64** (1958), 217-274.

2. J.-J. Sansuc and V. Tkachenko, *Spectral parametrization of non-selfadjoin Hill's operators*, J. Differential Equations **125** (1996), 366-384.

3. F. Gesztesy and V. Tkachenko, *A criterion for Hill operators to be a spectral operators of scalar type*, J. d'Analyse Mathematique **107** (2009), 287-353.