Operators on partial inner product spaces

C. Trapani

Many families of function spaces, such as L^p spaces, Besov spaces, amalgam spaces or modulation spaces, exhibit the common feature of being indexed by one parameter (or more) which measures the behavior (regularity, decay properties) of particular functions. All these families of spaces are, or contain, scales or lattices of Banach spaces and constitute special cases of the so-called *partial inner product spaces (*PIP*-space s)* that play a central role in analysis, in mathematical physics and in signal processing (e.g. wavelet or Gabor analysis). The basic idea for this structure is that such families should be taken as a whole and operators, bases, frames on them should be defined globally, for the whole family, instead of individual spaces.

In this talk, we shall give an overview of PIP-space s and operators on them, illustrating the results by families of spaces of interest in mathematical physics and signal analysis. In particular, an operator on a PIP-space is a *coherent* collection of linear maps, each one of them acting on one space of the family: they are often regular objects when considered on the global structure of a PIP-space but possibly singular when considered in an individual space. Various classes of operators will be considered and the link between (partial) *-algebras of operators on a PIP-space and (partial) *-algebras of unbounded operators acting in Hilbert spaces will be briefly discussed.

The talk is based on the joint research monograph with J.-P. Antoine, *Partial inner product spaces: Theory and applications*, (Lecture Notes in Mathematics #1989, 2009)