Spectral problem for a class of non-self-adjoint Jacobi matrices

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We study the distribution of eigenvalues of tridiagonal matrices of the form

$$J_n^{(k)} = \begin{pmatrix} a & b_1 & 0 & \dots & 0 & 0 \\ c_1 & 0 & b_2 & \dots & 0 & 0 \\ 0 & c_2 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & b_{n-1} \\ 0 & 0 & 0 & \dots & c_{n-1} & 0 \end{pmatrix}, \tag{1}$$

where $a \in \mathbb{R}$, $c_j > 0$ and, for some number k, $0 \leqslant k \leqslant n-1$,

$$b_j < 0 \qquad \text{whenever} \quad j \leqslant k,$$

$$b_j > 0 \qquad \text{whenever} \quad j > k.$$

It is established that for any k, $0 \leqslant k \leqslant n-1$, the characteristic polynomial of the matrix $J_n^{(k)}$ is a generalized Hurwitz polynomial [1]. Conversely, for any generalized Hurwitz polynomial p of degree n, there exists a unique matrix of the form (1) whose characteristic polynomial is p. Special cases of $k = 0$ and $k = n-1$ are particularly discussed.

References

[1] M. Tyaglov, Generalized Hurwitz polynomials, *in preparation*.