Generalized Triebel-Lizorkin spaces and bounded H^∞-calculus for $\mathcal{R}_q$-sectorial operators

A. Ullmann

A set $\mathcal{T}$ of bounded operators on a Banach function space X is called $\mathcal{R}_q$-bounded if an estimate

$$\Big\| \Big(\sum_{j=1}^{n} |T_j x_j|^q \Big)^{1/q} \Big\|_X \lesssim \Big\| \Big(\sum_{j=1}^{n} |x_j|^q \Big)^{1/q} \Big\|_X$$

holds uniformly for $T_j \in \mathcal{T}, x_j \in X, n \in \mathbb{N}$. A sectorial operator A in X is called $\mathcal{R}_q$-sectorial, if the set $\{\lambda R(\lambda, A) | \lambda \notin \overline{\Sigma}_\omega\}$ is $\mathcal{R}_q$-bounded outside some closed sector $\overline{\Sigma}_\omega$. We will associate certain homogeneous and inhomogeneous intermediate spaces $\dot{X}^\theta_{q,A}, X^\theta_{q,A}$ to A which correspond to the classical Triebel-Lizorkin spaces if $X = L^p$ and $A = -\Delta$. We will show that the part of A has always a bounded H^∞-calculus in the homogeneous space $\dot{X}^\theta_{q,A}$ and that these spaces are stable under certain kinds of perturbation. As an application we will obtain a bounded H^∞-calculus for uniformly elliptic operators with Hölder-continuous coefficients in the classical Triebel-Lizorkin spaces.