Commutative algebras of Toeplitz operators, hyperbolic geometry, and Berezin quantization

N. Vasilevski

We will discuss a quite unexpected phenomenon in the theory of Toeplitz operators on the Bergman space: the existence of a reach family of commutative C*-algebras generated by Toeplitz operators with non-trivial symbols. As it tuns out the smoothness properties of symbols do not play any role in the commutativity, the symbols can be merely measurable. Everything is governed here by the geometry of the underlying manifold, the hyperbolic geometry of the unit disk. At the same time the complete classification of such commutative algebras involves the Berezin quantization procedure.

These commutative algebras come with a powerful research tool, the spectral type representation for the operators under study. This permit us to answer to many important questions in the area. As an example we consider a kind of the semi-classical analysis of the spectra of Toeplitz operators.