Calculating Hausdorff dimensions of invariant sets using spectral theory

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The dimension of an invariant set of a dynamical system is one of the most important characteristics. In this talk we present a new approach to compute the Hausdorff dimension of conformally self-similar invariant sets. The approach is based on a direct spectral analysis of the transfer operator associated with the dynamical system. This operator theoretic approach relies on the theory of trace class operators and their determinants and is a tribute to one of the fundamental contributions of Israel Gohberg to operator theory. In the case that the maps defining the dynamical system are analytic, our method yields a sequence of successive approximations that converge to the Hausdorff dimension of the invariant set at a super-exponential rate. This allows us to estimate the dimension very precisely. We illustrate our approach with examples from dynamical systems and from number theory via Diophantine approximations.