Realization of noncommutative functions

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A theorem of Ball–Groenewald–Malakorn (BGM) describes various incarnations of the d-variable noncommutative Schur class as characteristic functions of unitary colligations associated to appropriate noncommutative multidimensional systems. BGM view the characteristic (transfer) function as a formal power series in d noncommuting indeterminates that can be then evaluated on appropriate d tuples of operators on a Hilbert space (e.g., forming a row contraction). However it can be also viewed as a *noncommutative function* as introduced by Kaliuzhnyi-Verbovetskyi–Vinnikov; i.e., it is a function defined on appropriate d-tuples of complex matrices of all sizes which satisfies certain compatibility conditions as we vary the size of matrices — it respects direct sums and simultaneous similarities. The d-tuples of matrices where this function is defined form in fact the noncommutative unit ball over $\mathbb{C}^d$ endowed with an appropriate operator space structure. This indicates a possible generalization towards realization theorems for contractive noncommutative functions on the noncommutative unit ball over a general operator space.