On sufficient conditions for the total positivity of matrices and applications

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We will discuss the following sufficient condition of the total positivity of matrices.

Theorem (O. Katkova, A. Vishnyakova). *Suppose $M = (a_{i,j})$ be a $k \times k$ matrix with positive entries and $a_{i,j}a_{i+1,j+1} \geq 4\cos^2\frac{\pi}{k+1} \; a_{i,j+1}a_{i+1,j}$ ($1 \leq i \leq k-1$, $1 \leq j \leq k-1$). Then the matrix M is totally positive.*

The constant $4\cos^2\frac{\pi}{k+1}$ in this Theorem is the smallest possible not only in the class of $k \times k$ matrices with positive entries but in the classes of $k \times k$ Toeplitz matrices and of $k \times k$ Hankel matrices. Some applications and generalizations of the above result to almost strictly totally positive matrices, Pólya frequency sequences, moment problem, hyperbolic polynomials, Hurwitz stable polynomials and positive polynomials will be presented.

The talk is based on a joint work with O. Katkova.