Positive extensions of matrices indexed by a homogeneous tree

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Let T be a homogeneous tree of order q – that is, an acyclic, undirected, connected graph such that every node belongs to exactly q + 1 edges. Let T_n be a maximal subgraph of T with the property that the distance between any two nodes of T_n does not exceed n, and let $A = [a_{t,s}]_{t,s\in T_n}$ be a square positive definite matrix indexed by the nodes of T_n . The matrix A is said to be *isotropic* if $a_{t,s}$ depends only on the distance between the nodes t and s(in the case q = 1 this means that A is a real symmetric Toeplitz matrix). The positive extension problem for the matrix A consists in finding all such isotropic positive definite matrices B indexed by the nodes of $T_{n+1} \supset T_n$ that the diagonal block of B corresponding to T_n coincides with A.

In this talk we shall discuss a solution of the positive extension problem based on the canonical form of isotropic matrices, obtained in a joint work with D. Alpay.