Rational discrete analytic functions

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A function $f : \mathbb{Z}^2 \longrightarrow \mathbb{C}$ is said to be discrete analytic if

$$\frac{f(x+1,y+1)-f(x,y)}{1+i} = \frac{f(x,y+1)-f(x+1,y)}{i-1} \quad \forall x,y \in \mathbb{Z}.$$

This notion of discrete analyticity, developed by J. Ferrand (Lelong), R.J. Duffin and others, has many analogies with the classical continuous theory. However, there is also a notable difference: the pointwise product of two discrete analytic functions is not necessarily discrete analytic. For example, the functions $z = x + yi$ and $z^2 = x^2 - y^2 + 2xyi$ are discrete analytic but the function $z^3 = x^3 - 3xy^2 + 3x^2yi - y^3i$ is not. Nevertheless, one can show that for every polynomial $p(x)$ there is a unique discrete analytic polynomial $P(x,y)$ such that $P(x,0) = p(x)$ $\forall x \in \mathbb{Z}$.

We shall discuss a generalization of this result for rational functions.

The talk is based on a joint work with D. Alpay.