Multicomponent transmission problems with spectral parameter in equations and boundary conditions

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We consider linear multicomponent boundary value transmission problems where we have several unknown functions defined on jointed boundary domains and connected with each other only by boundary transmission conditions. These functions satisfy elliptic equations in domains with Lipshitz boundaries and boundary conditions contained the spectral parameter. We consider two main situations when we have equal values of functions or normal derivatives on joint boundaries. Such problems arise in different applied problems from theory of diffraction, theory of elasticity, mechanics, hydrodynamics and others (see e.g. [1], [2]).

To obtain qualitative properties of these problems, i.e. discreteness of spectrum, basis properties of eigenfunctions, we research the abstract auxiliary boundary value problems. We use such name for generalized problems formulated in terms of operators from abstract Green's formula. Last one can be constructed by given triple of Hilbert spaces and abstract trace operator (see [3] and [4], [5]). For problems of transmission we use abstract Green's formula for mixed boundary value problems proved by N. Kopachevsky. It can be constructed by given triple of Hilbert spaces and set of abstract traces operators acting to special boundary spaces (to parts of boundary) with additional imbedding properties (see [6]). Finding weak solutions of abstract boundary value problems as elements satisfying variational identities we can introduce and research properties of corresponding linear operators acting in some Hilbert spaces. As a result we find operator statement of initial boundary value problem that is eigenvalue problem for an operator-matrix or eigenvalue problem for a pencil of operators.

The talk is based on a joint work of N. Kopachevsky, P. Starkov and V. Voytitsky (see [6]).

References

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