

# Products of Nevanlinna functions with symmetric rational functions

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To a symmetric rational function  $r$  associate the class  $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  of generalized Nevanlinna functions by the formula

$$\mathcal{N}_\kappa^{\tilde{\kappa}}(r) = \{Q \in \mathcal{N}_\kappa : rQ \in \mathcal{N}_{\tilde{\kappa}}\}.$$

Here  $\mathcal{N}_\kappa$ ,  $\kappa \in \mathbb{N}$ , denotes the class of generalized Nevanlinna function with  $\kappa$  negative squares.

With  $\tilde{\kappa} = 0$  the classes  $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  extend the Kreĭn-Langer classes  $\mathcal{N}_\kappa^+$  of generalized Nevanlinna functions, see [4], and with  $\kappa = 0$  the classes  $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  extend the classes  $\mathcal{S}^{\pm\tilde{\kappa}}(\alpha, \beta)$  ( $-\infty < \alpha < \beta < \infty$ ) of Nevanlinna functions introduced by V.A. Derkach and M.M. Malamud in [2], as a generalization of the Stieltjes and inverse Stieltjes classes  $\mathcal{S}$  and  $\mathcal{S}^{-1}$ , which were originally introduced by M.G. Kreĭn, see e.g. [3]. Furthermore, the classes  $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  contain the classes  $\tilde{\mathcal{N}}_k^{\pm\kappa}$  introduced by V.A. Derkach in [1].

By means of a characterization of Nevanlinna functions with gaps by their behavior at the endpoints of the gaps, the classes  $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  can be completely described. Furthermore, for a function  $Q \in \mathcal{N}_\kappa^{\tilde{\kappa}}(r)$  the realization of  $rQ$  is connected to the realization of  $Q$ .

This talk is based on joint work with S. Hassi.

## References

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