The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J–dilations and canonical forms

M. Wojtylak

We will describe a procedure of dilating an operator in an infinite dimensional Krein space. Namely, let $\mathcal{H}_0$, $\mathcal{K}_0$, $\mathcal{H}$, $\mathcal{K}$ be Krein spaces. We say that an operator $T \in \mathbf{B}(\mathcal{H}, \mathcal{K})$ is an *J–dilation of* $T_0 \in \mathbf{B}(\mathcal{H}_0, \mathcal{K}_0)$ if the following three conditions are satisfied:

(i) $\mathcal{H}_0$ is a subspace of $\mathcal{H}$, $\mathcal{K}_0$ is a subspace of $\mathcal{K}$.

(ii) There exist subspaces $\mathcal{H}_i$ of $\mathcal{H}$ and $\mathcal{K}_i$ of $\mathcal{K}$ $(i = 1, 2, 3)$ such that

$$\mathcal{H} = \mathcal{H}_0 \boxplus \mathcal{H}_1 \boxplus (\mathcal{H}_2 \dot{+} \mathcal{H}_3), \qquad \mathcal{K} = \mathcal{K}_0 \boxplus \mathcal{K}_1 \boxplus (\mathcal{K}_2 \dot{+} \mathcal{K}_3),$$

where $\mathcal{H}_1$ and $\mathcal{K}_1$ are Krein spaces, $\mathcal{H}_2$ and $\mathcal{H}_3$ ($\mathcal{K}_2$ and $\mathcal{K}_3$) are skewly linked neutral spaces such that $\mathcal{H}_2 + \mathcal{H}_3$ ($\mathcal{K}_2 + \mathcal{K}_3$) is a Krein space.

(iii) The operator T has a following representation with respect to the above decomposition

$$T = \begin{pmatrix} T_0 & 0 & T_{02} & 0 \\ 0 & 0 & 0 & 0 \\ T_{20} & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1}$$

It appears that the dilated operators $T^{[*]}T$ and $TT^{[*]}$ inherit many spectral properties from the operators $T_0T_0^{[*]}$ and $T_0^{[*]}T_0$, respectively. We use the J-dilation procedure to study and compare the canonical forms of matrices $T^{[*]}T$ and $TT^{[*]}$ in a finite dimensional Krein space.

The talk is based on a joint work

[1] Ran A.C.M., Wojtylak M., The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J–dilations and canonical forms, *Integral Equations and Operator Theory*, to appear.