Spectral properties of nonselfadjoint one-dimensional singular perturbations of unbounded selfadjoint operators

D. Yakubovich

A (linear unbounded) operator A is called a finite-dimensional singular perturbation of an operator A_0 if their graphs differ in a finite-dimensional space. We study spectral properties of a one-dimensional singular perturbation A of an unbounded selfadjoint operator A_0 with compact resolvent. Our approach is based on a functional model of this operator, similar to a model by V. Kapustin. We assume that the spectrum of A is real. We show that for any operator A of our class there exist an inner function $\Theta(z)$ and an outer function $\varphi(z)$ in the upper half plane $\mathbb{C}_+$ with $\frac{\varphi}{z+i} \in H^2$ and

$$\Theta = \frac{\varphi}{\overline{\varphi}} \qquad \text{a.e. on } \mathbb{R}$$

such that A is unitarily equivalent to the operator $T = T_{\Theta,\varphi}$ which acts on the model space $K_\Theta = H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+)$, with the domain defined as

$$\mathcal{D}(T) = \{f \in K_\Theta : \qquad \exists\, c = c(f) \in \mathbb{C} : zf - c\varphi \in K_\Theta\},$$

and

$$Tf = zf - c\varphi, \qquad f \in \mathcal{D}(T).$$

We give criteria for completeness of eigenvectors and for the possibility to remove the whole spectrum by an adequate perturbation of the type considered in terms of the sparsity of the spectrum of the unperturbed operator. The proofs use entire functions of Hermite–Biehler and Cartwright classes.

This work was supported by the grant MTM2008-06621-C02-01/MTM from the Ministry of Science and Innovation and FEDER.

The talk is based on a joint work with A. Baranov.