Convergent interpolation to Cauchy integrals over analytic arcs with Jacobi-type weights

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We design uniformly convergent sequences of rational interpolants to Cauchy integrals of the form $f_\mu(z) = \int (z-t)^{-1} d\mu(t)$, where

$$d\mu(t) = (1-t)^\alpha (1+t)^\beta h(t) dt, \quad \alpha, \beta > -1,$$

and h is a non-vanishing smooth function on an analytic arc Δ with endpoints ± 1. Namely, we show that for any analytic arc there exist probability Borel measures that make it symmetric in the sense of Stahl [1] (that is, the normal derivatives, taken from the left and right-hand sides of Δ, of the Green potential of each such measure coincide). Proper discretization of these measures produces sought interpolation schemes. The proof of convergence of the corresponding rational interpolants proceeds via $\bar{\partial}$-extension [2] of the Riemann-Hilbert approach [3] applied to the underlying boundary value problem.

This talk is based on a joint work with L. Baratchart.

References

[1] H. Stahl. Structure of extremal domains associated with an analytic function. *Complex Variables Theory Appl.*, 4:339–356, 1985.

[2] K.T.-R. McLaughlin and P.D. Miller. The $\bar{\partial}$ steepest descent method for orthogonal polynomials on the real line with varying weights. *Int. Math. Res. Not. IMRN*, 2008:66 pages, 2008.

[3] A.B. Kuijlaars, K.T.-R. McLaughlin, W. Van Assche, and M. Vanlessen. The Riemann-Hilbert approach to strong asymptotics for orthogonal polynomials on $[-1, 1]$. *Adv. Math.*, 188(2):337–398, 2004.