Complete sets of metrics in solvable PT-symmetric models

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The concept of the observable coordinate of a point particle moving in a 1D potential $V(q)$ is often generalized to a not necessarily measurable real argument x entering a concrete representation $\psi(x) \in L^2(R)$ of an abstract ket-vector $|\psi\rangle$. As a consequence, the "friendly" space $L^2(R) := \mathcal{H}^{(F)}$ may (and, for the so called PT-symmetric Hamiltonians H, does) prove unphysical. Fortunately, a return to textbook theory can be mediated by the replacement of linear functionals in $\mathcal{H}^{(F)}$ [i.e., dual vectors called, usually,"Dirac's" bra-vectors $\langle\psi|$] by "brabras" $\langle\langle\psi| = \langle\psi|\Theta$. They are defined in terms of a suitable "metric" $\Theta \neq I$ and form the linear functionals in a unitarily inequivalent and potentially physical Hilbert space $\mathcal{H}^{(P)}$. The nature and elimination of the ambiguity of the assignment of metric Θ to a given Hamiltonian H will be studied. We shall take advantage of the possibility of construction of all of the admissible, $H-$compatible metrics $\Theta = \Theta(H)$ in a few simplified, schematic models.