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Book of Abstracts

Some boundary value problems for operator-differential equations of mixed type and their applications to inverse problems

N.L. Abasheeva

First we consider the questions on existence and uniqueness of a solution to the boundary value problem

$$B(t)u_t = L(t)u + f(t),\ t \in (0, T),\ T < \infty,$$

$$u(T) - u(0) = u_0.$$

Here $B(t)$ are self-adjoint operators and $L(t)$ are uniformly dissipative operators acting in a complex Hilbert space E with the inner product $(\cdot, \cdot)$ and the norm $\|\cdot\|$. The operator $B(t)$ can be noninvertible and has arbitrary arrangement of the spectrum.

Then we apply obtained results to the following linear inverse problem: to find a function $u(t)$ and an element φ satisfying the equation

$$B(t)u_t = Lu + f(t) + B(0)\varphi$$

and the boundary conditions

$$u(0) = u_0,\ u(T) = u_1.$$

Trace formula in perturbation theory

V. Adamyan

In this talk we discuss some old and recent results on the trace formula of perturbation theory in Hilbert and Krein spaces and the contribution of Heinz Langer to the elaboration of this issue.

Quasiconformal mappings associated with Gahov's equation in inverse boundary value problems

A. Akhmetova

The question on the quantity of roots of so called Gahov's equation is of importance for the theory of inverse boundary value problems, because it determines the quantity of decisions of that problems. As known, it coincides with the number $\mathcal{N}$ of extreme points of conformal radius.

$$R \equiv R(D, z) = |f'(\zeta)|(1 - |\zeta|^2), \tag{1}$$

where $z = f(\zeta),\ \zeta \in E = \{\zeta : |\zeta| < 1\},\ D = f(E)$.

Here equlity (1) is understood as the graph of function $R(D, z)$, i.e., a surface in $\mathbb{R}^3$ over disk E or domain D. The value N coincides with the number of coverings of origin by the range of gradient of the conformal radius

$$\nabla R(D, z) = 2R_{\bar{z}}. \tag{2}$$

In this connection there arises an intrinsic problem of classification of diffeomorphisms (in particular, quasiconformal mappings) of form (2).

The present work is a review of results of the papers [4], [5] and [6], supplemented by new effects on quasiconformal mappings (2) for domains $f(rE),\ 0 < r < 1$, and on calculation of *Gahov's radiuses* for solution of inverse boundary value problems.

The talk is based on a joint work with L. Aksent'ev.

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Reduction of the plasticity model to an operator equation and regularity

H.-D. Alber

We show that the model equations of plasticity and viscoplasticity, which use internal variables to model history dependent material behavior, can be reduced to an evolution equation with a nonlinear monotone evolution operator. Based on standard properties of this evolution equation we prove interior and boundary regularity of the solutions. The most interesting part is the investigation of the boundary regularity, since one cannot apply the standard method to prove regularity of the tangential derivatives and subsequentially extend this result to the normal derivatives by solving the equations for this derivative. Accordingly, one can show that the stress field belongs to $H^{1,\mathrm{loc}}$, but at the boundary one only gets $H^{1/2-\varepsilon}$-regularity. Recently this result could be improved in [3] to $H^{1/2+\varepsilon}$.

The talk is based on joint work with S. Nesenenko.

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The dynamics of the multiple tunnel effect

F. Ali Mehmeti

We consider the Klein-Gordon equation on a star-shaped network composed of n half-axes connected at their origins, adding a potential which is constant but different on each branch. Our main result [3, 4] is an explicit construction of a spectral representation of the corresponding spatial operator using generalized eigenfunctions, exhibiting what we call the multiple tunnel effect.

Results in experimental physics [7, 8], theoretical physics [6] and functional analysis [1, 5] describe new phenomena created by the dynamics of the (simple) tunnel effect: the delayed reflection and advanced transmission near nodes issuing two branches. It is of major importance for the comprehension of the vibrations of networks to understand these phenomena near ramification nodes i.e. nodes with at least 3 branches, motivating our interest for the multiple tunnel effect.

Related possible applications are the $L^\infty-$time decay, the extension to coupled transmission conditions and, in the case of semi linear equations, global existence and causality (cf. [2] for the case of two branches).

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The talk is based on a joint work with R. Haller-Dintelmann and V. Régnier.

Linear stochastic systems: a white noise space approach

D. Alpay

We present a new approach to input-output systems

$$y_n = \sum_{m=0}^{n} h_m u_{n-m}, \quad n = 0, 1, \ldots,$$

and state space equations

$$\begin{aligned} x_{n+1} &= Ax_n + Bx_n, \\ y_n &= Cx_n + Du_n, \quad n = 0, 1, \ldots \end{aligned}$$

when randomness is allowed both in the input sequence (u_n) and in the impulse response (h_n) or in the matrices A, B, C, D, which determine the state space equations. We use Hida's white noise space setting, and the Kondratiev spaces of stochastic test functions and stochastic distributions. *The key to our approach is that the pointwise product between complex numbers is now replaced by the Wick product $\lozenge$ between random variables.* Thus we have systems of the form

$$y_n = \sum_{m=0}^{n} h_m \lozenge u_{n-m}, \quad n = 0, 1, \ldots,$$

which can be shown to be (in an appropriate basis) *double convolution systems*. The Hermite transform maps the white noise space onto the reproducing kernel Hilbert space $\mathcal{F}$ with reproducing kernel $e^{\langle z,w\rangle_{\ell_2}}$ with $z, w \in \ell_2$, that is, onto the Fock space. This allows us to transfer most, if not all, problems from classical system theory into problems for analytic functions where now there is a countable number of variables. We will describe some stability theorems. The image of the Kondratiev space of distributions under the Hermite transform is a commutative ring without divisors, and we will also explain links with the theory of linear systems over commutative rings. Finally, we will describe the parallels with double convolution systems associated to a new approach to multiscale systems, developed with M. Mboup.

The talk is based on various joint works with H. Attia, D. Levanony, M. Mboup and A. Pinhas.

A Schur algorithm for a large class of functions

D. Alpay

We present a framework within the indefinite Schur algorithm can be set, and which allows to consider generalized Schur functions, generalized Nevanlinna functions, and the like in a unified way. We consider a wide class of kernels which includes all such classical kernels as the kernels associated to Schur functions and Nevanlinna functions, and for which a Schur algorithm and general theory of interpolation can be developed.

The talk is based on a joint work with A. Dijksma, H. Langer and D. Volok.

On singular and Marcinkiewicz integral operators

A. Al-Salman

Singular integral operators and Marcinkiewicz integral operators have been an active research topic since their appearance in the works of Littlewood, Paley, Zygmund, Marcinkiewicz, and Stein. In recent years, the theory of such operators has attracted the attention of many mathematicians. In this paper, we discuss the L^p mapping properties of various classes of singular integral operators and Marcinkiewicz integral operators. In particular, we shall derive a multiplier formula characterizing the L^2 boundedness of Hörmander's parametric Marcinkiewicz integral operator. As a consequence, we prove the optimality of the kernel size condition $L(\log L)^{1/2}(\mathbf{S}^{n-1})$ as well as the optimality of the Block size condition. Furthermore, by the aid of such multiplier formula, we shall show the strong relation between the class of Marcinkiewicz integral operators and the corresponding class of Calderón- Zygmund singular integral operators.

Minimal and maximal invariant Banach spaces of holomorphic functions on bounded symmetric domains

J. Arazy

Let D be a *Cartan domain* of rank r and genus p in $\mathbb{C}^d$. The group G of all holomorphic automorphisms of D acts projectively and irreducibly on holomorphic functions on D via

$$U^{(\lambda)}(g)(f)(z) := (J(g^{-1})(z))^{\lambda/p} f(g^{-1}(z)), \quad z \in D, \quad g \in G,$$

where "J" stands for the complex Jacobian and λ is the "Wallach parameter".

We study the minimal and maximal Banach spaces of holomorphic functions on D which are isometrically invariant under $U^{(\lambda)}(G)$. In particular - we prove their existence and uniqueness, establish their duality with respect to the (unique) $U^{(\lambda)}(G)$ - invariant inner product, and identify them concretely as Besov-type spaces. It follows that the maximal space $M^{(\lambda)}$ consists of all holomorphic functions F on D for which $F(z)h(z,z)^{\lambda/2}$ is bounded, where $h(z,w)$ is the "Jordan determinant" (given by $det(I-zw^*)$ in the matrix domains), and the minimal space $\mathfrak{M}^{(\lambda)}$ is the space of all holomorphic functions f on D for which $f(z)h(z,z)^{\lambda/2}$ is integrable with respect to the (unique) G-invariant measure on D.

This study is extended to points λ in the *pole set* of D in the cases where D is of tube type and the highest quotient of the composition series of the $U^{(\lambda)}(G)$-invariant spaces is unitarizable (i.e. $s := d/r - \lambda$ is a positive integer). The main tool in this study is the *intertwining formula*

$$N^s(\frac{\partial}{\partial z})U^{(\lambda)}(g) = U^{(p-\lambda)}(g)\, N^s(\frac{\partial}{\partial z}), \quad \forall g \in G,$$

where $N(z)$ is the *determinant* polynomial of the Euclidean Jordan algebra associated with D. For instance, if $\lambda = 0$ (and thus $U^{(0)}(g)f := f \circ g^{-1}$) and $s := d/r$ is a positive integer, then

$$M^{(0)} = \{F \text{ holomorphic in } D;\ \|F\| := \sup_{z\in D} |N^s(\frac{\partial}{\partial z})F(z)|h(z,z)^s < \infty\},$$

$$\begin{aligned}\mathfrak{M}^{(0)} =&\{f \text{ holomorphic in } D;\\ &\qquad \|f\| := \int_D |N^{2s}(\frac{\partial}{\partial z})(N^s(z)f(z))|\, dm(z) < \infty\}.\end{aligned}$$

These results generalize the well-known facts concerning the maximality of the Bloch space and the minimality of the Besov space $B^1_{1,1}$ among Möbius-invariant Banach spaces of holomorphic functions on the unit disk.

From forms to semigroups

W. Arendt

Sesquilinear forms are a powerful tool to generates one-parameter semigroups on Hilbert space. We will give a survey on the method with emphasis on operator theoretical aspects.
A first part concerns recent results on sectorial forms obtained in collaboration with Tom ter Elst. To each sectorial form a holomorphic semigroup can be associated in a natural way. The notion of closability is shown to be superfluous. A variety of examples are obtained by forms. We will give a characterization of those operators which are associated with a form in terms of their H^∞-infinity functional calculus. Perturbation will be discussed and forms having a numerical range in a parabola are considered. This leads us to second order problems and cosine functions.

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Bounded quasi-selfadjoint operators, their Weyl functions, and special block operator Jacobi matrices

Yu. Arlinskiĭ

A bounded operator T in a separable Hilbert space $\mathfrak{H}$ is called quasi-selfadjoint if $\ker(T - T^*) \neq \{0\}$ and $\mathfrak{N}$-quasi-selfadjoint if $\mathfrak{N} \supseteq \operatorname{ran}(T - T^*)$, where $\mathfrak{N}$ is a subspace of $\mathfrak{H}$. An $\mathfrak{N}$-quasi-selfadjoint operator T is called $\mathfrak{N}$-simple if the linear hull of $\{T^n\mathfrak{N},\ n = 0, 1, \ldots\}$ is dense in $\mathfrak{H}$. We study the $\mathfrak{N}$-Weyl function $M(z) = P_{\mathfrak{N}}(T - zI_{\mathfrak{H}})^{-1}\upharpoonright\mathfrak{N}$ of an $\mathfrak{N}$-quasi-selfadjoint operator and define Schur transformation and Schur parameters of $M(z)$. The main result is that any $\mathfrak{N}$-quasi-selfadjoint and $\mathfrak{N}$-simple operator is unitarily equivalent to an operator given by a special block operator Jacobi matrix constructed by means of the Schur parameters of its $\mathfrak{N}$-Weyl function.

The talk is based on a joint work with L. Klotz.

Localization of the spectrum of a quadric pencil with a strong damping operator

N. Artamonov

In Hilbert space H we consider a quadric pencil

$$L(\lambda) = \lambda^2 I + \lambda D + A$$

with self-adjoint positive definite operator A. By H_s denote a collection of Hilbert spaces generated by operator $A^{1/2}$, $\|\cdot\|_s$ is a norm on H_s. We will assume that D is a bounded operator acting from H_1 to H_{-1} and

$$\inf_{x\in H_1, x\neq 0} \frac{\mathrm{Re}(Dx,x)_{-1,1}}{\|x\|_1^2} = \delta > 0.$$

(here $(\cdot,\cdot)_{-1,1}$ is a duality pairing on $H_{-1}\times H_1$). We obtain a localization of the spectrum of $L(\lambda)$:

$$\sigma(L) \subset \{\lambda \in \mathbb{C} \mid \mathrm{Re}\lambda \leq -\omega, |\mathrm{Im}\lambda| \leq \kappa|\mathrm{Re}\lambda|\}$$

for some positive $\omega, \kappa > 0$.

The work is supported by the Russian Fund for Basic Research (grant No. 08–01–00595).

Resolvent kernel for the Kohn Laplacian on the Heisenberg group by means of a Schrödinger operator with magnetic field

N. Askour

In this communication, we give some spectral tools of the Kohn Laplacian on the Heisenberg group. We have found its heat kernel, resolvent kernel, wave kernel and the associated Poisson semi-group kernel explicitly trough its connection with a Schrodinger operator with magnetic laplacian. In more, for the isotropic case, we have found the spectral density. Also the Green kernel associated to the fractional powers of the Kohn laplacian was computed.

Completely positive definite functions on a commutative semigroup with involution

D. Atanasiu

In this talk we give a Stinespring type characterization of a class of completely positive definite functions defined on a commutative semigroup with involution and without unity. This characterization is a extension of a result on the completely positive definite functions defined on a commutative semigroup with involution and unity proved by Berg and Maserick. Our proof is independent of Naimark's theorem. We also use this characterization to prove a dilation theorem.

On negative eigenvalues of a linear pencil

T.Ya. Azizov

Let A and K be selfadjoint operators with $N(A) < \infty$ and $N(K) < \infty$, counting multiplicity, negative eigenvalues, respectively. Let $N(L)$ denote the number of negative eigenvalues of the linear pencil $L(\lambda) = A - \lambda K$. If A is a boundedly invertible operator and K is a bounded operator with a trivial kern then

$$|N(A) - N(K)| \leq N(L) \leq N(A) + N(K).$$

The main result of this talk is:

(a) if there exists a $\gamma > 0$ such that $A \geq \gamma K$ or $K \geq \gamma A$ then

$$N(L) = |N(A) - N(K)|;$$

(b) if there exists a $\gamma < 0$ such that $A \geq \gamma K$ then $N(L) = N(A) + N(K)$.

The talk is based on the joint works with M.V. Chugunova.

The work is supported by the Russian Foundation for Basic Researches, grant 08-01-00566-a.

Canonical model theory for Hilbert space row contractions

J.A. Ball

Given any completely nonunitary Hilbert-space contraction operator T, the Sz.-Nagy model theory associates a contractive operator-valued function holomorphic on the unit disk (the characteristic function $\Theta_T(\lambda)$

of T) which has the additional property of being *pure* (there is no nonzero vector e such that $\|\Theta_T(0)e\| = \|e\|$). Conversely, any pure Schur-class function arises as the characteristic function of a completely nonunitary contraction operator. Moreover, the correspondence between completely nonunitary contraction operators T and pure Schur-class operator functions on the unit disk Θ is bijective, as long as one considers contraction operators up to unitary equivalence and Schur-class functions up to an equivalence relation known as *coincidence* (independent unitary change of basis on the input space and the output space). There have now appeared impressive extensions of this theory to multivariable settings, specifically, by Popescu to freely noncommutative row contractions, by Bhattacharyya, Eschmeier, Sarkar to commuting row contractions, and by Popescu and others to more general operator varieties. However, this model theory falls short of the completely nonunitary case and handles only the so-called completely noncoisometric case in the Sz.-Nagy-Foias theory. This talk reports on extensions of this multivariable model theory to cases beyond the completely noncoisometric case. Part of the ingredients is the the study of transfer-function realization for Schur-class functions on the noncommutative ball (contractive multianalytic functions in the sense of Popescu) and on the commutative ball (contractive multipliers of the Drury-Arveson space).

This talk is based on joint work with V. Vinnikov and V. Bolotnikov.

Truncated Toeplitz operators: existence of bounded symbols

A. Baranov

Truncated Toeplitz operators are compressions of usual Toeplitz operators to star-invariant (model) subspaces of H^2 in the disc: if Θ is an inner function and $K_\Theta = H^2 \ominus \Theta H^2$, then, for a function ϕ in L^2 on the circle, the truncated Toeplitz operator A_ϕ is defined by the formula $A_\phi f = P_\Theta(\phi f)$ for functions f in K_Θ such that ϕf is square integrable. Here P_Θ is the projection onto K_Θ.

A systematic study of truncated Toeplitz operators was started recently by D. Sarason. In contrast to the classical Toeplitz operators, a truncated Toeplitz operator may be sometimes extended to a bounded operator on K_Θ even for an unbounded symbol ϕ. The question, posed by Sarason, is whether boundedness of the operator implies the existence of a bounded symbol. We show that in general the answer to this question is negative. Moreover, we give a description of those inner functions Θ for which the answer is positive. In particular, we show that bounded symbols always exist in the case when Θ is a so-called one-component inner function, that is, the sublevel set $\{z : |\Theta(z)| < \varepsilon\}$ is connected for some $\varepsilon \in (0, 1)$.

The talk is based on joint works with I. Chalendar, E. Fricain, J. Mashreghi and D. Timotin, and with R. Bessonov and V. Kapustin.

Spectral regularity of Banach algebras and non-commutative Gelfand theory

H. Bart

Let $\mathcal{B}$ be a Banach algebra with unit element. If D is a bounded Cauchy domain in the complex plane and f is an analytic $\mathcal{B}$-valued function taking invertible values on the boundary ∂D of D, the contour integral

$$\frac{1}{2\pi i}\int_{\partial D} f'(\lambda)f(\lambda)^{-1}d\lambda \tag{1}$$

is well-defined. By Cauchy's theorem, it is equal to the zero element in $\mathcal{B}$ when f has invertible values on all of D. The Banach algebra $\mathcal{B}$ is said to be *spectrally regular* if the converse of this is true. This means that (1) can only vanish in the trivial case where f takes invertible values on all of D. Matrix algebras are always spectrally regular, Banach algebras of bounded linear operators on an infinite dimensional Banach space generally not. The talk focusses on criteria for Banach algebras to be spectrally regular. These involve new aspects of non-commutative Gelfand theory using families of matrix homomorphisms.

The work reported on has been done joint with T. Ehrhardt and B. Silbermann.

The state space method in analysis and Schur complements

H. Bart

In past years, notions from system theory have been used to analyze such issues in analysis as linearization and inversion of analytic operator functions, inverse Fourier transforms, the Riemann-Hilbert boundary value problem, minimal factorization of rational matrix functions, Wiener-Hopf factorization, Wiener-Hopf and Toeplitz equations, transport theory, model reduction, Szegö limits etc. A central feature in all of this is the use of *realizations*, that is of expressions of the type

$$D + C(\lambda I - A)^{-1}B \tag{1}$$

where λ is a complex variable, A, B, C, D are operators (sometimes represented by matrices) and I is an appropriate identity operator. The basis for the successful application of realizations lies in the fact that (1) turns out to be ideally suited for manipulating rational matrix functions and, more generally, analytic operator functions. The main results that demonstrate this will be reviewed briefly. Further it will be pointed out that these results can be seen as λ-versions of certain observations on Schur complements involving equivalence, extension, multiplication, inversion, and factorization. Together these observations form an algebraic framework which is of independent interest and gives rise to an intriguing open problem on Schur complements.

The talk is based on joint work done with I. Gohberg, M.A. Kaashoek and A.C.M. Ran.

Representations of non local C^* algebras of singular operators and C^* crossed products

M.A. Bastos

Representations are constructed for classes of non local C^* algebras of singular integral operators with piecewise slowly oscillating coefficients when the associated group is an amenable discrete group of homeomorphisms with a set of fixed or periodic points. The non local algebras are related with C^* crossed products and the construction of their representations with C^* crossed products representations.

Operator splitting for evolution equations

A. Bátkai

Operator splitting is a widely used method of numerical analysis based on the Lie-Trotter product formula

$$\left(e^{\frac{t}{n}A}e^{\frac{t}{n}B}\right)^n \to e^{t(A+B)}. \tag{1}$$

The idea is to decompose complicated operators into simpler ones and then use this to approximate the solutions of the corresponding Cauchy problem. In the talk we present a general abstract framework and present a list of examples (e.g., nonautonomous equations, abstract boundary value problems, inhomogeneous problems, semigroups associated to admissible control and observation systems, etc.) where the abstract theory can be applied. Joint work with P. Csomós, K.-J. Engel, B. Farkas and G. Nickel. Supported by the Alexander von Humboldt-Stiftung.

Quasi-hyperbolic operators and semigroups

C. Batty

This talk will describe a class of operators which are necessarily hyperbolic but behave similarly. For C_0-semigroups, the failure of spectral mapping theorems prevents a simple characterisation of quasi-hyperbolicity in terms of the generator, so we discuss properties of semigroups which can be deduced from the appropriate conditions on the generator.

The talk is based on a joint work with Y. Tomilov.

Remarks on commuting Toeplitz operators

W. Bauer

Consider two Toeplitz operators T_g, T_f on the Segal-Bargmann space over the complex plane.Let us assume that g is a radial function and both operators commute. Under certain growth condition at infinity of f and g we show that f must be radial, as well. In case of Toeplitz operators with bounded symbols acting on the Bergman space over the unit disc the corresponding result has been proven earlier by Ž. Čučković and N.V. Rao. In the Segal-Bargmann space setting we give a counterexample of this fact by choosing an appropriate fast growing radial symbol g. In this case the vanishing commutator $[T_g, T_f] = 0$ does not imply the radial dependence of f. Finally, we replace the complex plane by $\mathbb{C}^n$, $n > 1$ and give some remarks on the commuting property of Toeplitz operators. This talk is based on a joint work with Y.J. Lee.

Nonautonomous flows in networks

F. Bayazit

Motivated by an example in Air Traffic Flow Management we study nonautonomous flows in networks in the spirit of M. Kramar and E. Sikolya (Math. Z. 249, 139162, [2005]). We show wellposedness of the corresponding nonautonomous Abstract Cauchy Problem and discuss asymptotic behavior in the time periodic case.

Realizations of extremal classes of Stieltjes and inverse Stieltjes functions

S. Belyi

We consider extremal classes of Stieltjes and inverse Stieltjes functions. It is shown that each function from the above mentioned classes can be realized as the impedance function of a singular L-system. Moreover, we establish the connection between the above mentioned classes and the Friedrichs and Krein-von Neumann boundary value problems. Applications to the inverse problems of singular L-systems with accretive Schrödinger operator are presented.

The talk is based on a joint work with E. Tsekanovskiĭ.

Spectral and oscillation properties for fourth-order boundary value problems $Ny = \lambda Py$

J. Ben Amara

Many equations arising in elasticity theory and hydrodynamics lead to eigenvalue problems of the form

$$\tilde{P}y = \lambda \tilde{Q}y.$$

Here $\tilde{P}$ and $\tilde{Q}$ are linear operators generated by the differential expressions

$$P = (p(x)y'')'' \quad and \quad Q = -y'' + cr(x)y,$$

respectively, where $p > 0$, $r > 0$ are continuous functions on the interval $[0, 1]$, and $c \in \mathbf{R}$ (e.g., see [1, 2, 3]). In the case of self-adjoint boundary conditions, we show that the spectrum of these problems is real, it consists of a finite number of negative simple eigenvalues and a sequence of positive semi-simple eigenvalues tending to $+\infty$:

$$\mu_{-p} < \cdots < \mu_{-1} < 0 < \mu_1 < \mu_2 \leq \cdots \leq \mu_n \to +\infty.$$

The corresponding eigenfunction y_{-n}, $1 \leq n \leq p$, has $n - 1$ zeros in $(0, 1)$, and y_1 has no zeros in $(0, 1)$. In the case when the boundary conditions are linearly depend of λ, it is shown that the negative eigenvalues of this problem (which are simple) interlace with those of the problem with self-adjoint boundary conditions. Furthermore, The corresponding eigenfunctions y_{-n} have similar oscillation properties.

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Extended Weyl type theorems

M. Berkani

In this talk we will consider the new properties (b), (gb), (ab), (gab), (aw) and (gaw) introduced recently and jointly by the author and H. Zariouh, as a new extended Weyl type theorems. We will characterize these properties and we will study their preservation under finite rank or nilpotent perturbations. As an example of the results obtained, we show that if T is a bounded linear operator acting on a Banach space X, then T possesses property (gb) if and only if T possesses property (gab) and $\mathrm{ind}(T - \lambda I) = 0$ for all $\lambda \in \sigma_a(T) \setminus \sigma_{SBF_+^-}(T)$; where $\sigma_{SBF_+^-}(T)$ is the essential semi-B-Fredholm spectrum of T and $\sigma_a(T)$ is the approximate spectrum of T.

A new inverse scattering transform

S. Bernstein

A.S. Fokas introduced a new transform method for solving initial boundary value problems for linear and for integrable nonlinear PDEs in two independent variables. This method is based on the fact that linear and integrable nonlinear equations possess a Lax pair formulation. The implementation of this method involves performing a simultaneous spectral analysis for both parts of the Lax pair and solving a Riemann-Hilbert problem (see for example [1]).

Un this talk we will use Clifford analysis and certain Dirac-type operators to construct Lax pairs. We also highlight the connection to Riemann-Hilbert-type problem in Clifford analysis. In this way we can generalize the Lax pair method to linear PDE with more than two variables.

The talk is based on joint work with U. Kähler and P. Cerejeiras.

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Coincidence of Schur Multipliers of the Drury-Arveson space

A. Bhattacharya

In a purely multi-variable setting we show that the coincidence of two operator valued Schur class multipliers, admitting functional model realizations, on the Drury-Arveson space is characterized by the fact that the associated colligations (or a variant, obtained canonically) are 'unitarily coincident' in a sense to be made precise in the talk.

The talk is based on a joint work with T. Bhattacharyya.

Completely positive kernels, Hilbert C^*-modules and coherent states

T. Bhattacharyya

The concept of coherent states has been important in physics for a long time. The coherent states are intimately related to positive definite kernels. The theory of positive definite kernels has been generalized to take into account kernels which takes values in the space of bounded liner operators between C^*-algebras. In view of this, we propose Hilbert C^*-module valued coherent states. We relate it to physical examples.

This talk is based on joint work with S. T. Ali and S. S. Roy.

Some Sturm-Liouville type results for the p-Laplacian

P. Binding

A review will be given of some recent work on nonlinear (but homogeneous) generalisations of the Sturm-Liouville problem involving the p-Laplacian.

Asymptotic behavior of Bergman kernels with logarithmic weight

P. Blaschke

A description of the boundary behavior of Bergman kernels on strongly pseudoconvex domains is vitally important for many applications. Unfortunately, results for kernels with respect to non-trivial weights are still unknown. There exist only partial results for weights of the form $w = d^\alpha e^g$, where d denotes the distance from the boundary of the domain, $\alpha > -1$, and g is a function smooth on the closure; and, more generally, also for weights of the same form, but with “logarithmic” singularities allowed in terms of higher order. We present a result for weights that have logarithmic singularity in the main term, in the simplest case of a radially symmetric weight on the unit disc.

Quantum graphs with singular two-particle interactions

J. Bolte

Single quantum particles on graphs have proven to provide interesting models in quantum chaos, and their spectral properties have been studied in great detail. We now consider many-particle models on graphs with singular two-particle interactions that are either localised in the vertices of the graph or along the edges. These delta-type interactions are realised in terms of suitable self adjoint extensions of two-particle Laplacians. The talk is based on joint work with J. Kerner.

A behavioral interpretation of Livšic systems

G. Boquet

In the behavioral approach to (discrete-time) multidimensional linear systems, one views solution trajectories simply as the set of all solutions of a homogeneous linear system of difference equations. The duality shared by the ring of operators (polynomials with constant coefficients) and the signal space (where indeterminates act via the backward shift in their respective directions) allows one to related linear systems to commutative algebra. In this setting the transfer matrix is identified as the unique rational matrix function H satisfying $QH = P$ where $R = \begin{bmatrix} -Q & P \end{bmatrix}$ is a partitioning of the kernel representation R (i.e., the behavior is given as the kernel of the polynomial matrix R) for the behavior such that P has full column rank equal to the rank of R. This transfer matrix can be seen as a more fundamental and unifying formalism capturing the transfer functions associated with the older Givone-Roesser and Fornasini-Marchesini input/state/output approaches to multidimensional linear systems.

A quite different type of input/state/output linear system having original motivation from operator theory is the Livšic linear system, where the state-evolution equations, arising from a commuting pair of operators, are overdetermined and lead to compatibility constraints on both the input and output signals; in this case the admissible input signals are not free but form their own behavior. The main point of the present work is to identify how Livšic systems fit into the behavioral framework. In particular, we extend the transfer matrix to the setting of autonomous behaviors lacking any free variables (in which case the standard transfer matrix is trivial with no columns) by letting the reduced ring (the quotient of the polynomial ring by the annihilator of the behavior) act on the behavior. We then make explicit identifications between the transfer matrix over the reduced ring and the Livšic Joint Characteristic Function. This connection not only allows one to consider Livšic systems in a behavioral setting, but brings to light the different input/output structure that Livšic systems exhibit in comparison to i/s/o systems with free (over the reduced ring) signals.

On the approximate K-spherical functions on the Heisenberg group

B. Bouikhalene

Let G be a locally compact group, K a compact subgroup of $Aut(G)$ and dk the normalized Haar measure on K. A function $f \in \mathcal{C}(G)$ is a K-spherical function ([1]) if it satisfies the functional equation

$$\int_K f(xk \cdot y)dk = f(x)f(y), \quad x, y \in G \tag{1}$$

where $k \cdot y$ denotes the action of $k \in Aut(G)$. On the Heisenberg group $H = \mathbb{R}^2 \times \mathbb{R}$, equipped with the composition rule

$$(x, y, t)(x', y', t') = (x + x', y + y', t + t' + \frac{1}{2}(xy' - yx'),$$

if K is a two-element subgroup of $Aut(H)$, consisting of the automorphism i given by $i(x, y, t) = (y, x, -t)$ and the identity mapping, the equation (1) becomes the Stetkær functional equation ([2])

$$f(x + x', y + y', t + t' + \frac{xy' - yx'}{2}) + f(x + y', y + x't - t' + \frac{xx' - yy'}{2}) \\ = f(x, y, t)f(x', y', t'), \quad (2)$$

where $f : H \longrightarrow \mathbb{C}$ is a complex-valued function. The continuous solutions of (2) are given by Stetkær in [2]. In [3], Sinopoulos determined measurable, with respect to the first variable, solutions of this equation. In this work, we give the approximate general solutions of equation (2).

This is a joint work with E. Elqorachi.

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On the operator-valued K-spherical functions

B. Bouikhalene

Let G be a locally compact group, K be a compact subgroup of $Aut(G)$ and dk the normalized Haar measure on K.A continuous function $f \in$

$\mathcal{C}(G)$ is said to be K-positive definite, if for any $n \in \mathbf{N}$, $x_1, ..., x_n \in G$ and arbitrary complex numbers $c_1, ..., c_n$, we have

$$\sum_{i=1}^{n} \sum_{j=1}^{n} c_i \overline{c_j} \int_K f(x_j^{-1} k \cdot x_i) dk \geq 0. \tag{1}$$

where $k \cdot y$ denotes the action of $k \in Aut(G)$. By a K-representation of G, on a Hilbert space $\mathcal{H}$, we mean a weakly continuous mapping $\pi : G \longrightarrow B(\mathcal{H})$ (the algebra of all bounded operators on $\mathcal{H}$) such that

i) $\int_K \pi(xk \cdot y) dk = \pi(x)\pi(y),\ x, y \in G$,

ii) $\pi(e) = I$ (I is the identity operator),

iii) $\pi(x)^* = \pi(x^{-1})$, $x \in G$, ($\pi(x)^*$ is the adjoint operator of $\pi(x)$).

In this work, we deal with the relationship between K-positive definite functions and K-representation of G, this extends classical harmonic analysis and theory of representations. Related results are given by Chojnaki in [5] and Stetkær in [6]. In [4] M. Ait Si Baha studied μ-representations and μ-positive definite functions, where μ is a Gelfand measure ([1],[2],[3]).

This is a joint work with M.A. Sibaha and A. Bakali.

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Eigenvalue asymptotics of a family of Sturm-Liouville operators with boundary and interior singularities

L. Boulton

In this talk we will examine spectral asymptotics of a family of periodic singular Sturm-Liouville problems which are highly non-self-adjoint but have only real eigenvalues. The problem originated from the study of the lubrication approximation of a viscous fluid film in the inner surface of a rotating cylinder and has received a substantial amount of attention in recent years. Our main focus of interest will be the determination of sharp Schatten class inclusions for the associated resolvent operator.

The talk is based on a joint work with M. Levitin, M. Marletta and D. Rule.

On the reflexivity of the kernel of an elementary operator

J. Bračič

The notion of a reflexive linear space of operators is closely related with the invariant subspace problem for complex Banach spaces. There are several generalizations of this notion. One of them is k-reflexivity, where k is an arbitrary positive integer. One can show that a linear space of operators is k-reflexive if and only if it is an intersection of kernels of a set of elementary operators of length at most k. Thus, it is natural to ask when is the kernel of a given elementary operator k-reflexive. We will present some results related to this question.

Fischer decompositions in Hermitean Clifford analysis

F. Brackx

A result of E. Fischer states that, given a homogeneous polynomial $q(x)$, $x \in \mathbf{R}^m$, every homogeneous polynomial $P_k(x)$ of degree k can be uniquely decomposed as $P_k(x) = Q_k(x) + q(x)R(x)$, where $Q_k(x)$ is homogeneous of degree k, satisfying $q(D)Q_k = 0$, D being the differential operator corresponding to x through Fourier identification, and $R(x)$ is a homogeneous polynomial of suitable degree. If in particular $q(x) = ||x||^2$, then $q(D)$ is the Laplacian and Q_k is harmonic, leading to the well–known decomposition of the spaces of complex valued homogeneous polynomials into spaces of complex valued harmonic homogeneous polynomials.

Clifford analysis is a higher dimensional function theory, constituting a refinement of harmonic analysis. It studies monogenic functions, i.e. null solutions of the rotation invariant, vector valued, first order Dirac operator $\underline{\partial}$, which factorizes the Laplacian. Fischer decompositions of the spaces of Clifford algebra valued homogeneous polynomials into spaces of monogenic homogeneous polynomials were obtained as a result. In the more recent branch Hermitean Clifford analysis, the rotational invariance has been broken by introducing a complex structure J on Euclidean space and a corresponding second Dirac operator $\underline{\partial}_J$, leading to the system of equations $\underline{\partial} f = 0 = \underline{\partial}_J f$ expressing so-called Hermitean monogenicity. The invariance of this system is reduced to the unitary group. In this talk the spaces of homogeneous monogenic polynomials are further decomposed into $\mathrm{U}(n)$-irrucibles involving ho mogeneous Hermitean monogenic polynomials.

This is joint work with H. De Schepper and V. Souček.

Spectral analysis of an one-term irregular symmetric differential operator

I. Braeutigam

We consider a minimal closed symmetric operator L_0^{pq} which is induced by an irregular ordinary differential expression $l_{2m}, (m = 1, 2, \ldots)$ in $\mathcal{L}_2(I)$. We shall call the differential expression $l_{2m}[y]$ an *irregular differential expression* (see [1, ch.XIII]) if its leading coefficient vanishes at some points in I.
Let

$$l_{2m}[y](x) = (-1)^m (c(x)y^{(m)})^{(m)}(x), \quad x \in I := [-1; 1].$$

We suppose that the coefficient $c(x)$ is defined on I and has on this set only one zero $x_0 = 0$ *right-sided* order p (see [2]) and *left-sided* order q,

$p, q \in \{1, 2, \ldots, 2m-1\}$, that is

$$c(x) = \begin{cases} x^p a(x), & \text{if } x \in [0;1]; \\ (-x)^q b(x), & \text{if } x \in [-1;0], \end{cases}$$

and functions $a(x), b(x)$ have only real values as $x \in I$ and can be represent as convergent series when $|z| < 1$

$$a(z) := a_0 + \sum_{j=1}^{+\infty} a_j z^j, \quad a_0 \neq 0, \quad b(z) := b_0 + \sum_{j=1}^{+\infty} b_j z^j, \quad b_0 \neq 0.$$

The deficient numbers of the operator L_0^{pq} in the upper and lower open complex planes are equal and we denote their common value as n_{pq}.

The following result holds.

Theorem 1. *The deficient numbers of the operator L_0^{pq} are defined by formula:*

$$n_{pq} = \begin{cases} 4m - \max\{p, q\}, & \textit{if } p, q \in \{m+1, m+2, \ldots, 2m-1\}; \\ 2m + \min\{p, q\}, & \textit{if } p, q \in \{1, 2, \ldots, m\}; \\ 3m + p - q, & \textit{if } p \in \{1, 2, \ldots, m\}, \\ & \quad q \in \{m+1, m+2, \ldots, 2m-1\}. \end{cases}$$

Let's notice that in this case the deficient numbers of the operator are greater-than an order of the generative differential expression.

Theorem 2. *Spectrum of each self – adjoint expansion of the operator L_0^{pq} is discrete.*

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Schatten-von Neumann norm estimates for resolvent differences of Dirichlet operators

J. Brasche

We discuss perturbations of regular Dirichlet operators by killing terms. Estimates for the Schatten von Neumann-norms of the difference between the resolvents of the unperturbed and perturbed operators are derived and used for the investigation of the spectra of the perturbed operators. The talk is based on joint work with H. BelHadjAli and A. Ben Amor.

Lie ideals in algebra and analysis

M. Bresar

A linear subspace L of an algebra A is called a Lie ideal of A if $al - la \in L$ for all $l \in L$, $a \in A$. Lie ideals naturally appear in various topics in algebra and analysis, and we will briefly review some of them. At the end we will indicate their meaning in the problem of describing values of a noncommutative polynomial on an algebra. The latter is based on a joint work with I. Klep.

The C^*-algebra of Toeplitz operators associated with a polyhedral cone

N. Buyukliev

We consider the C^*-algebra $\mathcal{T}$ generated by the multivariable Toeplitz operators associated with a polyhedral cone. We present $\mathcal{T}$ as a groupoid C^*-algebra: $\mathcal{T} \cong C^*(\mathcal{G})$. The groupoid $\mathcal{G}$ is a reduction of a transformation group $\mathcal{G} = (Y \times Z^n)|X$, where Y and X are suitable topological spaces. This enables us to describe the ideal structure of $\mathcal{T}$.

We use cyclic cohomology to give a formula for computing the index of Fredholm operators in $\mathcal{T}$.

Fredholm properties of Toeplitz operators and meromorphic corona problems

M.C. Câmara

A meromorphic analogue to the corona problem is formulated and studied and its solutions are characterized as having a left inverse in a space of meromorphic functions. It is used to study the Fredholm properties of Toeplitz operators with 2×2 symbol G, assuming that an associated Riemann-Hilbert problem admits a left invertible meromorphic solution. These results are applied to obtain Fredholmness and invertibility conditions for a class of Toeplitz operators, as well as partial index estimates for the corresponding matrix symbols.

The talk is based on a joint work with C. Diogo and L. Rodman.

On truncated Wiener-Hopf operators and $BMO(\mathbb{Z})$

M. Carlsson

Let $\Phi : (0,2) \to \mathbb{C}$ be a function and consider the operator $\Gamma_\Phi : L^2((0,1)) \to L^2((0,1))$ given by

$$\Gamma_\Phi(F)(x) = \int_0^1 \Phi(x+y)F(y)dy, \quad 0 < x < 1.$$

Surprisingly, there seems to be very few studies on it. It goes under names like "Truncated Wiener-Hopf operator" "Toeplitz operator on the Paley Wiener space" or "Truncated Hankel operator on $\mathbb{R}$". We consider norm estimates and show that it shares many properties with Hankel operators, in particular we provide a Nehari type norm characterization, as well as norm and compactness characterization in terms of the sequential BMO-space. Time allowing, we will discuss a number of conjectures and open problems relating to non-harmonic Fourier analysis and approximation of functions by sums of exponential functions.

An index formula for classes of pseudodifferential operators on non-compact spaces

C. Carvalho

The Atiyah-Singer index formula gives a topological formulation for the Fredholm index of elliptic pseudodifferential operators on compact spaces, generalizing the fundamental Gohberg-Krein index theorem for operators on the circle. On non-compact spaces, many difficulties arise: for a start, ellipticity does not suffice, in general, to ensure Fredholmness. In this talk, we consider a class of pseudodifferential operators on $\mathbb{R}^n$, or more generally, on a Lie manifold, that extend at infinity to a suitable compactification, in that they are generated by a given class of vector fields that are tangent to the boundary of the compactification. As particular cases, one obtains several well-known pseudodifferential calculi on $\mathbb{R}^n$. In case the vector fields actually vanish at infinity, we give Fredholm conditions depending on ellipticity and invertibility of a complete symbol on spheres at the boundary. We then extend Atiyah-Singer's index formula to this class; to this effect, we use homotopy to asymptotically constant symbols. This result can be applied to perturbed Dirac operators $\not{D} + V$, where V is an unbounded potential.

The talk is based on joint work with V. Nistor.

Electromagnetic scattering by cylindrical orthotropic waveguide irises

L. Castro

We will present a mathematical analysis of scattered time-harmonic electromagnetic waves by an infinitely long cylindrical orthotropic waveguide iris. This is modeled by an orthotropic Maxwell system in a cylindrical waveguide iris for plane waves propagating in the x_3-direction, imbedded in an isotropic infinite medium. The problem is equivalently reduced to 2-dimensional boundary-contact problem with the operator $\operatorname{div} M \operatorname{grad} + k^2$ inside the domain and the (Helmholtz) operator $\Delta + k^2 = \operatorname{div}\operatorname{grad} + k^2$ outside the domain. Here M is a 2×2 positive definite, symmetric, matrix with constant, real valued entries. The unique solvability of the appropriate boundary value problems is proved and regularity of solutions is established in Bessel potential spaces.

The talk is based on a joint work with R. Duduchava and D. Kapanadze.

Phase space analysis in Dunkl setting

P. Cerejeiras

Phase space describes simultaneously the *momentum* p of a given particle and its position q. Moreover, canonical transformations of the phase-space give the time-evolution of the related system, together with its symmetries, or invariants, of the sympletic form $[p, q]$. However, if in classical mechanics the observable is fully determined once the value of the state of the system is known the same is no longer true in quantum mechanics, the observables being probability distributions and, hence, one requires $\mathbb{R}^{2n+1}$ as phase-space.

In this setting, the basic operators are given as multiplication with the coordinate and by partial derivatives. In recent years, it appeared in the study of quantum n-body systems a new type of operators, the so-called *Dunkl operators*. In this talk we will show that one can replace the classic partial derivatives by the Dunkl operators and obtain a theory analogous to the classic one. Moreover, we will show that the usage of Weyl relations allows to obtain a higher dimension function theory invariant under reflection groups.

The trace formula for a non-selfadjoint Friedrichs' model

E. Cheremnikh

There are different directions known as "trace formula". Some idea apropos of trace formula, its applications and references one can find in [1–3]. The trace formula for selfadjoint Friedrichs' model and spectral identities was considered in [4] under sufficiently general suppositions.

We consider non-selfadjoint model in some particular case, our aim is the study of the role of spectral singularities. We work in the space $H =$

$L^2_\rho(0,\infty)$, the model is $T = S + A^*B$ where $(S\varphi)(\tau) \equiv \tau\varphi(\tau)$, $\tau > 0$ and $A, B : H \to G$ — integral operators which act from H in auxiliary Hilbert space G. The conditions on the perturbation A^*B permit to consider Sturm-Liouville operator with exponentially decreasing potential, but we suppose that the set of spectral singularities is finite.

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Berezin transform and compactness of Toeplitz operators on the harmonic Bergman space

B.R. Choe

For an operator which is a finite sum of products of finitely many Toeplitz operators on the harmonic Bergman space over the half-space, we study the problem: *Does the boundary vanishing property of the Berezin transform imply compactness?* This is motivated by the Axler-Zheng theorem for analytic Bergman spaces, but the answer would not be yes in general, because the Berezin transform annihilates the commutator of any pair of Toeplitz operators. Nevertheless we show that the answer is yes for certain subclasses of operators. We first find a sufficient condition on general operators and use it to reduce the problem to whether the Berezin transform is one to one on the subclass of operators under consideration.

Then we obtain positive results for three subclasses. We also announce some open problems naturally raised by this work.

This talk is based on a joint work with K. Nam.

Determinacy of the Hamburger moment problem of a determinate Stieltjes moment problem in the matrix case

A.E. Choque Rivero

A moment problem is said to be determinate if it has a unique solution. Otherwise the moment problem is called indeterminate.

We generalize to the matrix case the classical theorem of Hamburger which enables to determinate whether a determinate Stieltjes moment problem is determinate or indeterminate Hamburger moment problem with the same moments.

The talk is based on a joint work with Yu. Dyukarev.

Non-commutative real algebraic geometry for a unital associative $*$-algebra

J. Cimprič

We study non-commutative real algebraic geometry for a unital associative $*$-algebra $\mathcal{A}$ viewing the points as pairs (π, v) where π is $*$-representation of $\mathcal{A}$ on a pre-Hilbert space which contains the vector v. These general results are illustrated in the cases where $\mathcal{A}$ is one of the following $*$-algebras: $M_n(\mathbb{R})$, $M_n(\mathbb{R}[X_1, \ldots, X_d])$ and the free $*$-algebra in d variables.

For comparison, non-commutative real algebraic geometry where the points are $*$-representation of $\mathcal{A}$ - an approach of K. Schmüdgen - is well developed.

Generic reflexivity for spaces of matrices

N. Cohen

The properties of algebraic reflexivity and local linear dependence, for a vector space of linear operators, are weak analogues of span and linear dependence, respectively. Both have established connection with the structure of invariant subspaces, Banach algebra derivations, and vector spaces of singular matrices.

Recently some authors (in particular Meshulam-Šemrl and Bračič-Kuzma) have obtained new bounds on rank and structure of low-dimensional spaces of linear operators with these properties.

In spite of these efforts, the exact relationship between the two properties remains unclear. Are they really closely related?

In the talk I introduce a new type of reflexivity property, called generic reflexivity, which presents a much closer affinity with local linear dependence. For example, a space W of matrices is locally linearly dependent iff it is contained in the generic reflexive hull of a strict subspace $W_1 \subset W$. This condition becomes only sufficient if the word "generic" is replaced by "algebraic".

In the case of spaces of matrices, I will survey and partially improve some of the recent results on algebraic reflexivity and local linar dependence, and describe their generic analogues. The Kronecker-Weierstrass canonical form was central in obtaining these results.

Recent notions of spectra for n-tuples of operators in Clifford Analysis

F. Colombo

Slice monogenic functions have a Cauchy integral formula whose kernel can be expressed in two different ways. One of the two expressions of the kernel allows to construct a functional for n-tuples of non necessarily commuting operators. This calculus, called S-functional calculus, is based on the notion of S-spectrum. In the case of commuting operators, it is possible to use the second expression of the kernel and to construct a functional calculus based on the notion of F-spectrum. The advantage of the F-spectrum is that it is easier to compute than the S-spectrum. The two notions of spectrum coincide in the case of commuting operators.

The talk is based on a joint work with I. Sabadini.

Explicit Factorization of Matrix Functions with Mathematica 6.0

A. Conceição

The aim of this talk is to present our progress in the explicit factorization of some special matrix-valued functions. We construct an algorithm to obtain the solutions of certain singular integral equations related with a self-adjoint operator. Using these solutions we construct another algorithm to determine an effective factorization of the matrix functions. Using Mathematica 6.0 symbolic computation package we implement the two algorithms on a digital computer, automating the factorization process as a whole. We present some examples, both in the real line and in the unit circle, obtained with the Mathematica application.

The talk is based on a joint work with J. Pereira and V. Kravchenko.

Chaotic shift operators coming from polynomials

J.A. Conejero

An operator $T \in L(X)$ is said to be hypercyclic if there exists some $x \in X$ such that $\{T^n x : n \in \mathbb{N}_0\}$ is dense in X. The first example of a hypercyclic operator on a Banach space was given by Rolewicz in 1969 [5]. He proved that the multiples λB of the backward shift operator $B(x_1, x_2, \dots) := (x_2, x_3, \dots)$ are hypercyclic on the space l_1 of absolutely summable sequences if and only if $|\lambda| > 1$. In fact, these operators are chaotic in the sense of Devaney, i.e. they are hypercyclic and have a dense set of periodic points.

Ansari proved that every power of a hypercyclic operator is also hypercyclic [1], therefore λB^n, where B^n is the n-iteration of B and $|\lambda| > 1$, it is also hypercyclic. On the other hand, Salas showed that $I + B$ is also hypercyclic [6]. The hypercyclic and chaotic behaviour of weighted backward shifts has been considered in [2, 3, 4].

Given a polynomial $p(z) = \sum_{k=0}^{n} a_k z^k$, we can consider the operator $p(B) = \sum_{k=0}^{n} a_k B^k$. We present some sufficient conditions for the hypercyclicity and chaos of these operators in terms of the coefficients of the polynomial $p(z)$.

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The talk is based on a joint work with F. Martínez.

Potential operators for differential forms on Lipschitz domains

M. Costabel

Potential operators are right inverses of the exterior derivative on the space of differential forms satisfying the appropriate integrability conditions. They are generalizations of the operators that determine a scalar potential for a conservative vector field or a vector potential of a divergence free magnetic field. Such operators R_k satisfying the algebraic homotopy relation

$$d_{k-1}R_k + R_{k+1}d_k = id_k$$

where d_k and id_k are the exterior derivative and the identity on k-forms, were recently constructed for bounded and unbounded Lipschitz domains.

For bounded Lipschitz domains, one can use line integrals as in Poincaré's lemma, regularized with respect to the origin of the line integral. It can be shown that these regularized Poincaré operators and their adjoints, the Bogovskiĭ operators, are classical pseudodifferential operators of order -1. They have applications in vector analysis, computational electromagnetics and in fluid dynamics. On unbounded Lipschitz epigraphs, potential operators can be constructed as convolutions with a function supported in any given cone.

This talk is based on joint work with A. McIntosh and R. Taggart.

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Differential operators on graphs - an overview

S. Currie

In this talk I will give a collection of results obtained for differential operators on graphs. In particular, for Sturm-Liouville operators on graphs. The results will consist of finding eigenvalue asymptotics and solving three inverse problems.

The talk is based on various joint works with B.A. Watson.

Truncated moment problems, positive linear functionals, and finite algebraic varieties arising from cubic column relations

R. Curto

Let $\beta \equiv \beta^{(2n)} = \{\beta_i\}_{|i|\leq 2n}$ denote a d-dimensional real multisequence, let K denote a closed subset of $\mathbb{R}^d$, and let $\mathcal{P}_{2n} := \{p \in \mathbb{R}[x_1, \ldots, x_d] : \deg\ p \leq 2n\}$. Corresponding to β, the *Riesz functional* $L \equiv L_\beta : \mathcal{P}_{2n} \longrightarrow \mathbb{R}$ is defined by $L(\sum a_i x^i) := \sum a_i \beta_i$. We say that L is *K-positive* if whenever $p \in \mathcal{P}_{2n}$ and $p|_K \geq 0$, then $L(p) \geq 0$. In joint work with L.A. Fialkow, we prove that β admits a K-representing measure if and only if L_β admits a K-positive linear extension $\tilde{L} : \mathcal{P}_{2n+2} \longrightarrow \mathbb{R}$. This provides a generalization (from the full moment problem to the truncated moment problem) of the Riesz-Haviland Theorem.

Truncated moment problems (TMP) as above for which the support of a representing measure is required to lie inside a closed set K are called truncated K-moment problems (TKMP). In case K is a semi-algebraic set determined by polynomials q_1,...,q_m, the study of TKMP is dual to determining whether a polynomial nonnegative on K belongs to the positive cone consisting of polynomials of degree at most $2n$ which can be expressed as sums of squares, and of squares multiplied by one or more distinct q_i's. Thus, our results also show that a semialgebraic set solves the truncated moment problem in terms of natural degree-bounded positivity conditions if and only if each polynomial strictly positive on that set admits a degree-bounded weighted sum-of-squares representation.

For the multisequence β to have a representing measure μ it is necessary for the associated moment matrix $M(n)$ to be positive semidefinite, and for the algebraic variety associated to β, V_β, to satisfy rank $M(n) \leq$ card V_β as well as the following *consistency condition*: if a polynomial $p(x) \equiv \sum_{|i|\leq 2n} a_i x^i$ vanishes on V_β, then $p(\beta) := \sum_{|i|\leq 2n} a_i \beta_i = 0$. In joint work with L.A. Fialkow and H.M. Möller, we proved that for the extremal case (rank $M(n) =$ card V_β), positivity and consistency are sufficient for the existence of a (unique, rank $M(n)$-atomic) representing measure.

In recent joint work with S. Yoo we have considered cubic column relations in $M(3)$ of the form (in complex notation) $Z^3 = itZ + u\bar{Z}$, where u and t are real numbers. For (u, t) in the interior of a real cone, we prove that the algebraic variety V_β consists of exactly 7 points; we can then apply the above mentioned solution of the extremal moment problem to obtain a necessary and sufficient condition for the existence of a representing measure. This requires a new representation theorem for sextic polynomials in Z and $\bar{Z}$ which vanish in the 7-point set V_β.

The talk is based in part on joint work with L.A. Fialkow, with H.M. Möller and with S. Yoo.

Commuting pairs of multiplication operators on reproducing kernel Hilbert spaces over Reinhardt domains

R. Curto

I will discuss joint work with J. Yoon on the spectral properties of commuting 2-variable weighted shifts. Due to their symmetry properties, these operators can be modelled as multiplication operators acting on reproducing kernel Hilbert spaces over Reinhardt domains. By contrast with all previously known results in the theory of (single and 2-variable) weighted shifts, we show that the Taylor essential spectrum can be disconnected. We do this by obtaining a simple sufficient condition that guarantees disconnectedness, based on the norms of the horizontal slices of the shift. We also show that for every $k \geq 1$ there exists a k-hyponormal 2-variable weighted shift whose horizontal and vertical slices have 1- or 2-atomic Berger measures, and whose Taylor spectrum is disconnected.

We use tools and techniques from multivariable operator theory, and from the groupoid machinery developed by the author and P. Muhly to analyze the structure of the C*-algebra generated by multiplication operators acting on the Bergman space of an arbitrary Reinhardt domain. As a by-product, we show that, for 2-variable weighted shifts, the Taylor essential spectrum is not necessarily the boundary of the Taylor spectrum.

Cosine of angle and center of mass of an operator

G. Das

We consider the notion of real center of mass and total center of mass of a bounded linear operator relative to another bounded linear operator and explore their relation with cosine and total cosine of a bounded linear operator acting on a complex Hilbert space. We give another proof of the Min-max equality and then generalize it using the notion of orthogonality of bounded linear operators. We also illustrate with examples an alternative method of calculating the antieigenvalues and total antieigenvalues for finite dimensional operators.

The talk is based on a joint work with K. Paul.

Some problems related with frequent hypercyclicity and chaos

M. De la Rosa

It will be pointed out that any complex separable infinite-dimensional Fréchet space with a continuous norm and an unconditional Schauder decomposition supports an operator T which is chaotic and frequently hypercyclic, also it will be seen that complex Fréchet spaces with an unconditional basis support that kind of operators too. By this mean, it will be given a partial positive answer to a problem posed by J. Bonet. Also it will be presented a non-chaotic hypercyclic operator on $\omega = \mathbb{C}^{\mathbb{N}}$ which solves in the negative a problem posed by J. Bès and K. Chan.

The talk is based on a joint work with L. Frerick, S. Grivaux and A. Peris.

Metrics and Geodesics in Control and Identification of Shapes and Geometries

M.C. Delfour

1 Courant Metrics

A natural way to construct a family of variable domains is to consider the images of a fixed subset of $\mathbf{R}^N$ by some family of transformations of $\mathbf{R}^N$. The structure and the topology of the images can be specified via the natural algebraic and topological structures of the space of transformations or equivalence classes of transformations for which the full power of function analytic methods is available. There are many ways to do that and specific constructions and choices are very much problem dependent.

In 1972 [6] introduced complete metric topologies on a family of domains of class C^k that are the images of a fixed open domain (locally a C^k-epigraph) through a family of C^k-diffeomorphisms of $\mathbf{R}^N$. There the natural underlying algebraic structure is the *group structure* of the composition of transformations with the identity transformation as the neutral element. Her analysis culminates with the construction of a complete metric on the quotient of the group by an appropriate closed subgroup of transformations leaving the fixed subset unaltered. She called it the *Courant metric.* She introduced this terminology because it is proved in the book of CourantHilbert [3, p. 420], that the n-th eigenvalue of the Laplace operator depends continuously on the domain Ω, where $\Omega = (I + f)\Omega_0$ is the image of a fixed domain Ω_0 by $I + f$ and f is a smooth mapping (cf. [7]). But there is no notion of a metric in that book. Her constructions naturally extend to other families of transformations of $\mathbf{R}^N$ or of fixed hold-alls D.

In this paper we first extend her generic constructions associated with the space $C_0(\mathbf{R}^N, \mathbf{R}^N)$ of mappings from $\mathbf{R}^N$ into $\mathbf{R}^N$ to a larger family of Banach spaces of mappings such as $C^k(\overline{\mathbf{R}^N}, \mathbf{R}^N)$, $C^{k,1}(\overline{\mathbf{R}^N}, \mathbf{R}^N)$, or $\mathcal{B}^{\|}(\mathbf{R}^{\mathcal{N}}, \mathbf{R}^{\mathcal{N}})$ (cf. Delfour and Zolésio [5]), and beyond to Fréchet spaces such as $\mathcal{B}(\mathbf{R}^{\mathcal{N}}, \mathbf{R}^{\mathcal{N}})$ or $C_0^\infty(\mathbf{R}^N, \mathbf{R}^N)$ of infinitely continuously differentiables mappings. We emphasize the *geodesic character* of the construction of the metric and its interpretation as trajectories of bounded variation on the group.

The next step in the construction is the choice of the closed subgroup of transformations of $\mathbf{R}^N$ that is very much problem dependent. Originally, it was chosen as the set of transformations that leave the underlying set or pattern unaltered. However, in some applications, it could be unaltered up to a translation, a rotation, or a flip. The underlying set or pattern can be a closed set or an open crack free set. This includes closed submanifolds of $\mathbf{R}^N$. It is shown that, as long as the subgroup is closed, we get a complete Courant metric on the quotient group. In this section we also characterize the tangent space to the group of transformations of $\mathbf{R}^N$ that leads to the Courant metric. It is an example of an infinite dimensional manifold where the tangent space is independent of the point.

Finally, we free the constructions from the framework of bounded continuously differentiable transformations to reach the spaces of all homeomorphisms or C^k-diffeomorphisms of $\mathbf{R}^N$ or an open subset D of $\mathbf{R}^N$. Again it is shown that they are complete metric spaces. Hence, their quotient by a closed subgroup yields a Courant metric and a complete metric topology. With such larger spaces, it now becomes possible to consider subgroups involving not only translations but also isometries, symmetries, or flips in $\mathbf{R}^N$ or D.

2 Velocity Method

The quotient groups of transformations $\mathcal{F}(\Theta)/\mathcal{G}$ and their associated complete Courant metrics are neither linear nor convex spaces. We specialize the constructions to subspaces of transformations that are generated at time $t = 1$ by the flow of a *velocity field* over a generic time interval $[0, 1]$ with values in the tangent space Θ. The main motivation is to introduce a notion of semiderivatives in the direction $\theta \in \Theta$ on such groups as well as a tractable criterion for continuity via C^1 or continuous paths in the quotient group endowed with the Courant matric.

This point of view was adopted by Zolésio [26, 27]] as early as 1973 and considerably expanded in his *thèse d'état* in 1979. One of his motivations was to solve a *shape differential equation* of the type $\mathcal{A}\,V(t)+G(\Omega_t(V)) = 0$, $t > 0$, where G is the *shape gradient* of a functional and $\mathcal{A}$ a duality operator[1]. At that time most people were using a simple perturbation of the identity to compute shape derivatives. The first comprehensive book systematically promoting the *velocity method* was published in 1992 by Sokolowski and Zolésio [12]. Structural theorems for the Eulerian Shape Derivative of smooth domains were first given in 1979 in [27] and generalized to non-smooth domains in 1992 in [4]. The velocity point of view was also adopted in 1994 by R. Azencott [1] and his team (cf. for instance Trouvé [16, 14, 15]] in 1995 and 1998) to construct complete metrics and *geodesic paths* in spaces of diffeomorphisms generated by a velocity field with a broad spectrum of applications to imaging. The reader is referred to the forthcoming book of Younes [23] for a comprehensive exposition of this work and beyond to related papers such as [8], [9, 10], [24]. In view of the above motivations, this paper specializes the results to transformations generated by velocity fields. It also explores the connections between the constructions of Azencott and Micheletti that implicitly uses a notion of *geodesic path with discontinuities*.

[1]Cf. [27] and the recent book [11].

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On the distribution of the eigenvalues for non-selfadjoint operators

M. Demuth

Let A be a selfadjoint operator. We are interested in the discrete spectrum of $B = A + M$ where B is non-selfadjoint. If the resolvent difference $R = (s - B)^{-1} - (s - A)^{-1}$ is in the Schatten class S_p then

$$\sum_{\lambda \in \sigma_{\text{disc}}(B)} \frac{\text{disc}(\lambda, \sigma(A))^\gamma}{|\lambda|^{\gamma/2}(1 + |\lambda|)^\gamma} \leq c\ ||R||_p$$

where $\gamma \geq \max\ (1 + p,\ 2p)$. By means of this estimate we can give qualitative estimates for the number of eigenvalues of B or their moments. That can be applied to Schrödinger operators with complex potentials.

On the spectral properties of the product of two selfadjoint operators

M. Denisov

Let $\mathcal{H}$ be a Hilbert space. Linear operators A and B are selfajoint. The aim of this talk is to describe spectral properties of operator $C := AB$. In particular we will investigate when operator C has spectral function.

The talk is based on a joint work with T. Ya. Azizov and F. Phillip. The work is supported by the Russian Foundation for Basic Researches grant 08 – 01000566-a.

Resolvent estimates for mixed-order systems

R. Denk

For boundary value problems which are parabolic in the sense of Petrovskij, uniform a priori estimates are well-known. This gives results on the spectrum of the L^p-realization of the boundary value problem and estimates for the norm of the resolvent. The situation is more complicated if the operator is studied in Sobolev spaces of higher order instead of L^p or if we have a mixed-order (Douglis-Nirenberg) system of operators. In the talk we discuss uniform a priori estimates in parameter-dependent norms, resolvent estimates and the generation of an analytic semigroup for mixed-order systems in higher order Sobolev spaces. The operators considered are differential operators defined in the whole space or in bounded domains. In the whole space, we obtain even the existence of a bounded H^∞-calculus for general mixed-order systems of pseudodifferential operators.

Borg-type theorems for finite generalized Jacobi matrices

M. Derevyagin

We will discuss two types of inverse spectral problems for a class of generalized Jacobi matrices associated with P-fractions. Namely, analogs of the Hochstadt-Lieberman theorem and the Borg theorem for the finite generalized Jacobi matrices will be presented.

The Jacobi matrices approach to Nevanlinna-Pick problems

M. Derevyagin

A modification of the well-known step-by-step process for solving Nevanlinna-Pick problems in the class of $\mathbf{R}_0$-functions gives rise to a linear pencil $H - \lambda J$, where H and J are Hermitian tridiagonal matrices. First, we show that J is a positive operator. Then we prove that the corresponding Nevanlinna-Pick problem has a unique solution if and only if a densely defined symmetric operator $J^{-\frac{1}{2}} H J^{-\frac{1}{2}}$ is self-adjoint. Besides, some criteria for $J^{-\frac{1}{2}} H J^{-\frac{1}{2}}$ to be self-adjoint are obtained. In the self-adjoint case, we get that multipoint diagonal Padé approximants converge to a unique solution of the Nevanlinna-Pick problem locally uniformly in $\mathbb{C} \setminus \mathbb{R}$.

To show the relation of our approach to the classical one we should notice that, roughly speaking, if all interpolation points tend to infinity simultaneously (that is, the Nevanlinna-Pick problem is approaching a moment problem) then the corresponding matrix J tends to the identity I elementwise. Thus, the proposed scheme extends the classical Jacobi matrix approach to moment problems and Padé approximation for $\mathbf{R}_0$-functions.

For the sake of completeness, let us recall that $\varphi \in \mathbf{R_0}$ if it has the following integral representation

$$\varphi(\lambda) = \int_{\mathbb{R}} \frac{d\sigma(t)}{t - \lambda},$$

where σ is a finite positive Borel measure (i.e. $\int_{\mathbb{R}} d\sigma(t) < \infty$).

Regularity of critical points of nonnegative operators in Kreĭn spaces

V. Derkach

We consider a nonnegative linear operator $\widetilde{A}$ in a Kreĭn space, which can be represented as a coupling of two Hilbert space symmetric and nonnegative operators S_+ and S_- with deficiency indicies $(1, 1)$. Starting from the Veselič criterion of regularity of critical points 0 and ∞, we reformulate it in terms of the abstract Weyl functions m_+ and m_- of the operators S_+ and S_-. By Abelian and Tauberian theorems for Stieltjes integrals the problem of regularity of critical points is reduced in essential to the study of asymptotics of m_+ and m_-. The results are illustrated on some differential operators.

The talk is based on joint work with B. Ćurgus.

The Cauchy–Kovalevskaya extension in Hermitean Clifford analysis

H. De Schepper

The Cauchy–Kovalevskaya extension theorem is well–known: in particular, it follows from this theorem that a holomorphic function in an appropriate region of the complex plane is completely determined by its restriction to the real axis. This holomorphic CK–extension principle has been elegantly generalized to higher dimension in the framework of Clifford analysis, a higher dimensional function theory centered around the notion of a monogenic function, i.e. a Clifford algebra valued null solution of the Dirac operator $\underline{\partial} = \sum_{j=1}^{m} e_j \, \partial_{x_j}$, where $(e_1, \ldots, e_m)$ is an orthonormal basis of $\mathbf{R}^m$ underlying the construction of the real Clifford algebra $\mathbf{R}_{0,m}$. Completely similar to the holomorphic case, a monogenic function in an appropriate region of $\mathbf{R}^m$ will be completely determined by its restriction to the hyperplane $x_1 = 0$.

More recently Hermitean Clifford analysis has emerged as a new branch of Clifford analysis, which focusses on the simultaneous null solutions, called Hermitean monogenic functions, of two Hermitean conjugate complex Dirac operators. The functions considered now take their values in the complex Clifford algebra $\mathbf{C}_m$. In this talk we establish a Cauchy–Kovalevskaya extension theorem for Hermitean monogenic polynomials. The minimal number of initial polynomials needed to obtain a unique extension is determined, along with the compatibility conditions they have to satisfy.

This is joint work with F. Brackx, R. Lavička and V. Souček.

Combining Schur interpolation and Schur elimination to produce new types of matrix conditioners

P. Dewilde

Matrix conditioners are approximations to the inverse of a matrix. They are instrumental in solvers for large systems of linear equations in which the original matrix is either very sparse or can be represented with a limited amount of data so that the product of the matrix with a vector is of low computational complexity. To exploit this low complexity for solving the system with an iterative algorithm, one needs in addition an approximation of the inverse with the same property of low complexity matrix-vector products. I consider the case of (large) positive definite matrices and the use of both Schur type interpolation as well as Schur type elimination to produce new conditioners (the computation of the conditioner itself must also be of low computational complexity, of course.) Classically, a Schur matrix interpolation can only be used on data that forms a band around the main diagonal. We show how the combination of the two Schur methods succeeds in producing conditioners for multi band and hierarchically multi band matrices - a goal that has proven elusive so far. I shall also discuss applications and extensions briefly.

Boundary interpolation and rigidity for generalized Nevanlinna functions

A. Dijksma

We solve a boundary interpolation problem at a real point for generalized Nevanlinna functions, and use the result to prove uniqueness theorems for generalized Nevanlinna functions.

Joint work with D. Alpay and H. Langer.

Self-adjoint linearizations of eigenvalue problems with boundary conditions which depend polynomially on the eigenvalue parameter

A. Dijksma

The eigenvalue problems we consider are of the form

$$S^* f = \lambda f, \quad P(\lambda)\mathrm{b}(f) = 0,$$

where S^* is the adjoint of a densely defined symmetric operator S in a Hilbert space with both defect numbers equal to d, b is a boundary mapping on the domain of S^* with values in $\mathbb{C}^{2d}$ and corresponding $2d \times 2d$ Gram matrix Q, and $P(z)$ is a $d \times 2d$ matrix polynomial such that

1. $P(z)Q^{-1}P(z^*)^* = 0$ for all $z \in \mathbb{C}$,
2. $P(z)$ has rank d for all $z \in \mathbb{C}$, and
3. for some nonnegative integers $\mu_1, \ldots, \mu_d$ the limit

$$P_\infty := \lim_{z\to\infty} \operatorname{diag}\left(z^{-\mu_1}, \ldots, z^{-\mu_d}\right) \mathcal{P}(z)$$

   exists and is a $d \times 2d$ matrix of rank d.

In the lecture we define and investigate the self-adjoint linearization of this problem and discuss corresponding eigenfunction expansions.

The lecture is based on joint work in progress with T. Azizov and B. Curgus.

Some properties of Fredholm type for Toeplitz operators with 2×2 symbols

C. Diogo

Several relations between the kernels and cokernels of Toeplitz operators with symbols differing by factors of a particular type, which are known to hold in the case of existence of a Wiener-Hopf factorization of the symbol, are obtained here for general symbols in $(L_\infty(\mathbb{R}))^{n\times n}$. These results are applied to establish some properties of Fredholm type for 2×2 symbols.

This is a joint work with C. Benhida and M.C. Câmara.

Compact perturbations of linear bounded operator that preserve spectral continuity

S.V. Djordjević

For an infinite dimensional Banach space X, with $B(X)$ we denote the algebra of all linear bounded operator on X and $K(X)$ the ideal of all compact operators. For $T \in B(X)$, let $\sigma(T)$ be the spectrum of T, $\sigma_p(T)$ set of all eigenvalues of T, and $\pi_0(T)$ the set of all isolated eigenvalues of finite geometric multiplicity.

The perturbation of an operator by compact operators is usual technic in areas of operators equations. Our interest is finding such compact operators that the continuity of the spectrum move from T to $T+K$, $T \in B(X)$ and $K \in K(X)$. Moreover, it is given condition that isolated points in $\sigma_p(T)$ with finite geometrical or algebraic dimension still are like that in $\sigma(T+K)$ or out of the spectrum.

The talk is based on a joint work with S. Sánchez-Perales.

Essential spectrum of multivariable matrix-valued Toeplitz operators

R. Douglas

Classical Toeplitz operators were defined on the Hardy space for the unit disk with a major focus being on the invertibility or Fredholmness of them. For a Toeplitz operator with a continuous, matrix-valued symbol, the essential spectrum was determined more than fifty years ago by Gohberg and Krein. Since then, many researchers have considered such questions for Toeplitz operators with more general symbols and on other Hilbert spaces of holomorphic functions such as the Bergman space. More recently, generalizations to the multivariable context have been considered with new phenomena present and many different techniques involved.

In joint work with J. Sarkar, we obtain necessary conditions for the semi-Fredholmness of a Toeplitz operator with a matrix-valued holomorphic symbol on a Hilbert space of holomorphic functions on the unit ball. Examples of such spaces are the Hardy or Bergman spaces on the unit ball or the Drury-Arveson space. The techniques used include the Berezin transform in this context as well as the Taylor spectrum defined in terms of the Koszul complex. Relations of these questions to the Corona problem are also discussed.

Similarity, dilations, and commutant lifting in the DA-Space

R. Douglas

During the past fifty years, an extensive and refined theory has been developed based on a canonical model for contraction operators on Hilbert space. A key ingredient in this development was the commutant lifting theorem (CLT), which grew out of a seminal paper of Sarason. In the last decade or two, there has been a systematic effort to extend much of this theory to the Drury-Arveson (DA) space in m variables, including the CLT. The latter development is due to a number of authors including Arveson, McCullough-Trent, Muller-Vasilescu, and Popescu.

In this talk, based on joint work with Foias and Sarkar, and expressed in the language of Hilbert modules, I will discuss results on similarity of quotient Hilbert modules of vector-valued DA-modules with the splitting of the corresponding short exact sequences and the complementation of the submodules, including connections to the CLT. Furher, I will show that all isometries on vector-valued DA-modules are trivial, in a sense to be defined, and use this fact to begin the analysis of more general resolutions of such modules and discuss their relation to dilations.

Preorders and a realization theorem for the Schur class

M.A. Dritschel

Inspired by recent work of Grinshpan, Kalyuzhni-Verbovetski, Vinnikov and Woerdeman, as well ideas from real algebraic geometry, we give a realization theorem which extends well known results of Agler and others to, among more notable examples, the Schur class of the polydisk, and discuss various applications.

Extension of functions from hypersurfaces with the boundary

R. Duduchava

A coretraction, a right inverse to the trace operator to a smooth hypersurface $\mathcal{S}$ with the smooth boundary $\Gamma = \partial\mathcal{S} \neq \emptyset$, is constructed. The coretraction extends m-tuples of functions from the Besov spaces on each face of $\mathcal{S}$ into the Bessel potential space on the domain slit by the hypersurface $\mathbb{R}^n_{\mathcal{S}} := \mathbb{R}^n \setminus \overline{\mathcal{S}}$, provided these tuples satisfy a compatibility conditions on the boundary Γ. The traces are defined by arbitrary Dirichlet system of boundary operators and extension is performed by two different methods, one implicit and one explicit. Explicit extension, based on the solution to the Dirichlet BVP for the poly harmonic equation, permits the extension of distributions from the Besov space $\mathbb{B}^s_{p,p}(\mathcal{S})$ with a negative $s < 0$. Moreover, it permits to establish some additional features of the extended functions, which are useful in applications.

Coretractions have essential applications in boundary value problems for partial differential equations when, for example, it is necessary to reduce a BVP with non-homogeneous boundary conditions to a BVP with the homogeneous boundary conditions.

For a pair of Besov spaces we introduce the following shortcut

$$\overset{2}{\mathbb{B}}{}^s_{p,p}(\mathcal{S}) := \mathbb{B}^s_{p,p}(\mathcal{S}) \otimes \widetilde{\mathbb{B}}^s_{p,p}(\mathcal{S})$$

and denote $p_s := p$ if $s \neq 0, \pm 1, \ldots$, $p_s < p$ if $s = 0, \pm 1, \ldots$. The notation $[s]^- \in \mathbb{Z}$ is used for the largest positive or negative integer less than s, i.e., $s - 1 \leqslant [s]^- < s$.

Here is one of the principal results obtained in the present investigation,

THEOREM. *Let $\mathcal{S}$ be a smooth hypersurface with the boundary, $\mathbf{A}(x, D)$ be a PDO of order $k \in \mathbb{N}_0$ and of normal type, $1 < p < \infty$ and $k \in \mathbb{N}_0$, $s > 0$, $k \leqslant s + 1$. Further let*

$$\vec{\mathbf{B}}^{(k)}(x, D) := \{\mathbf{B}_0(x, D), \ldots, \mathbf{B}_{k-1}(x, D)\}^\top$$

be a Dirichlet system of boundary operators and $\left\{\varphi_j^\pm\right\}_{j=0}^{k-1}$ be vector functions such that

$$\Phi_j := (\varphi_j^+ + \varphi_j^-, \varphi_j^+ - \varphi_j^-) \in \overset{2}{\mathbb{B}}{}^{s-j}_{p,p}(\mathcal{S}), \qquad \text{for all} \quad j = 0, 1, \ldots, k-1.$$

Then there exists a continuous linear operator

$$\mathcal{P}_\mathbf{A} \ : \ \bigotimes_{j=1}^{k-1} \overset{2}{\mathbb{B}}{}^{s-j}_{p,p}(\mathcal{S}) \to \mathbb{H}^{s+1/p_s}_{p_s,loc}(\mathbb{R}^n_\mathcal{S})$$

which has the prescribed traces on the boundary

$$\gamma_{\mathcal{S}^\pm} \mathbf{B}_j \mathcal{P}_\mathbf{A} \Phi = \varphi_j^\pm, \quad j = 0, 1, \ldots, k-1, \qquad \mathbf{A}\mathcal{P}_\mathbf{A}\Phi \in \widetilde{\mathbb{H}}^{s-k+1/p_s}_{p_s,loc}(\mathbb{R}^n_\mathcal{S}),$$

where $\Phi := \left\{\Phi_j\right\}_{j=0}^{k-1}$.

Some inverse problems for Krein-Dirac systems

H. Dym

In this talk we shall survey some of the results on inverse problems for Dirac-Krein systems that have been obtained in the past few years in collaboration with D. Z. Arov. The talk will be expository.

Bitangential Interpolation in Generalized Schur Classes

H. Dym

A description of the ranges of linear fractional transformations of generalized J-inner matrix valued functions will be presented and then applied to obtain linear fractional representations of the solutions to some bitangential interpolation problems in the generalized Schur class. The talk is based on joint work with Vladimir Derkach. It will be expository.

Diffusive wavelets and the Heisenberg group

S. Ebert

One important subject of the time-frequency analysis on groups is the decomposition of functions into matrix coefficients of irreducible representation a further discussion leads to decomposition by a different concept, namely wavelets. Powerful tools of harmonic analysis can be utilized to investigate wavelets on compact groups. Thanks to the Peter-Weyl theorem (giving a decomposition of L^2-functions in terms of irreducible representations) on compact Lie Groups $\mathcal{G}$ exists a elegant way to express the heat kernel, which is the main ingredient for the concept of *diffusive wavelets.*

In the case of noncompact groups arise difficulties, which end up in the fact that there is no equivalent of Peter-Weyl theorem in that case.

In many different fields of mathematics and physics the Heisenberg group H_n plays an important role. In wavelet theory on $\mathbb{R}^n$ it arises naturally.

I will discuss diffusive wavelets for the case of compact groups to introduce the general idea. The main point then will be to transfer the concept to H_n.

The concept of diffusive wavelets can be established on H_n thanks to the fact it is a nilpotent group, which posses a Plancherel measure. Because of this enormous interest to the Heisenberg group it is well investigated and the Plancherel measure of it is explizite known.

Considering the Sub-Riemannian structure on H_n we will discuss the spectral decomposition of the corresponding sub-heat kernel which leads to the construction of *diffusive wavelets on H_n*.

Szegö-Widom Limit theorems for band-dominated operators with almost periodic diagonals

T. Ehrhardt

The classical Szegö-Widom Limit Theorem describes the asymptotics of the determinants of block Toeplitz matrices

$$T_{nN}(A) = (A_{j-k}), \quad 0 \le j,k \le n-1, \quad A_k \in \mathbb{C}^{N\times N}$$

as n goes to infinity. An equivalent way of describing these block Toeplitz matrices is to say that all their diagonals are periodic sequences with period N and that the matrix size is a multiple of N.

The goal of the work presented is to generalize the limit theorem to matrices whose diagonals are almost periodic sequences. The most prominent example arises from the finite sections of the almost Mathieu operator,

$$M = U_1 + aI + U_{-1}$$

where $U_{\pm 1}$ are the forward and backward shift on ℓ^2 and a stands for the sequence $a(n) = \alpha + \beta \sin(2\pi\xi n + \delta)$, determining the entries of the main diagonal. As suggested by the block case, the size of the finite sections has to be chosen in a suitable way to yield reasonable results.

A somewhat surprising feature is that the asymptotics may be significantly different. For the explanation of this fact, one has to resort to some deep results in diophantine approximation theory. For example, in the case of the almost Mathieu operator the transcendence properties of ξ play a crucial role.

As all classical methods of proving the Szegö-Widom Theorem fail to generalize, a new algebraic approach is used to compute the determinants asymptotically.

This talk is based on joint work with S. Roch and B. Silbermann.

On non-commutative analoga of multiple ergodic theorems

T. Eisner

We consider the classical multiple ergodic theorems in the context of non-commutative von Neumann algebras and discuss which properties concerning convergence and multiple recurrence remain valid. This is a joint work with T. Austin and T. Tao.

A class of weighted holomorphic Bergman spaces

A. El-Sayed

In this talk, we introduce the class $\mathcal{N}_{K,\varphi_a,p}$ of weighted holomorphic functions on the unit disc $\mathbb{D}$ of the complex plane $\mathbb{C}$. Moreover, we prove some essential properties of functions belonging to this general class. Further, we represent functions from the class $\mathcal{N}_{K,\varphi_a,p}$ by the help of their lacunary Taylor series. Finally, we study composition operators acting between $\mathcal{N}_{K,\varphi_a,p}$-type classes.

This talk is based on a joint work with H. Al-Amri.

Boundary control of flows in networks

K.-J. Engel

We present some results concerning control and asymptotic behavior of flows in networks. Our approach is based on an explicit variation of constants-type formula for the solution of an abstract boundary control system and makes use of the theory of strongly continuous semigroups.

The talk is based on joint work with M. Kramar Fijavž, B. Klöss, R. Nagel, and E. Sikolya.

On strong solutions to the equation of motion of compressible viscous fluid flow on an exterior domain

Y. Enomoto

I would like to talk about a global in time unique existence theorem of solutions to the equation describing the motion of compressible viscous fluid flow in a two dimensional exterior domain for small initial data. This is an extension of the work due to Matsumura-Nishida (Commun. Math. Phys. **89** (1983), 445–464), where they proved a global in time unique existence theorem for small initial data in a three dimensional exterior domain. I also present some decay properties of solutions in the Banach space setting.

The talk is based on a joint work with Y. Shibata.

Hyperbolic Function Theory

S.-L. Eriksson

The aim of this talk is to consider the hyperbolic version of the standard Clifford analysis. The need for such a modification arises when one wants to make sure that the power function x^m is included. H. Leutwiler noticed in 1990 that the power function is the conjugate gradient of a harmonic function, defined with respect to the hyperbolic metric of the upper half space. The theory was extended to the total Clifford algebra valued functions called hypermonogenic in 2000 by H. Leutwiler and S.-L. Eriksson. Hypermonogenic functions have integral formulas, proved in 2004 and 2005. We present the kernel functions in term of hyperbolic metric. Using this new intepretation of the kernels we obtain power series presentations of hypermonogenic functions and related results.

The periodic decomposition problem for operator semigroups

B. Farkas

It was asked by I. Z. Ruzsa whether the identity function $\mathrm{id} : \mathbb{R} \to \mathbb{R}$ can be written as the sum of n periodic functions with prescribed periods $a_1, \ldots, a_n \in \mathbb{R}$. The answer to this question turns out to be dependent on the periods. One can reformulate and generalize this problem as follows. Consider the shift operators $T_a : \mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$ defined by $(T_a f)(x) := f(x + a)$. Is it true that

$$\mathrm{id} \in \ker(T_{a_1} - I) + \cdots + \ker(T_{a_n} - I)?$$

(A function belongs to the right hand side if and only if it is a sum of n periodic functions.) Or, more generally, does the equality

$$\ker(T_{a_1} - I) \cdots (T_{a_n} - I) = \ker(T_{a_1} - I) + \cdots + \ker(T_{a_n} - I)$$

hold true? (Here $I : \mathbb{R}^{\mathbb{R}} \to \mathbb{R}^{\mathbb{R}}$ is the identity mapping.) Note that for $n \geq 2$ the id belongs to the left hand side and that the inclusion "$\supseteq$" is trivial. Of course, this question is now meaningful for general linear operators T_i replacing the shifts T_{a_i}. We give some answers in the case when T_i are given commuting, bounded linear operators on a Banach space. Both algebraic and analytic properties of the semigroup generated by the operators $T_1, \ldots, T_n$ play an important role here. The talk is partially based on a joint work with T. Keleti, V. Harangi and Sz. Gy. Révész.

Quantum mechanical methods in electromagnetism

N. Faustino

In physics, Wigner Quantum Systems play an important role in the study of the physical model of the harmonic oscillator described as a superposition of n-independent Hamiltonians of the form $-\frac{1}{2m}\mathbf{a}_j 2 + \frac{m\omega 2}{2}(\mathbf{a}_j^\dagger)2$, where m denotes the mass and ω denotes the frequency.

According to Wigner approach [5], Wigner Quantal Systems are canonically equivalent to the Lie superalgebra of the type $\mathrm{osp}(1|2n)$. This, in particular, allows a meaningful description of discrete function theory based in terms of Wigner Quantal Systems as representations of $\mathrm{osp}(1|2)$ [2].

There is another approach developed by G-C. Rota and collaborators, where the description of umbral calculus was obtained by means of bosonic calculus in interplay with the second quantization approach [1]. Indeed, the algebra of multivariate polinomials is isomorphic to the free algebra generated by position and momentum operators, $\mathbf{a}_j^\dagger$ and $\mathbf{a}_j$, respectively, satisfying the Heisenberg-Weyl relations

$$[\mathbf{a}_j, \mathbf{a}_k] = 0, \quad [\mathbf{a}_j^\dagger, \mathbf{a}_j^\dagger] = 0, \quad [\mathbf{a}_j, \mathbf{a}_k^\dagger] = \delta_{jk}\mathbf{id}$$

In this talk, it will be presented a similar approach to the approach recently proposed by N. Faustino and G. Ren in [3] for the time-harmonic Maxwell equations.

We will start to make an overview for the formulation of the Maxwell equations in the 3D case as well as the topological and material laws encoded.

Next, using the machinery already developed in [2], we will describe the polynomial solutions of the Maxwell time-harmonic equations by means of Wigner Quantal Systems.

The talk is based on a joint work with D. Constales and R.S. Kraußhar.

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The role of positivity in moment and polynomial optimization problems

L. Fialkow

Let $y \equiv y^{(2d)} = \{y_i\}_{i \in \mathbb{Z}_+^n, |i| \leq 2d}$ denote an n-dimensional real multisequence of degree $2d$ with $y_0 > 0$, and let K denote a closed subset of $\mathbb{R}^n$. We consider connections between the following two classical problems: 1) (Truncated K-Moment Problem) Find conditions for the existence of a positive Borel measure μ, *supp* $\mu \subseteq K$, such that $y_i = int_K x^i d\mu \quad (|i| \leq 2d)$; 2) (Polynomial Optimization Problem) For a given polynomial $p \in \mathbb{R}[x_1, \ldots, x_n]$, compute (or estimate) $p_* := \inf_{x \in K} p(x)$. These problems lead naturally to various notions of positivity: positive semidefiniteness of the moment matrix and K-localizing matrices associated with y; K-positivity of the Riesz functional L_y, defined by

$$L_y \sum_{|i| \leq 2d} a_i x^i = \sum a_i y_i;$$

and positivity of polynomials on K. We discuss recent results which shed new light on the interconnectedness of these concepts and on their roles in solving moment and optimization problems. This talk is based in part on joint work with R. Curto, with C. Easwaran, and with J. Nie.

Equivalence of the indefinite Sturm-Liouville Riesz basis property with a HELP-type inequality

A. Fleige

For an indefinite weight function $r \in L^1[-1,1]$ with $xr(x) > 0$ we consider two problems which at first glance seem to be different. The first is the Riesz basis property of the indefinite Sturm-Liouville eigenvalue problem

$$-f'' = \lambda r f, \qquad f(-1) = f(1) = 0.$$

The second problem is the validity of the HELP-type inequality on $[0,1]$

$$\left(\int_0^1 |h'|^2 \frac{1}{r} \, dx \right)^2 \leq K \left(\int_0^1 |h|^2 dx \right) \left(\int_0^1 | \left(\frac{h'}{r} \right)' |^2 dx \right)$$

for a certain class of functions h. We show that for so-called strongly odd dominated functions r (including at least all odd r) both problems are equivalent. This allows us to apply known results from the theory of one problem to the other. The talk is based on a joint work with P. Binding.

Tensor products of GB^*-algebras and applications

M. Fragoulopoulou

GB^*-algebras are generalizations of C^*-algebras. They were introduced and studied first by G.R. Allan, in 1967. In 1970, P.G. Dixon extended the concept of a GB^*-algebra, in order to include also topological $*$-algebras which are not locally convex. The importance of GB^*-algebras is mainly due to the fact that being algebras of unbounded operators have interesting applications in mathematical physics and this gives a strong impetus for studying them. Typical examples of GB^*-algebras are pro-C^*-algebras (i.e, inverse limits of C^*-algebras), C^*-like locally convex $*$-algebras (introduced by A. Inoue - K.-D. Kürsten), the Arens algebra $L^\omega[0,1] = \bigcap_{1\leq p<\infty} L^p[0,1]$ equipped with the topology of the L^p-norms, $1 \leq p < \infty$ (G.R. Allan) and the algebra $M[0,1]$ of all measurable functions on $[0,1]$ (modulo equality a.e.), endowed with the topology of convergence in measure, which is not necessarily locally convex (P.G. Dixon).

To our knowledge, up to now, there is nothing in the literature about tensor products of GB^*-algebras. So, this talk is devoted to the investigation of this subject matter. First, we can show that if X is a Hausdorff locally compact space and A a C^*-like locally convex *-algebra (with continuous multiplication), then the complete locally convex $*$-algebra of all A-valued continuous functions on X, $C(X,A) \cong C_c(X)\widehat{\otimes}A$, is a GB^*-algebra, under the injective tensorial topology (note that "c" denotes the topology of compact convergence on the algebra $C(X)$ of all $\mathbb{C}$-valued continuous functions on X). Furthermore, sufficient and necessary conditions will be given, such that the compltеted tensor product of two GB^*-algebras under a "$*$-admissible" topology is again a GB^*-algebra. Applications concerning (unbounded) $*$-representation theory of tensor product GB^*-algebras will be presented.

The talk is based on a joint work with A. Inoue and M. Weigt.

Optimal H^2 solutions to a rational Bezout type equation

A.E. Frazho

Let G be a stable rational $m \times p$ matrix function where $m < p$. In particular, G is a rational function in $H^2_{m,p}$. We study Bezout type equations of the form

$$G(z)X(z) = I_m, \quad z \in \mathbb{D},$$

where the solution X is a rational function in $H^2_{p,m}$. Given a stable state space realization of G, we present necessary and sufficient conditions for $G(z)X(z) = I_m$ to be solvable in terms of solutions of associated Riccati and Stein equations. In this case, a state space formula is given for the optimal H^2 solution. Another state space formula for a special inner function yields the set of all solutions. A state space space solution for the Tolokonnikov completion appearing in the rational corona problem is also given. The proofs use operator theory techniques. Finally, numerical examples with applications to systems theory are presented.

This is joint work with M.A. Kaashoek and A.C.M. Ran.

Sampling theorems associated with regular and irregular eigenvalue problems

G. Freiling

We present sampling expansions associated with arbitrary Birkhoff-regular and non-regular boundary value problems, including problems with multiple eigenvalues. As a kernel of the sampled transforms the Green's function, multiplied by an adequate entire function, is used. The sampling expansions obtained are Lagrange- or Hermite-type interpolation series.

The talk is based on a joint work with M. Annaby and S. Buterin.

$\overline{\partial}$ and the Dirac operator

K. Gansberger

We will establish a connection between the Dirac operator in $\mathbb{R}^2$ and $\overline{\partial}$ acting in a certain weighted space on $\mathbb{C}$ showing that resolvent properties of the Dirac operator are closely related to properties of the solution operator to $\overline{\partial}$ and certain weighted Sobolev imbeddings. From this we derive a non-compactness result for the resolvent of the Dirac operator.

Hankel determinants, Coxeter-Toda lattices and cluster algebras

M. Gekhtman

We explore an interplay between inverse problems for generalized Jacobi matrices and the Poisson geometry of directed networks to construct a cluster algebra structure in the space of rational functions. We then interpret Bäcklund-Darboux transformations between various integrable lattices of Toda type in terms of cluster transformations.

The talk is based on a joint work with M. Shapiro and A. Vainshtein.

Riemann-Hilbert problems on compact Riemann surfaces

G. Giorgadze

The local theory of ordinary differential equations in one dimensional case is the object of intensive study starting from Gauss to nowadays. In an unfinished paper "Zwei allgemeine Satze uber lineare Differentialgleichungen mit algebraischen Koeffizienten" B.Riemann formulated the problem of constructing ordinary differential equations with prescribed branching points and monodromy.

This problem appeared fundamental and very stimulating in the analytic theory of ordinary differential equations. Later on D.Hilbert made important contributions to the topic and included it in the list of his famous problems under number 21. For this reason it is often called Riemann-Hilbert problem or Hilbert's 21st problem. It should be noted that Riemann himself suggested a fruitful approach to this problem and, in particular, showed that it can be reduced to another problem which also appeared very interesting and useful. The problem consists in finding piecewise holomorphic matrix functions which satisfy certain boundary condition on the unit circle. This problem is called the problem of linear conjugation for analytic functions or simply Riemann problem.

In course of work over classification of linear differential equations with singular points, L.Fuchs introduced an important class of such equations, which in the modern literature are called Fuchsian equations. It turned out that the Riemann-Hilbert problem is especially interesting in the class of Fuchsian systems and research on this topic continues up to now.

Considerable progress in solving Riemann-Hilbert problem and linear conjugation problem was achieved by J.Plemelj [1].

Plemelj reduced linear conjugation problem with piecewise continuous coefficients to the case of continuous coefficients and applied this for solving Riemann-Hilbert problem. For a long time it was believed that he gave a complete solution of Riemann-Hilbert problem but later it turned out that there was a gap in his argument and eventually A.Bolibruch found a counterexample showing that Riemann-Hilbert problem is not always solvable in the class of Fuchsian systems [2].

It should be noted that an important contribution to the theory of Riemann-Hilbert problem was made by I.Lappo-Danilevsky [3], who developed a theory of functions of matrices and applied it to effective construction of solutions of certain Riemann-Hilbert problems. A big progress in the theory of linear conjugation problem was achieved by N.Muskhelishvili and N.Vekua (see [4] who, in particular, successfully used matrix calculus and introduced the so-called partial indices of matrix functions on smooth contours.

We consider analogical problems on compact Riemann surfaces (see [5]).

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Sampling problems and oblique projections

J. Giribet

To sample a signal f means to obtain a sequence $\{f_n\}_{n\in\mathbb{N}}$ of instantaneous values of a particular signal characteristic, this sequence are called the samples of f. The classical sampling scheme is based on the Whittaker-Kotelnikov-Shannon theorem. Given a signal $f \in \mathcal{PW}$ (the Paley-Wiener space), the Whittaker-Kotelnikov-Shannon theorem establishes that it is possible to reconstruct the signal f from its samples $\{f_n\}_{n\in\mathbb{N}}$. When a signal $f \in L^2(\mathbb{R})$ does not belong to the Paley-Wiener space, a common strategy in signal processing applications is to apply a low pass filter (certain bounded linear operator) to the signal f obtaining a new signal g. Then, the filtered signal g is sampled giving the sequence $\{g_n\}_{n\in\mathbb{N}}$. Although, the signal recovered by the samples $\{g_n\}$ will not generally coincide with the original signal f, approximates it. In fact, the recovered signal is the best approximation, i.e., the orthogonal projection, of the original signal in $\mathcal{PW}$. A common way to represent the samples of a signal f, is by means of the inner product of f with some given vectors $\{v_n\}_n \in \mathbb{N}$ that spans a closed subspace $\mathcal{S}$, called the *sampling subspace*. By the other hand, given the the samples $\{f_n\}_{n\in\mathbb{N}}$, the reconstructed signal $\hat{f}$ is given by $\hat{f} = \sum_{n\in\mathbb{N}} f_n w_n$, where $\{w_n\}_{n\in\mathbb{N}}$ spans a closed subspace $\mathcal{R}$, called the *reconstruction subspace*.

In the classical sampling scheme the reconstruction and the sampling subspaces are assumed to be the same. In signal processing applications, this not always the case, and then it is not always possible to recover the best approximation of the original signal. Thus, different sampling techniques must be used. M. Unser and A. Aldroubi introduced the idea of consistent sampling, it means that the reconstructed signal $\hat{f}$ is not supposed to be the best approximation of the original signal, but f and $\hat{f}$

have the same samples. The main goal of this talk is to give an interpretation of the consistent sampling in terms of the notion of compatibility between a closed subspace $\mathcal{S}$ of a Hilbert space $\mathcal{H}$ and a positive semidefinite operator A acting on $\mathcal{H}$. This notion has a completely different origin. Z. Pasternak-Winiarski studied, for a fixed subspace $\mathcal{S}$, the analiticity of the map $A \to P_{A,\mathcal{S}}$ which associates to each positive invertible operator A the orthogonal projection onto $\mathcal{S}$ under the (equivalent) inner product $< \xi, \eta >_A = < A\xi, \eta >$, for $\xi, \eta \in \mathcal{H}$. Later E. Andruchow et al. presented a simplification of Pasternak-Winiarski's arguments and further geometrical results on the map $(A, \mathcal{S}) \to P_{A,\mathcal{S}}$. The notion of compatibility appears when A is allowed to be any positive semidefinite operator, not necessarily invertible (and even, a selfadjoint bounded linear operator). More precisely, A and $\mathcal{S}$ are said to be *compatible* if there exists a (bounded linear) projection Q with image $\mathcal{S}$ which satisfies $AQ = Q^*A$ (i.e., Q is Hermitian with respect to the semi-inner product $< .,. >_A$). Unlike what happens for invertible A's, it may happen that there is no such Q. These perturbations of the inner product occur quite frequently in applications. The consistent sampling scheme has not been studied as acting on perturbed inner spaces. But, studying the consistent sampling scheme in the semi-inner product spaces allows a simpler way to study some problems related with this notion.

The talk is based on a joint work with G. Corach.

Noncommutative Markov chains and multivariate operator theory

R. Gohm

We explain some recently found connections between mathematical models for open quantum systems and constructions for noncommuting tuples of operators. We illustrate these ideas in a model of repeated interaction between quantum systems which can be thought of as a noncommutative Markov chain. It is shown that there exists an outgoing Cuntz scattering system (as considered by Ball and Vinnikov) associated to this model. This induces an input-output formalism with a transfer function corresponding to an analytic intertwining operator. Finally we show that observability for this system is closely related to a scattering theory of noncommutative Markov chains (as considered by Kümmerer and Maassen).

A REPRESENTATION OF THE HEISENBERG GROUP BY OPERATORS ACTING ON PHASE SPACE; APPLICATIONS

M. de Gosson

The Heisenberg–Weyl operators $\widehat{T}(z_0) = e^{-\frac{i}{\hbar}\sigma(\hat{z},z_0)}$, which act on $L^2(\mathbb{R}^n)$, lead to an irreducible representation (the "Schrödinger representation") on $L^2(\mathbb{R}^n)$ of the Heisenberg group $\mathbb{H}_n$. The Schrödinger representation leads to the Weyl quantization procedure associating to a symbol a an operator $\widehat{A} = a(x, -i\partial_x)$. The Stone–von Neumann theorem is often invoked to claim that this is the only possible irreducible representation of $\mathbb{H}_n$. In this talk we show that there is a class of operators $\widetilde{T}(z_0)$ acting on $L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$ and corresponding to infinitely many intertwined representations of $\mathbb{H}_n$ on closed subspaces of $L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$. The operators $\widetilde{T}(z_0)$ lead to phase-space pseudo-differential operators formally given by $\widetilde{A} = a\left(x + \frac{1}{2}i\partial_y, y - \frac{1}{2}i\partial_x\right)$ ("Bopp operators"). These operators are intertwined with $\widehat{A} = a(x, -i\partial_x)$ by an infinite family of partial isometries $L^2(\mathbb{R}^n) \longrightarrow L^2(\mathbb{R}^n \oplus \mathbb{R}^n)$. We study the spectral properties of $\widetilde{A}$ when the symbol a belongs to a certain Shubin class, and apply our results to two examples: magnetic operators and Moyal's star-product.

This talk is based on the papers:

(1) M. de Gosson: Spectral Properties of a Class of Generalized Landau Operators. *Communications in Partial Differential Equations*, **33**(11) 2008.

(2) M. de Gosson and F. Luef: Spectral and Regularity properties of a Pseudo-Differential calculus Related to Landau Quantization. *Journal of Pseudo-Differential Operators and Applications* **1**(1), 2010.

MAXIMAL L^p-REGULARITY FOR A FLUID-SOLID INTERACTION PROBLEM

K. Götze

We consider the system of equations governing the coupled motion of a rigid body in a viscous incompressible fluid. Using maximal regularity of the Stokes problem, the linearized system can be reduced to a linear equation in $W^{1,p}(0, T; \mathbb{R}^6)$.

Oka's principle in the Fredholm theory for Frechet algebras of Fourier operators

B. Gramsch

The classical result of the homotopy transition from continuous to holomorphic maps (Oka) from holomorphy regions to Fredholm operators on Banach spaces (1971, 1984) is extended in several directions. We include in the theory the symmetric Hrmander class (1,1) of pseudodifferential operators. This class is known to be not spectrally invariant; it has not an open group of invertible elements; but using commutator methods involving hard analysis the submultiplicativity of this class(1,1) can be shown. This is the key to apply infinite products and meromorphic decompositions. The Arens- Royden theorem, a version of the Oka principle for Banach algebras, is extended for maps with values in various sets of Fredholm operators. The fundamental contributions of Grauert (1957) and Gromov (1989) to the Oka principle had an essential revival during the last decade by the work of Forstneric et al. and Lempert et al. (see e.g. Notices AmS,vol.57 2010,p. 50- 52 and Ann. Sc. Ec. Norm. Sup, vol. 40, 2007). Also the book of I. Gohberg and J. Leiterer (Birkhuser 2009) is an excellent starting point for the extension of the Oka principle to maps with values in the set of Fredholm operators. These contributions lead to a series of challenging problems also for operators in Hilbert spaces with maps from infinite dimensional holomorphy regions into Oka manifolds (see above Notices AMS). The operator methods can be applied to treat microlocal Frchet algebras on ramified or stratified manifolds using Lie group representations and commutator procedures with vector fields. For this purpose the Leibniz - Nelson product rule for closed operators and there commutators is analysed with an appropriate differentiability producing submultiplicative norms. There are relations to the dissertation of J. Ditsche and the work of M. Denz (Mainz 2007, 2008).

Reduced resolvent formula and weak operator equations, a bridge between block operator matrices and numerical linear algebra

L. Grubišić

We study the stability of eigenvalues and eigenvectors of self-adjoint operators which are defined by quadratic forms under a large class of singular perturbations. We divide the spectrum of an operator into the target component and the unwanted component of the spectrum. This dichotomy induces a natural block operator matrix representation of the associated resolvent. The associated spectral projections form a partition of unity. Furthermore, a similar block operator is induced by any partition of unity such that the intersection of the range of projection and the form domain of the operator is dense. We study such resolvent-s using basic matrix factorizations of numerical linear algebra and show how in an infinite dimensional setting the nonexistence of a factorization can be used to characterize a certain operator theoretic phenomenon. We revisit the use of techniques such as Schur complements, operator Riccati and Sylvester equations to quantitatively study the perturbation theory of spectra of operators defined as quadratic forms. Some applications will be presented which include both a study of the asymptotic exactness of error estimators employed in modern adaptive finite element approximations as well as asymptotic study of spectral properties in the limit of large penalty (e.g. as employed in the study of Maxwell or Stokes eigenvalue problems). This is a joint work with V. Kostrykin, K. Makarov, J. Ovall and K. Veselic.

Transformation operators for Schrödinger operators on infinite-gap backgrounds

K. Grunert

Transformation operators which preserve the asymptotic behavior at infinity are the main tool for considering different kinds of direct and inverse scattering problems. We present an investigation of the transformation operators for one-dimensional Schrödinger operators with potentials, which are asymptotically close to almost periodic infinite-gap potentials. At the end we will give an outlook on scattering theory in that case.

$\mathcal{PT}-$symmetry, Cartan decompositions, Lie triple systems and Krein space related Clifford algebras

U. Günther

Gauged $\mathcal{PT}$ quantum mechanics (PTQM) and corresponding Krein space setups are studied. For models with constant non-Abelian gauge potentials and extended parity inversions compact and noncompact Lie group components are analyzed via Cartan decompositions. A Lie-triple structure is found and an interpretation as $\mathcal{PT}-$symmetrically generalized Jaynes-Cummings model is possible with close relation to recently studied cavity QED setups with transmon states in multilevel artificial atoms. For models with Abelian gauge potentials a hidden Clifford algebra structure is found and used to obtain the fundamental symmetry of Krein space related $J-$selfadjoint extensions for PTQM setups with ultra-localized potentials.

The talk is based on arXiv:1006.1134[math-ph] – a joint work with S. Kuzhel.

On some questions of operator theory

M. Gürdal

In the present talk we mainly give some new applications of Berezin symbols technique. In particular, the Berezin symbol is used in approximation problem for H^{∞}-functions. We also study asymptotic multiplicativity of the Berezin symbols. Moreover, we investigate solvability of some Riccati operator equations of the form $XAX + XB - CX = D$ on the Toeplitz algebra $\mathcal{T}$, which is the C^*-subalgebra of the operator algebra $\mathcal{B}\left(L_a^2\right)$ generated by Toeplitz operators $\{Tg : g \in H^{\infty}(\mathbb{D})\}$ on the Bergman space $L_a^2(\mathbb{D})$ over the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$. We characterize compact truncated Toeplitz operators $A_{\varphi} := P_{K_{\theta}} T_{\varphi} | K_{\theta}$,

$\varphi \in L^{\infty}(\partial \mathbb{D})$, in terms of Berezin symbols. The spectrum of model operators $\varphi(M_\theta)$, $\varphi \in H^{\infty}(\mathbb{D})$, is localized in terms of so-called Berezin set by proving that $\sigma(\varphi(M_\theta)) \subset closBer(\varphi(M_\theta))$. Some other questions are also discussed.

This is a joint work with M.T. Karaev.

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Series expansions of monogenic functions

K. Gürlebeck

Recently complete orthonormal Appell systems of monogenic polynomials were introduced by several authors. These systems are used to define Taylor type series expansions based on the hypercomplex derivability of monogenic functions. With these results we study in the talk weighted function spaces of monogenic functions and characterize the functions belonging to such spaces equivalently by properties of their Taylor or Fourier coefficients, respectively. It will be shown that the relations between the coefficients can be used also to prove some function theoretic theorems, like the Bohr theorem and the Bloch theorem for monogenic functions.

The talk is based on a joint work with S. Bock and J. Morais.

Admissibilty for Volterra systems with scalar kernels

B.H. Haak

The purpose of this article is to present conditions for the admissibility of observation operators to parabolic Volterra equations, that is, we consider the 'observed' system

$$\begin{aligned} x(t) &= x_0 + \int_0^t a(t-s)\, Ax(s)\, ds, \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $t \geq 0$. Here, the operator A is supposed to be a closed operator with dense domain on a Banach space X, $x_0 \in X$, the kernel function

$a \in L^1_{\rm loc}$ is supposed to be of sub-exponential growth and 1-regular, and it is assumed that (1) is parabolic. In Prüss [7] it is shown that under these assumptions, equation (1) admit a unique *solution family*, i.e. a family of bounded linear operators $(S(t))_{t\geq 0}$ on X.

For some results we need in addition that $-A$ a sectorial operator of type $\omega \in (0,\pi)$ or that the kernel a is *sectorial of angle* $\theta \in (0,\pi)$. The kernel a is called *sectorial of angle* $\theta \in (0,\pi)$ if

$$\widehat{a}(\lambda) \in S_\theta \qquad \text{for all } \lambda \text{ with positive real part.}$$

In particular, when $-A$ and a are both sectorial in the respective sense with angles that sum up to a constant strictly inferior to π, the Volterra equation is parabolic.

The operator C is supposed to be an operator from X into another Banach space Y that acts as a bounded operator from $X_1 \to Y$ where $X_1 = \mathcal{D}(A)$ is endowed by the graph norm of A. In order to guarantee that the output function lies locally in L_2 we are interested in the following property.

Definition 1. A bounded linear operator $C : X_1 \to Y$ is called *finite-time admissible* for the Volterra equation (1) if there are constants $\eta, K > 0$ such that

$$\left(\int_0^t \|CS(r)x\|^2\, dr\right)^{\frac{1}{2}} \leq Ke^{\eta t}\|x\|$$

for all $t \geq 0$ and all $x \in \mathcal{D}(A)$.

The notion of admissible observation operators is well studied in the literature for Cauchy systems, that is, $a \equiv 1$, see for example [3], [8], and [9]. Admissible observation operators for Volterra systems are studied in [2], [4], [5] and [6].

The Laplace transform of S, denoted by H, is given by

$$H(\lambda)x = \frac{1}{\lambda}(I - \hat{a}(\lambda)A)^{-1}x, \qquad \operatorname{Re}\lambda > 0.$$

Our first main result, Theorem 2 provides a subordination argument to obtain admissibility for the observed Volterra equation from the admissibility of the observation operator for the underlying Cauchy problem.

Theorem 2. Let A generate an exponentially stable strongly continuous semigroup $(T(t))_{t\geq 0}$ and let $C : X_1 \to Y$ be bounded. Further we assume that the kernel $a \in L^1_{loc}(\mathbb{R}_+)$ is of sub-exponential growth, 1-regular and sectorial of angle $\theta < \pi/2$. Then finite-time admissibility of C for the semigroup $(T(t))_{t\geq 0}$ implies that of C for the solution family $(S(t))_{t\geq 0}$.

This allows a large number of corollaries, based on positive results for the Weiss conjecture. Here we only mention the following.

Corollary 3. Assume in addition to the hypotheses of the theorem that A admits a Riesz basis of eigenfunctions (e_n) on a Hilbert space X with corresponding eigenvalues λ_n. If $Y = \mathbb{C}$ and if

$$\mu = \sum_n |Ce_n|^2 \delta_{-\lambda_n}$$

is a Carleson measure on $\mathbb{C}_+$, then C is finite-time admissible for the solution family $(S(t))_{t\geq 0}$.

This corollary improves a direct Carleson measure criterion from Haak, Jacob, Partington and Pott [2]. Our second main result provides a sufficient condition for admissibility.

Theorem 4. Assume that A is a closed operator with dense domain on X, the kernel function $a \in L^1_{\mathrm{loc}}$ is of sub-exponential growth, 1-regular, and (1) is parabolic. Let $C : X_1 \to Y$ be bounded and assume that for some $\alpha > \frac{1}{2}$,

$$\sup_{r>0} \left\| \ (1 + \log^+ r)^\alpha r^{\frac{1}{2}} CH(r) \right\| < \infty. \qquad (2)$$

Then C is finite-time admissible for $(S(t))_{t\geq 0}$.

The obtained results are applied to time-fractional diffusion equations of distributed order and are compared with other results on Volterra systems known so far.

This is a joint work with B. Jacob.

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Besov class calculi for discrete and continuous operator semigroups

M. Haase

It is well-known that the class of power-bounded operators on a Hilbert space is less well-behaved than the class of contractions. For example, for a contraction T on a Hilbert space H von Neumann's inequality states that one can estimate

$$\|p(T)\|_{\mathcal{L}(H)} \leq \|p\|_{H^\infty(\mathbb{D})}$$

for every polynomial $p \in \mathbb{C}[z]$, but even the weaker inequality

$$\|p(T)\|_{\mathcal{L}(H)} \lesssim \|p\|_{H^\infty(\mathbb{D})}$$

fails for a general power-bounded operator. Peller (1982) has proved that for a power-bounded operator T on a Hilbert space one can estimate

$$\|p(T)\|_{\mathcal{L}(H)} \lesssim \|p\|_{B^0_{\infty,1}} := \sum_{n\geq 0} \|\varphi_n * p\|_{H^\infty(\mathbb{D})},$$

where $(\widehat{\varphi_n})_{n\geq 0}$ is a dyadic partition of unity of $\mathbb{Z}_+$, and $*$ is convolution on $\mathbb{T}$. This Besov class estimate is considerably stronger than the obvious estimate $\|p(T)\|_{\mathcal{L}(X)} \lesssim \|\widehat{p}\|_{\ell^1}$, best possible in the class of all power-bounded operators on Banach spaces.

Based on recently discovered *transference principles* for semigroups we present a completely different proof of Peller's result and establish several generalizations. First, we show that an analogous theorem holds for C_0-semigroups on Hilbert spaces. Second we establish general Banach space versions of these results involving the concept of γ-radonifying operators, introduced by Kalton and Weis in 2004. Finally, we sketch generalizations to operators/semigroups on L_p-spaces or, more general, UMD Banach spaces. In particular, it follows from our results that the singular integral

$$\mathrm{PV} - \int_0^{t+1} \frac{T(s)x}{s-t}\, ds \qquad (x \in X,\, t > 0).$$

converges whenever $(T(s))_{s\geq 0}$ is a C_0-semigroup on a UMD Banach space.

The talk is based on our paper [Math. Ann. 345 (2) (2009), 245-265] and the recent preprint *Transference Principles for Semigroups and a Theorem of Peller.*

The parabolic Harnack inequality for metric graphs

S. Haeseler

We consider energy forms with general weights on metric graphs. We study the intrinsic metric, volume doubling and a Poincar inequality. This enables us to prove a parabolic Harnack inequality. The proof involves various techniques from the theory of strongly local Dirichlet forms.

Non-negativity analysis for exponential-polynomial-trigonometric functions

B. Hanzon

The class of exponential-polynomial-trigonometric (EPT) functions is ubiquitous in the mathematical sciences. It is the class of functions that appear as solutions to linear differential equations with constant coefficients. In linear dynamical systems theory they appear as impulse response functions and that context are often represented as $y(t) = ce^{At}b$, where c is a row vector, A a square matrix and b a column vector. The name EPT functions draws on the fact that these functions can be written in the form $\sum_{i=0}^{d} p_i(t)e^{\lambda_i t}\cos(\theta_i t)$, where the p_i are polynomials and λ_i and θ_i are real numbers. These functions also appear in probability theory and financial mathematics (e.g. as forward rate curves). In such applications non-negativity is often required. In the present paper we address the question of how to characterize non-negative EPT functions. We describe necessary conditions and we present a new sufficient condition and methods for verification of this sufficient condition. The main idea is to represent an EPT function as the product of a row vector $b(t)$ of EP functions and a column vector $F(t) := f(e^{i\theta_1 t}, \ldots, e^{i\theta_m t})$ of multivariate trigonometric polynomials with unimodular exponential functions $e^{i\theta_k t},\ k = 1, 2, \ldots, m$ as arguments; here $\theta_k,\ k = 1, 2, \ldots, m$ are chosen to be real numbers which are linearly independent over the set of rational numbers $\mathbf{Q}$. From the theory of almost periodic functions it follows that the closure of the set of vectors $F(t)$ obtained by varying t over the interval $[T, \infty)$ for any $T > 0$, is equal to the set $f(\mathbf{T}^m)$, where $\mathbf{T}^m$ denotes the $m-$dimensional unit torus in $\mathbf{C}^\mathbf{m}$. This will be used to describe a sufficient condition. Two methods for verifying whether the sufficient condition is satisfied are presented, one based on minimization over the torus of a continuous level function which is non-negative iff the sufficient condition is satisfied; the other based on algebraic optimization techniques, involving Groebner basis and Sturm chain calculations. The methods are based on the availability of a generalized Budan-Fourier sequence technique to determine the minimum of an EPT function on a given finite interval $[0, T]$ which is presented elsewhere by the authors.

The presentation is based on joint work with F. Holland.

On some applications of pseudo-differential calculus in quantum mechanic

G. Harutyunyan

We study a new approach to determine the asymptotic behaviour of quantum many-particle systems near coalescence points of particles which interact via singular Coulomb potentials. This problem is of fundamental interest in electronic structure theory in order to establish accurate and efficient models for numerical simulations. Within our approach, coalescence points of particles are treated as embedded geometric singularities in the configuration space of electrons. Based on a general singular pseudo-differential calculus, we provide a recursive scheme for the calculation of the parametrix and corresponding Green operator of a nonrelativistic Hamiltonian. In our singular calculus, the Green operator encodes all the asymptotic information of the eigenfunctions. Explicit calculations and an asymptotic representation for the Green operator of the hydrogen atom and isoelectronic ions are presented.

This talk is based on a joint work with H. Flad, R. Schneider and B.-W. Schulze.

Compactness for the $\overline{\partial}$ - Neumann problem - a functional analysis approach

F. Haslinger

We characterize compactness of the $\overline{\partial}$-Neumann operator for a smoothly bounded pseudoconvex domain and in the setting of weighted L^2-spaces on $\mathbb{C}^n$. For this purpose we use a description of relatively compact subsets of L^2- spaces. We also point out how to use this method to show that property (P) implies compactness for the $\overline{\partial}$-Neumann operator on a smoothly bounded pseudoconvex domain.

Boundary relations and generalized resolvents of symmetric operators

S. Hassi

The Kreĭn-Naĭmark formula provides a parametrization of all selfadjoint exit space extensions of a, not necessarily densely defined, symmetric operator, in terms of maximal dissipative (in $\mathbb{C}_+$) holomorphic linear relations on the parameter space (the so-called Nevanlinna families). These parameter families can be interpreted as Weyl families of boundary relations, which were introduced in [1]. Using a coupling method of boundary relations (see [2]) the Kreĭn-Naĭmark formula for the generalized resolvents corresponding to the given parameter families can be easily constructed. This approach can be used to investigate various problems involving generalized resolvents and their applications; for instance, an old problem, going back to M.A. Naĭmark, concerning the analytical characterization of so-called Naĭmark extensions is solved.

The talk is based on a joint work with V.A. Derkach, M.M. Malamud, and H.S.V. de Snoo.

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A characterization of the bounded derivations from the disk algebra to its dual

M.J. Heath

We show that the space of all bounded derivations from the disk algebra into its dual can be identified with the Hardy space H^1; using this, we infer that all such derivations are compact. Also, given a fixed derivation D, we construct a finite, positive Borel measure μ_D on the closed disk, such that D factors through $L^2(\mu_D)$. Such a measure is known to exist, for any bounded linear map from the disk algebra to its dual, by results of Bourgain and Pietsch, but these results are highly non-constructive.

This talk is based on joint work with Y. Choi.

Hahn-Banach type theorems for normed modules

A.Ya. Helemskii

Let A be a normed algebra, $\mathcal{K}$ some class of left normed A-modules. A left normed A-module Z is called *extremely $\mathcal{K}$-injective* if, for every A-module Y and its submodule X, every bounded morphism $X \to Z$ can be extended to a morphism $Y \to Z$ of the same norm. (Thus, Z plays the role of $\mathbb{C}$ in the classical Hahn-Banach theorem).

In the following theorem we consider, as A, the algebra $\mathcal{B}(L)$ of all bounded operators on an infinite-dimensional Hilbert space L, and, as $\mathcal{K}$, the class of left Ruan modules (those X with the property $\|u+v\| \leq \sqrt{\|u\|^2+\|v\|^2}$, provided $u, v \in X$ satisfy $u = P \cdot u, v = Q \cdot v$ for some mutually orthogonal projections $P, Q \in \mathcal{B}(L)$). These modules were introduced in connection with attempts to obtain a transparent proof of the Arveson-Wittstock Extension Theorem, one of fundamental principles of quantum functional analysis.

Theorem 1. *Let H be an arbitrary Hilbert space, and $L \otimes H$ a Hilbert A-module with the outer multiplication $a \cdot (\xi \otimes \eta) := a(\xi) \otimes \eta$. Then such a module is extremely $\mathcal{K}$-injective.*

This theorem, combined with some general facts about Ruan modules, gives, as an easy corollary, Arveson-Wittstock Theorem.

Later Wittstock generalized and strengthened the formulated theorem in several directions. In particular, he proved that, with A and $\mathcal{K}$ as above, every dual to a Ruan module is $\mathcal{K}$-injective.

Turn to the opposite class of commutative algebras. What about modules over one of the simplest, the algebra c_0 of vanishing sequences? The following theorem describes extremely $\mathcal{K}$-injective modules within a certain reasonable class of c_0-modules. Namely, we call a c_0-module Z *homogeneous*, if, for $z', z'' \in Z$, the equalities $\|p^n \cdot z'\| = \|p^n \cdot z''\|; n = 1, 2, \ldots$, where $p^n = (0, \ldots, 0, 1, 0, \ldots)$, imply $\|z'\| = \|z''\|$.

Theorem 2. *Let $\mathcal{K}$ be a class of homogeneous c_0-modules, and Z is a non-degenerate homogeneous c_0-module. Then the module Z^* is extremely $\mathcal{K}$-injective if, and only if, for every $n = 1, 2, ..,$ the normed space $\{p^n \cdot z; z \in Z\}$ is, up to an isometric isomorphism, a dense subspace of $L_1(\Omega_n)$ for some measure space Ω_n.*

In particular, all c_0-modules $l_p; 1 \leq p \leq \infty$ are extremely $\mathcal{K}$-injective.

The condition of the non-degeneracy of Z can not be omitted: $Z := l_\infty$ provides the relevant counter-example.

One of basic tools of the proof of both theorems is the algebraic "law of adjoint associativity", properly modified to serve in functional analysis.

Convex Matrix Inequalities vs Linear Matrix Inequalities

B. Helton

A substantial advance in optimization starting in the 1990's was the realization that problems in many areas, like linear system control, combinatorics, statistics convert directly to matrix inequalities, abbreviated MIs, of which by dent of great cleverness some convert to Linear Matrix Inequalities, LMI's. A basic question is: which Matrix Inequalities are in fact Linear Matrix Inequalities? Clearly, LMIs are convex, but what about the converse?

How much more restricted are LMIs than Convex MIs?

There is getting to be a reasonable road map to this problem with much left to be proved. It involves use and development of techniques from areas like functional analysis, real algebraic geometry (polynomial inequalities) and matrix theory. In this talk we give results and conjectures on the answer to the LMI vs convexity question.

A further direction involves matrix variables and is the issue of transforming problems to convex ones. This leads to noncommutative versions of classical theorems in several complex variables.

A smoothing property of the Bergman projection

A.-K. Herbig

Let $\Omega \subset\subset \mathbb{C}^n$ be a smoothly bounded, strictly pseudoconvex domain.

We show that, measured in $L^2(\Omega)$, the derivatives of the output of the Bergman projection only depend on derivatives of the input in the so-called bad tangential direction. This lets us describe a family of functions on Ω, strictly larger than $C^\infty(\overline{\Omega})$, whose image under the Bergman projection is contained in $C^\infty(\overline{\Omega})$. This is joint work with J. McNeal.

Asymptotic behaviour of the Stokes-Coriolis-Ekman system

M. Hieber

In this talk we investigate the long time behaviour of the nonlinear Stokes-Coriolis-Ekman system on the halfspace or in an infinite layer. Of central importance are new strong stability results for the linear Stokes semigroup subject to lower order perturbations which allow furthermore to determine precise decay rates of the underlying system. Generalizations of these results to semigroups acting on arbitrary Banach spaces will be given in addition.

This talk is partly based on joint work with M. Hess, A. Mahalov and J. Saal.

The low-energy behaviour of Regge poles

A. Hiscox

We investigate the behaviour of Regge poles in the low-energy limit and show that for a potential such that $|(1+r)V(r)|$ is integrable, the associated Regge poles tend either to the spectral points of the limiting self-adjoint problem or to infinity. This confirms the experimental results which show that Regge poles formed during low-energy electron elastic scattering become stable bound states.

The talk is based on work supervised by M. Marletta and B. M. Brown.

Zeros of entire functions: from René Descartes to Mark Krein and beyond

O. Holtz

The central question of many classical investigations, going back to Descartes, Newton, Euler, and others, is finding zeros of entire and meromorphic functions, given some standard representation of such a function, e.g., its coefficients in some standard basis. Specific questions of this type include zero localization with respect to a given curve (e.g., stability and hyperbolicity), behavior of zeros under special maps (e.g., differentiation, Hadamard product), and relations among roots of function families (e.g., orthogonal polynomials). The point of this talk is to give an overview of matrix and operator methods in this area, emphasizing old and new connections between algebra and analysis. The novel results in this talk are joint with M. Tyaglov.

Inverse scattering on the line for Schrödinger operators with singular Miura potentials

R. Hryniv

We study direct and inverse scattering problems for one-dimensional Schrödinger operators with highly singular Miura potentials $q \in H^{-1}(\mathbb{R})$, i.e., potentials of the form $q = u' + u^2$ for some $u \in L_2(\mathbb{R})$. Under some additional assumptions there exist unique Riccati representatives u_+ and u_- that are integrable respectively at $+\infty$ and $-\infty$, and there is a well-defined reflection coefficient r that determines u_+ and u_- uniquely. We show that the map $(u_+, u_-) \mapsto r$ is continuous with continuous inverse and obtain an explicit reconstruction formula. Among potentials included are, e.g., potentials of Marchenko–Faddeev class and their perturbations by compactly supported distributions from $H^{-1}(\mathbb{R})$ (e.g., delta-functions and regularized Coulomb $1/x$-type interactions) and some highly oscillating unbounded potentials.

The talk is based on a joint project with Ch. Frayer (Lexington, KY, USA), Ya. Mykytyuk (Lviv, Ukraine), and P. Perry (Lexington, KY, USA).

Scattering problems in fluid-structure interaction

G.C. Hsiao

We are concerned with the direct and inverse scattering problems in fluid-structure interaction. Scattering problem in the fluid-structure interaction can be simply described as follows: an acoustic wave propagates in the fluid domain of infinite extent where a bounded elastic body is immersed. The direct problem is to determine the scattered pressure and velocity fields in the fluid domain as well as the displacement fields in the elastic body, while the inverse problem is to reconstruct the shape of the elastic scatterer from a knowledge of the far filed pattern of the fluid pressure or from the measured of scattered fluid pressure filed. As is well known, the inverse problems are generally nonlinear and highly ill-posed. For treating inverse problem of this kind, we reformulate the problem as a nonlinear optimization problem including special regularization terms. The precise formulation of the nonlinear objective functional will depend on the approaches of the direct problem. In this lecture we will present various approaches for the direct problem and their corresponding formulations of the inverse problem. Emphasis will be placed upon the mathematical foundations of the variational formulations of the corresponding problems. The talk is based on joint work with J. Elschner and A. Rathsfeld of WIAS in Berlin,

Decomposition of function space on non-rectifiable curve

L. Hua

After comparing with each other between Kats and Stiljies integrals we define the integrable function spaces on the curve. After we discuss the singular integral operator and the decompostion of function space on the snowflake curve. As the role of ax+b group in the singular integral theory of one real variable, it is studied for the representation of symmetry group of snowflake. Then we get the decomposition of the square integrable functions, from which the suitable definition of singular integral operator appears.

On a linear neutral integro-differential equation

A.D. Ioannidis

Suppose that we are given

- a Banach space X,
- linear operators A, B, $(K(t))_{t\geqslant 0}$ in X, and
- an $x \in X$.

We study the following initial value problem: to find a function $u : \mathbb{R}_+ \to X$ which satisfies

$$\begin{cases} \dfrac{\mathrm{d}}{\mathrm{dt}}\left(Bu(t) - \displaystyle\int_0^t K(t-s)u(s)\,ds \right) = Au(t)\ ,\ t \geqslant 0 \\ u(0) = x \end{cases} \tag{IVP}$$

We pay special attention in the case when A is the generator of a C_0-semigroup. Our aim is to find conditions on B, $K(\cdot)$ and x in order (IVP) to be well-posed. The idea is, by using a variation-of-constants procedure, to rewrite (IVP) as a Volterra integral equation in X.

This work uses results from the author's PhD thesis and is an abstraction of a problem arising in Electromagnetic Theory. Collaboration with G. Kristensson and I. Stratis is acknowledged.

Compact Toeplitz operators for weighted Bergman spaces on bounded symmetric domains

H. Issa

It is well known that the Berezin transform $\tilde{g}^t$ of the Toeplitz operator T_g^t acting on the Segal-Bargmann space of square integrable entire functions with respect to a time dependent Gaussian measure, is the solution of the heat equation at time $t > 0$ with initial data g. As it was recently shown, if g is a function on $\mathbb{C}^n$ with a bounded mean oscillation, $\tilde{g}^{t_0}$ vanishes at infinity for **some fixed** time t_0 if and only if $\tilde{g}^t$ vanishes at infinity for **any** time $t > 0$.

If we replace $\mathbb{C}^n$ by a bounded symmetric domain Ω , we can still define the weighted Berezin transform $\tilde{g}_\nu$ of the Toeplitz operator T_g^ν acting on the weighted Bergman space of Ω. This Berezin transform generalizes the heat transform on $\mathbb{C}^n$ where the time parameter is replaced by the weight parameter. In this talk we consider $\Omega \subset \mathbb{C}^n$ of type (r, a, b) in its Harish-Chandra realization. Under some conditions on the weights ν and ν_0 we show the existance of $C(\nu, \nu_0) > 0$, such that

$$\|\tilde{g}_{\nu_0}\|_\infty \leq C(\nu, \nu_0) \left\|T_g^\nu\right\|_\nu$$

for all g in a suitable class of symbols containing $L^\infty(\Omega)$. As a consequence we prove that the compactness of T_g^ν (or the vanishing of $\tilde{g}_\nu$ near the boundary of Ω) is **independent** of the weight ν, whenever $g \in L^\infty(\Omega)$ and $\nu > C$ where C is a constant depending on (r, a, b).

Weighted interpolation in Paley–Wiener spaces and finite-time controllability

B. Jacob

We consider the solution of weighted interpolation problems in model subspaces of the Hardy space H^2 that are canonically isometric to Paley–Wiener spaces of analytic functions. A new necessary and sufficient condition is given on the set of interpolation points which guarantees that a solution in H^2 can be transferred to a solution in a model space. The techniques used rely on the reproducing kernel thesis for Hankel operators, which is given here with an explicit constant. One of the applications of this work is to the finite-time controllability of diagonal systems specified by a C_0 semigroup.

The talk is based on a joint work with J.R. Partington (Leeds) and S. Pott (Lund).

H-expansive matrices in indefinite inner product spaces and their invariant subspaces

D. Janse van Rensburg

We considered indefinite inner products given by a square real invertible symmetric matrix $H = H^T : [x, y] = (Hx, y)$. On the Euclidean space equipped with this indefinite inner product, we consider matrices A for which $A^*HA - H$ is nonnegative. Such matrices are called H-expansive matrices.

We are interested in the construction of complex (as well as real) A-invariant maximal H-nonnegative and nonpositive subspaces. The complex case has already been shown if one uses a suitable Cayley transform. The problem arises when A is real and $A^T HA - H$ is nonnegative and A has both 1 and -1 as eigenvalues. The uniqueness and stability of these subspaces are also studied.

The talk is based on a joint work with J.H. Fourie, G.J. Groenewald and A.C.M. Ran.

Krein systems on a finite interval: accelerants and continuous potentials

M.A. Kaashoek

The notion of an accelerant has been introduced by M.G. Krein in the mid fifties. By definition it is a continuous function k on a finite interval $[-\mathbf{T}, \mathbf{T}]$ such that the convolution integral operator

$$(Tf)(t) = f(t) - \int_0^{\mathbf{T}} k(t-s)f(s)\,ds \quad (0 \le t \le \mathbf{T})$$

is positive definite on $L^2(0, \mathbf{T})$. For such a function k the corresponding canonical system of Krein type has a continuous potential on $[0, \mathbf{T}]$. In this talk we deal with the inverse problem: Is each continuous potential on $[0, \mathbf{T}]$ generated by an accelerant. We shall see that after an appropriate modification of the definition of an accelerant the answer is positive, even for matrix-valued kernel functions. Moreover, the accelerant is uniquely determined by the potential. The talk is based on joint work with D. Alpay, I. Gohberg (Z″L), L. Lerer, and A.L. Sakhnovich.

Canonical factorization of rational matrix functions on the unit circle revisited

M.A. Kaashoek

Canonical factorization of a rational $m \times m$ matrix function on the unit circle will be described explicitly in terms of a stabilizing solution of a discrete algebraic Riccati equation using a special state space representation of the given function. The corresponding difference equation and its relation with the finite section method will also be discussed. The talk is based on joint work with A.E. Frazho and A.C.M. Ran.

Discrete skew self-adjoint canonical systems with rational Weyl functions

M.A. Kaashoek

A discrete analog of a skew self-adjoint canonical system with a so-called pseudo-exponential potential will be presented. The associate Weyl function is a proper square rational matrix function. The corresponding direct and inverse problem will be solved explicitly using state space techniques from mathematical system theory. As an application explicit solutions will be given of a discrete integrable nonlinear equation related to the isotropic Heisenberg magnet model. The talk is based on joint work with A.L. Sakhnovich.

Special Skew-Weyl relations and discrete time-frequency analysis

U. Kähler

In recent years one can observe an increasing interest in obtaining discrete counterparts for various continuous structures. One of the principal topics is the construction of discrete Dirac operators and its corresponding function theory. If we look into the continuous case one of the principal approaches is the one developed by F. Sommen which is based on Weyl relations and on the study of the algebra of endomorphisms. But a direct application of this method by using discrete Weyl relations creates some rather difficult problems since one has to work with two different relations for forward and backward difference operators (instead of a single partial derivative operator). For instance, while it is possible to construct Dirac operators which factorize the star Laplacian, the resulting vector variable operators from the Weyl relation are not creating a second-order scalar operator as in the continuous case. Furthermore, they do not commute like foward/backward differences, thereby creating a different algebraic structure. One way to overcome this problem is to use a special type of skew-Weyl relations, so-called Sommen-Weyl relations. In this talk we will show the advantages of such an approach in constructing discrete function theories and study its links with Umbral calculus and (discrete) time-frequency analysis.

Noncommutative Functions: Algebraic and Analytic Results

D. Kaliuzhnyi-Verbovetskyi

Functions on d-tuples of square matrices of all sizes which take values on square matrices of same sizes and respect direct sums and joint similarities of matrices are called noncommutative functions over $\mathbb{C}^d$. A list of examples includes, but is not limited to, noncommutative polynomials or formal power series, and noncommutative rational expressions. A natural setting for the notion of a noncommutative function over a Banach space is provided in the context of operator spaces. Noncommutative functions arise in various areas of mathematics and its applications: noncommutative algebra, operator theory, free probability, noncommutative multidimensional system theory, robust control, optimization, etc. In this talk, a difference-differential calculus for noncommutative functions will be introduced, including a noncommutative version of the Taylor formula. Some algebraic and analytic applications of this calculus will be presented.

The talk is based on a joint work with V. Vinnikov.

Von Neumann inequalities with respect to weighted symmetric Fock spaces

H.T. Kaptanoğlu

The weighted symmetric Fock spaces considered can be realized as reproducing kernel Hilbert spaces of holomorphic functions in the unit ball of $\mathbb{C}^N$. Their kernels are $K_q(z,w) = (1-\langle z,w\rangle)^{-(1+N+q)}$ for $q > -(1+N)$, and hypergeometric functions for $q \le -(1+N)$. We also call them Dirichlet spaces, and the case $q = -N$ is the Drury-Arveson space. We obtain inequalities that place upper bounds on the norms of polynomials of row contractions of operators on arbitrary Hilbert spaces in terms of the norms of polynomials of shift operators on these spaces for all q.

Operator equalities as means for the study of singular integral operators with Carleman shift

A. Karelin

In the articles [1,2] we obtained a direct relation between singular integral operators A with a model involution and matrix characteristic singular integral operators: for an orientation-preserving shift it is a similarity transform $\mathcal{F}A\mathcal{F}^{-1}$ and for an orientation-reversing shift it is a transform by two invertible operators $\mathcal{H}A\mathcal{E}$. We will refer to the formulas as operator equalities.

Different applications of operator equalities to singular integral operators and to boundary value problems are considered.

In particular, in the space $L_2(\Gamma)$ we study a structure of the kernel of singular integral operators with involution

$$A_\Gamma = a_\Gamma I_\Gamma + c_\Gamma S_\Gamma + b_\Gamma W_\Gamma + d_\Gamma S_\Gamma W_\Gamma,$$

where Γ is the unit circle $\mathbb{T}$ o the real axis $\mathbb{R}$, coefficients are bounded measurable functions on Γ; $(W_\Gamma\varphi)(t) = \varphi(-t)$, $(I_\Gamma\varphi)(t) = \varphi(t)$, S_Γ is the Cauchy singular integral operator.

[1] A. A. Karelin, On a relation between singular integral operators with a Carleman linear-fractional shift and matrix characteristic operators without s hift, Boletin Soc. Mat. Mexicana Vol. 7 No. 12 (2001), pp. 235–246.

[2] A. Karelin, Aplications of operator equalities to singular integral operators and to Riemann boundary value problems, Math. Nachr. Vol. 280 No. 9-10 (2007), pp. 1108–1117.

The talk is based on a joint work with A. Tarasenko and G. Perez Lechuga

An algebra of convolution type operators with discontinuous data

Yu. Karlovych

Let $\mathfrak{B}_{p,w}$ be the Banach algebra of all bounded linear operators acting on the weighted Lebesgue space $L^p(\mathbb{R}, w)$ where $1 < p < \infty$ and w is a Muckenhoupt weight. We study the Banach subalgebra $\mathfrak{A}_{p,w}$ of $\mathfrak{B}_{p,w}$ generated by all operators of the form $a\mathcal{F}^{-1}b\mathcal{F}$ where $\mathcal{F}$ is the Fourier transform, the functions $a, b \in L^\infty(\mathbb{R})$ admit piecewise slowly oscillating discontinuities on $\mathbb{R}\cup\{\infty\}$ and b is the Fourier multiplier on $L^p(\mathbb{R}, w)$. Applying results on pseudodifferential operators with non-regular symbols, the Allan-Douglas local principle and the limit operators techniques, we construct a Fredholm symbol calculus for the Banach algebra $\mathfrak{A}_{p,w}$. As a result, a Fredholm criterion for the operators $A \in \mathfrak{A}_{p,w}$ in terms of their symbols is established.

The talk is based on a joint work with I. Loreto Hernández.

Structured Pseudospectra of Hamiltonian matrices

M. Karow

We consider the variation of the spectrum of Hamiltonian matrices under Hamiltonian perturbations. The first part of the talk deals with the associated structured pseudospectra. We show how to compute these sets and give some examples. In the second part we discuss the robustness of linear stability. In particular we determine the smallest norm of a perturbation that makes the perturbed Hamiltonian matrix unstable.

Total positivity and the Riemann zeta-function

O. Katkova

The sequence $\{a_k\}_{k=0}^{\infty}$ is called m-times positive, $m \in \mathbf{N}$, (totally positive), if all minors of order $\leq m$ (of any order) of the infinite matrix $A = (a_{j-i}), i, j = 0, 1, 2, \ldots$ ($a_k = 0$ for $k < 0$) are nonnegative. The class of corresponding generating functions $f(z) = \sum_{k=0}^{\infty} a_k z^k$ is denoted by $PF_m(PF_\infty)$.

In 1953 Aissen, Schoenberg, Whitney and Edrei obtained the full description of functions $f \in PF_\infty$: $f(z) = Cz^n e^{\gamma z} \prod_{k=1}^{\infty} \frac{1+\alpha_k z}{1-\beta_k z}$, where $C \geq 0, n \in \mathbf{Z}, \gamma \geq 0$, $\alpha_k \geq 0, \beta_k \geq 0, \sum(\alpha_k + \beta_k) < \infty$.

We consider the following function

$$\xi_1(z) = \frac{1}{2}(z - \frac{1}{4})\pi^{-\sqrt{z}/2-1/4}\Gamma(\sqrt{z}/2 + 1/4)\zeta(\sqrt{z} + 1/2),$$

where ζ is the Riemann ζ-function.

The function ξ_1 is an entire function of order $\frac{1}{2}$ and the Riemann Hypothesis is equivalent to the statement that ξ_1 has only real negative zeros. By Theorem of Aissen, Schoenberg, Whitney and Edrei it means that $\xi_1 \in PF_\infty = \cap_{m=1}^{\infty} PF_m$.

We obtain the following result.

Theorem. For all $m \in \mathbf{N}$ the sequence of coefficients of ξ_1 is asymptotically m-times positive, that is for all $m \in \mathbf{N}$ there exists a positive integer N such that all minors of matrix $A_N = (a_{N+j-i}), i = 0, 1, 2, \ldots, m-1, \quad j = o, 1, \ldots$ ($a_k = 0$ for $k < 0$) are nonnegative.

Riemann boundary value problem on non-rectifiable arcs and Cauchy transform

B.A. Kats

Let Γ be Jordan arc on the complex plane $\mathbb{C}$ with end points a_1 and a_2. We consider the Riemann boundary value problem on this arc, i.e., the problem on evaluation of holomorphic in $\overline{\mathbb{C}} \setminus \Gamma$ function $\Phi(z)$ satisfying equality

$$\Phi^+(t) = G(t)\Phi^-(t) + g(t), t \in \Gamma \setminus \{a_1, a_2\},$$

and certain restrictions on its behavior at the points a_1, a_2. Here $\Phi^{\pm}(t)$ are limit values of $\Phi(z)$ at a point $t \in \Gamma \setminus \{a_1, a_2\}$ from the left and from the right correspondingly, and functions G and g are given. In the simplest case $G \equiv 1$ the Riemann boundary value problem turns into so called jump problem.

This boundary value problem has a long history and a lot of applications, both traditional and new. The classic results on the problem are based on assumption, that the arc Γ is piecewise-smooth, or at least rectifiable. A solution of the jump problem under this assumption is Cauchy integral $\Phi(z) = (2\pi i)^{-1} \int_\Gamma g(t)(t-z)^{-1} dt$, and the Riemann boundary problem reduces to the jump problem by means of well-known factorization method.

In the present work we investigate this problem for non-rectifiable arcs. We introduce certain distributions with supports on non-rectifiable arc Γ, which generalize operation of weighted integration along this arc. Then we consider boundary behavior of Cauchy transforms of these distributions, i.e., their convolutions with $(2\pi i z)^{-1}$. As a result, we obtain description of solutions of the Riemann boundary value problem in terms of a new version of metric dimension of arc Γ, so called approximation dimension. It characterizes a rate of the best approximation of Γ by polygonal lines. For instance, if Γ is graph of a real function, then its approximation dimension is related with coefficients of Faber-Schauder expansion of this function.

Stieltjes functions and stable entire functions

V. Katsnelson

The function $\psi(z)$ belongs to the class $\boldsymbol{S}$ if ψ is holomorphic in the domain $\mathbb{C} \setminus (-\infty, 0]$ and satisfy the conditions: **1**. $\psi(x) \geq 0$ for $x > 0$; **2**. $\operatorname{Im} \psi(z) \leq 0$ for $\operatorname{Im} z \geq 0$. It is known that a function ψ belongs to the class $\boldsymbol{S}$ if and only if ψ admits the representation $\psi(z) = c + \int\limits_{-\infty}^{0} \dfrac{d\sigma(\lambda)}{\lambda + z}$, where $d\sigma$ is a non-negative measure satisfying the condition $\int\limits_{-\infty}^{0} \frac{d\sigma(\lambda)}{1+|\lambda|} < \infty$ and c is a non-negative constant.

If $\{a_j\}$, $\{b_j\}$ are two sequences of non-negative numbers which interlace: $0 \leq a_1 < b_1 < a_2 < b_2 < \ldots$, then the product $\psi(z) = \prod\limits_{j=1}^{\infty} \dfrac{z + a_j}{z + b_j}$

converges for $z \in \mathbb{C} \setminus (-\infty, 0)$ and defines a function of the class $\boldsymbol{S}$.

The following result is obtained: *If ψ is a function of the class $\boldsymbol{S}$, $\psi(+0) < \infty$, and*

$$f(z) = \sum_{k=0}^{\infty} \psi(k) \frac{z^k}{k!},$$

then f is a stable entire function of exponential type. In particular, all roots of f lie in the open left half plane.

This result allows to construct some new classes of meromorphic Laguerre multiplier sequences.

(The notion of meromorphic Laguerre multiplier sequence was introduced and discussed by T. Craven and G. Csordas in J. Math. Anal. Appl., **314** (2006), 109-125.)

B-evolution operators and its application for Partial differential equations on manifolds

S. Kerbal

In this talk we will give a brief introduction on B-evolution operators and their relation with semigroup of Operators. For illustration we will consider two mathematical problems: One arising in heat transfer problem in nuclear reactor and the other one in Structural vibration. The above problems will be written in an abstract setting, in other terms will be written as an ordinary differential equation in infinite dimensional space.

Operator semigroups of regular norm-behaviour

L. Kérchy

The regularity property, introduced, enables us to extend classical theorems on bounded representations of semigroups to a large setting. We are especially interested in stability. Criteria are provided in terms of spectra and cyclic properties.

Completeness of translates of entire functions and hypercyclic operators

V.E. Kim

Let $H(\mathbb{C})$ be the space of all entire functions with uniform convergence topology. For a wide class of entire functions (including, for example, Airy functions) we prove that the systems of their translates are complete in $H(\mathbb{C})$. As a corollary we obtain new classes of hypercyclic operators on $H(\mathbb{C})$.

Consider the translation operator on $H(\mathbb{C})$:

$$f(z) \to S_\lambda f(z) \equiv f(z+\lambda).$$

A well-known theorem of Godefroy and Shapiro [1] states that every continuous linear operator T on $H(\mathbb{C})$, satisfying $TS_\lambda - S_\lambda T = 0$ is hypercyclic, if it is not a scalar multiple of the identity. We obtain the following more general result:

Theorem 1. Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that 1)$\exists a \in \mathbb{C} : TS_\lambda f - S_\lambda Tf = a\lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$; 2) $\ker T \neq \{0\}$, $\ker T \neq H(\mathbb{C})$. Then T is hypercyclic.

This result is based on the following theorem.

Theorem 2. Let $T : H(\mathbb{C}) \to H(\mathbb{C})$ be a continuous linear operator such that $TS_\lambda f - S_\lambda Tf = \lambda S_\lambda f$, $\forall \lambda \in \mathbb{C}$, $\forall f \in H(\mathbb{C})$. Then the system $\{S_\lambda g, \lambda \in \Lambda \subset \mathbb{C}\}$ is complete in $H(\mathbb{C})$, if $g \in \ker T \setminus \{0\}$ and Λ contains a limit point.

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Matrix-valued truncated K-moment problems in several variables

D. Kimsey

In this presentation, the matrix-valued truncated K-moment problem on $\mathbb{R}^d$ and $\mathbb{C}^d$ will be considered. The matrix-valued truncated K-moment problem on $\mathbb{R}^d$ requires necessary and sufficient conditions for a multisequence of Hermitian matrices $\{S_\gamma\}_{\gamma\in\Gamma}$, where Γ is a finite subset of $\mathbb{N}_0^d$, to be the corresponding moments of a positive matrix-valued Borel measure σ and also the support of σ must lie in some given non-empty set $K \subseteq \mathbb{R}^d$, i.e.

$$S_\gamma = \int_{\mathbb{R}^d} \xi^\gamma d\sigma(\xi), \quad \gamma \in \Gamma, \tag{1}$$

and

$$\operatorname{supp} \sigma \subseteq K. \tag{2}$$

In a joint work with Hugo J. Woerdeman, given a non-empty set $K \subseteq \mathbb{R}^d$ and a finite multisequence, indexed by a certain family of finite subsets of $\mathbb{N}_0^d$, of Hermitian matrices we obtain necessary and sufficient conditions for the existence of a finitely atomic measure which satisfies (1) and (2). In particular, our result can handle the case when the indexing set that corresponds to the powers of total degree at most $2n+1$. We will also discuss a similar result in the complex setting.

Krein signature and eigencurves of non-conservative gyroscopic systems

O. Kirillov

Frequency loci crossing and veering phenomena are closely related to wave propagation and instabilities in fluids and structures. In engineering applications the crossings of the eigencurves are typically observed in gyroscopic or potential systems in the presence of symmetries, such as rotational or spherical one. The examples are perfect bodies of revolution that serve for modeling turbine wheels, disk and drum brakes, tires, clutches, vibrating gyroscopes, paper calenders and other rotating machinery. The frequency-rotational speed plot, or Campbell diagram, of such gyroscopically coupled systems consists of eigencurves that intersect each other, representing the frequencies of forward-, backward-, and reflected traveling waves. The intersections correspond to the double eigenfrequencies, or doublets. Zero doublets occur at the critical speeds of rotation corresponding to a stationary backward traveling wave in the non-rotating frame. It is well-known that the stationary conservative loads induce divergence and flutter instabilities either at the critical speeds or at the non-zero doublets above the critical speeds (mass and stiffness instabilities). In the vicinity of such doublets rings of complex eigenvalues originate (bubbles of instability). The same loads, however, only veer the eigencurves away near the crossings situated below the lowest critical speed without destabilization. MacKay (1986) explains this phenomenon using the notion of symplectic (Krein) signature of eigenvalues. He shows that the unfolding of the doublets due to Hamiltonian perturbations is represented by the conical eigenvalue surfaces of two types. Their one-dimensional slices fully describe all possible frequency loci veering scenarios in conservative gyroscopic systems. When damping and non-conservative forces act on a gyroscopically coupled system with symmetry, the frequency veering scenarios become more complicated. Both frequencies and growth rate loci can cross and avoid crossing. Moreover, flutter instabilities can occur below the first critical speed. In the present talk I show that as in the case of a conservative gyroscopic system, all possible frequency and growth rate loci veering and crossing scenarios in the presence of damping and non-conservative positional forces are described by means of the one-dimensional slices of singular eigenvalue surfaces of only two types. These surfaces are different from the MacKay's cones; they have two singular points locally equivalent to the Whitney umbrella. The type of the singular surface into which the MacKay's cones unfold under non-Hamiltonian perturbation is sharply determined by the symplectic (Krein) signature of the double eigenvalue of the unperturbed gyroscopic system. The singularities found, connect the problems of wave propagation in the rotating continua with that of electromagnetic and acoustic wave propagation in non-rotating anisotropic chiral media. As examples, eigencurves in a model of a rotating shaft under non-conservative loading and in a non-self-adjoint boundary value problem for a rotating circular string passing through the eyelet with friction are studied in detail.

Description of Helson-Szegő measures in terms of the Schur parameter sequences of associated Schur functions

B. Kirstein

Let μ be a probability measure on the Borelian σ-algebra of the unit circle. Then we associate a Schur function θ in the unit disk with μ and give characterizations of the case that μ is a Helson-Szegő measure in terms of the sequence of Schur parameters of θ. Furthermore, we state some connections of these characterizations with the backward shift.

The talk is based on joint work with V. K. Dubovoy and B. Fritzsche.

Hypercomplex representations of the Heisenberg group: p-mechanics

V.V. Kisil

This is a report on the ongoing work on unification of quantum and classical mechanical formalism, known as p-mechanics. Complex valued representations of the Heisenberg groups naturally provide a natural framework for quantum mechanics. This is the most fundamental example of the Kirillov orbit method and geometrical quantisation technique.

Following the pattern we consider representations of the Heisenberg group which are induced by hypercomplex characters of its centre. Besides complex numbers (which correspond to the elliptic case, $i^2 = -1$) there are two other types of hypercomplex numbers: dual (parabolic, $i^2 = 0$) and double (hyperbolic, $i^2 = 1$).

To describe dynamics of a physical system we use a universal equation based on inner derivations of the convolution algebra. The complex valued representations produce the standard framework for quantum mechanics with the Heisenberg dynamical equation.

The representations with value in dual numbers provide a convenient description of the classic mechanics. For this we do not take any sort of semiclassical limit, rather the nilpotency of the imaginary unit ($i^2 = 0$) removes the vicious necessity to consider the Planck constant tending to zero. The dynamical equation takes the Hamiltonian form. One of mathematical models uses $\mathbb{R}$-min ($\mathbb{R}$-max) algebras, which are studied within tropical mathematics and Maslov's dequantisation.

The double number valued representations, with the imaginary unit $i^2 = 1$, is a natural source of hyperbolic quantum mechanics developed recently by A. Khrennikov. The universal dynamical equation employs hyperbolic commutator. This can be seen as a version of the Moyal bracket based on the hyperbolic sine function.

The approach provides not only with three different types of dynamics, it also generates the respective rules for addition of probabilities. For example, the quantum interference is the consequence of the same structure which direct the Heisenberg equation. The absence of an interference (a wave behaviour) in the classical is again the consequence the nilpotency of the imaginary unit. Double numbers creates the hyperbolic law of additions of probabilities which were extensively investigates by A. Khrennikov.

The work clarifies foundations of quantum and classical mechanics. It also hinted that hyperbolic counterpart is (at least theoretically) as natural as classical and quantum mechanics are. The approach provides a framework for description of aggregate system which have say both quantum and classical components. This can be used to model quantum computers with classical terminals.

Covariant spectrum and Krein spaces

V.V. Kisil

The group $G = SL_2(\mathbb{R})$ consists of 2×2 matrices with real entries and the unit determinant. Let H be a Hilbert space, for a bounded linear operator T on H and a vector $v \in H$ we define the function on $SL_2(\mathbb{R})$ by

$$c(g) = (cT + dI)^{-1}v \qquad \text{where} \quad g^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL_2(\mathbb{R}).$$

Consider the space $L(G, H)$ of H-valued functions spanned by all left translations of $c(g)$. The left action of $SL_2(\mathbb{R})$ on this space is a linear representation of this group.

A covariant functional calculus is defined as a continuous linear intertwining map between two representations of a group: the first representation acts on scalar-valued functions, the second—on vector valued ones. The intertwining property replaces an algebra homomorphism in the standard construction of functional calculus. The above action of $SL_2(\mathbb{R})$ is an example of covariant calculus.

A covariant spectrum is defined as the support of covariant functional calculus, that is a decomposition of the intertwining map into primary subrepresentations. Such components associated with the above $SL_2(\mathbb{R})$ calculus fall into three large classes, with the pairings defined by the usual and indefinite inner products as well as degenerate one.

We discuss some aspects of covariant spectral theory and its connections with characteristic functions, the functional model and Krein spaces.

Relaxing linear matrix inequalities noncommutatively

I. Klep

Given linear matrix inequalities (LMIs) L_1 and L_2 it is natural to ask:

(Q_1) when does one dominate the other, that is, does $L_1(X) \succeq 0$ imply $L_2(X) \succeq 0$?

(Q_2) when are they mutually dominant, that is, when do they have the same solution set?

In this talk we describe a natural relaxation of an LMI, based on substituting matrices for the variables x_j. With this relaxation, the domination questions (Q_1) and (Q_2) have elegant answers. Assume there is an X such that $L_1(X)$ and $L_2(X)$ are both positive definite, and suppose the positivity domain of L_1 is bounded. For our "matrix variable" relaxation a positive answer to (Q_1) is equivalent to the existence of matrices V_j such that

$$L_2(x) = V_1^* L_1(x) V_1 + \cdots + V_\mu^* L_1(x) V_\mu. \tag{A_1}$$

As for (Q_2), L_1 and L_2 are mutually dominant if and only if, up to certain redundancies, L_1 and L_2 are unitarily equivalent.

Algebraic certificates for positivity, such as (A_1) for linear polynomials, are typically called Positivstellensätze. We shall also explain how to derive a Putinar-type Positivstellensatz for polynomials with a cleaner and more powerful conclusion under the stronger hypothesis of positivity on an underlying bounded domain of the form $\{X \mid L(X) \succeq 0\}$.

An observation at the core of this talk is that the relaxed LMI domination problem is equivalent to a classical problem in operator algebras. Namely, the problem of determining if a linear map from a subspace of matrices to a matrix algebra is *completely positive*.

The talk is based on joint work with J.W. Helton and S. McCullough; see

http://arxiv.org/abs/1003.0908

The flow approach to waves on networks

B. Klöss

We use a "non-standard method" to treat wave equations on networks, leading to a transport process on the doubled graph. From the node conditions, we derive a flow governed by a certain adjacency matrix. We show that the behavior of the infinite-dimensional wave process is essentially coded in this matrix and use it to derive stability criteria for systems of vibrating strings damped or delay-damped in the vertices.

Dislocation problems for periodic Schrödinger operators and mathematical aspects of small angle grain boundaries

M. Kohlmann

In many mathematical models for periodic crystals, the crystal atoms are assumed to be on the lattice sites of a periodic lattice Γ, under the influence of a Γ-periodic potential V, so that the energy levels of the crystal are described by the spectrum of the Schrödinger operator $-\Delta+V$. However, in real crystals, the regular periodic pattern of atomic arrangement is interrupted by crystallographic defects. In this talk, we study models for a translational defect as well as a small angle defect in two-dimensional lattices. We begin with dislocation problems in one dimension and on the strip $\mathbb{R}\times(0,1)$ to motivate our technical tools and to establish some basic results. Our first goal is to compute estimates for the density of surface states in dislocation problems on $\mathbb{R}^2$. Secondly, for a small angle grain boundary with angle $\vartheta > 0$, we show that the spectral gaps of the periodic problem fill with spectrum as $\vartheta \downarrow 0$. The talk is based on a joint work with R. Hempel.

New results in the theory of univalent subordination chains in several complex variables

G. Kohr

In this talk we present recent and modern results in the theory of univalent subordination chains (Loewner chains) and the generalized Loewner differential equation on the unit ball in $\mathbb{C}^n$. Various applications and examples will be also considered.

This talk is based on a joint work with I. Graham, H. Hamada and M. Kohr.

Recent results for boundary value problems concerning general Brinkman operators in Lipschitz domains. Applications

M. Kohr

In this talk we present recent results for boundary value problems concerning general Brinkman operators on arbitrary Lipschitz domains in the Euclidean setting or compact Riemannian manifolds. These results have been obtained by using the theory of pseudodifferential operators and potential theory. Applications to special problems in fluid mechanics are also considered.

This talk is based on a joint work with W. L. Wendland.

Direct and inverse problems for discrete Schrödinger operators on $Z^d, d \geq 2$

E. Korotyaev

We consider the discrete Schrödinger operator $H = \Delta + V$ on $Z^d, d \geq 2$ with real potential $V(n) = O(|n|^{-a}), a > d$ as $|n| = |n_1| + ... + |n_d| \to \infty$, where $n = (n_j)_1^d \in Z^d$. The following results are obtained:

1) Eigenvalues. We show the operator has finite number of eigenvalues including multiplicity. Moreover, the eigenvalues on the spectrum are absent, similar to the Kato result for the continuous case.

2) We solve few inverse problems. In particular, the S-matrix determines the potential uniquely, and we present the algorithm to recover the potential.

3) We study resonances for the operator H with finitely supported potential. We show that the Riemann surface for the operator has infinitely many sheets for any dim $d \geq 2$. Some results about the distribution of resonances are obtained. Recall that in the case $d = 1$ (and in the continuous case for any dimension) the Riemann surface for the Schrödinger operator has two sheets.

4) We determine the trace formulas for H.

The talk is based on a joint work with H. Isozaki.

Frequencies and action variables for periodic KdV and dNLS on the circle

E. Korotyaev

We consider the KdV and defocussing NLS equation on the circle. A new approach to study the Hamiltonian as a function of action variables is demonstrated. The problems for the KdV equation is reformulated as the problem of conformal mapping theory corresponding to quasimomentum of the Hill operator. In particular, we determine the asymptotics of the Hamiltonian for small action variables. Moreover, we determine the gradient of Hamiltonian with respect to action variables. This gives so called frequencies and determines how the angles variables depend on the time. The main tool is the Löwner type equation for the quasimomentum.

Weyl–Titchmarsh theory for Bessel operators

A. Kostenko

The main objective of the talk is the Weyl–Titchmarsh theory for perturbed spherical Schrödinger operators, also known as Bessel operators. It is known that one can still define a corresponding singular Weyl m-function for this type of Sturm–Liouville operators. Here we will investigate when this singular m-function is a generalized Nevanlinna function.

The talk is based on a joint work with G. Teschl and A. Sakhnovich.

On indefinite quadratic forms

V. Kostrykin

In this talk we revisit the representation theorems for sign-indefinite, not necessarily semibounded symmetric sesquilinear forms. In particular, we discuss new straightforward proofs of these theorems and a number of necessary and sufficient conditions ensuring the second representation theorem to hold. Furthermore we present a new simple and explicit example of a self-adjoint operator for which the second representation theorem does not hold is also provided. The talk is based on a joint work with L. Grubišić, K. A. Makarov, and K. Veselić.

Applications of parabolic Dirac operators to the instationary MHD equations in 3D

R.S. Kraußhar

In this paper we apply new techniques from quaternionic analysis using parabolic Dirac type operators to develop new analytic methods and computation schemes to compute solutions to the instationary incompressible viscous magnetohydrodynamic equations over bounded and unbounded time-varying Lipschitz domains in ℝ3. We also present explicit existence and uniqueness criteria as well as a priori estimates.

This talk is based on a joint work with K. Gürlebeck.

The Hörmander functional calculus

C. Kriegler

Let $d \in \mathbb{N}$ and $p \in (1, \infty)$. A theorem of Hörmander says that if $f : [0, \infty) \to \mathbb{C}$ satisfies

$$\sup_{r>0} \int_r^{2r} |s^k f^{(k)}(s)|^2 \frac{ds}{s} < \infty \quad (k = 0, 1, \ldots \lfloor \frac{d}{2} \rfloor + 1)$$

then f is a radial Fourier multiplier on $L^p(\mathbb{R}^d)$, i.e. the mapping

$$L^p(\mathbb{R}^d) \to L^p(\mathbb{R}^d),\ g \mapsto f(-\Delta)g = [f(|\cdot|^2)\hat{g}]\check{}$$

is bounded.

We put this into a more general framework. Consider a generator A of a semigroup with spectrum contained in $[0, \infty)$. We compare conditions on the semigroup with multiplier theorems modeled after Hörmander's one above.

Vanishing of the Lyapunov exponent

H. Krüger

A Jacobi operator is bounded linear operator on $\ell 2(\mathbb{N})$ acting by

$$Hu(n) = a(n+1)u(n+1) + b(n)u(n) + a(n)u(n-1),$$

where $u(0) = 0$ and $a(n)$, $b(n)$ are bounded real valued sequences. We will also assume that $a(n) > 0$ is bounded away from zero.

The Lyapunov Exponent $L(E)$ is given by the maximal exponential growth of solutions of $Hu = Eu$ ignoring the boundary condition at 0. I will describe consequences of the Lyapunov exponent vanishing on the essential spectrum of H.

The most basic result is that if the essential spectrum is $[-2, 2]$ and the Lyapunov exponent vanishes on it, then $a(n)-1$ and $b(n)$ Cesàro sum to zero.

Closures of sums of squares in various convex topologies

S. Kuhlmann

We consider the cone $\sum \mathbb{R}[\underline{x}]2$ of sums of squares in the polynomial ring $\mathbb{R}[\underline{x}] := \mathbb{R}[x_1, \cdots, x_n]$. We describe its closure in the various locally convex topologies on $\mathbb{R}[\underline{x}]$, such as the $\|\cdot\|_p$ and weighted l_p norm topologies for $1 \leq p \leq \infty$. This talk is based on joint work with M. Ghasemi and E. Samei.

On optimal L^p-L^q-estimates for parabolic boundary value problems

P.C. Kunstmann

We study boundary value problems that are elliptic with a parameter in an $L^q(\Omega)$-setting. We present a modification of Davies' method for these problems and show that the natural bound on the full inhomogeneous resolvent problem self-improves to exponential off-diagonal estimates for the solution operators where the coefficients of the domain operator are only bounded and measurable. These off-diagonal estimates are used to enlarge the L^q-scale and to establish optimal L^p-L^q-estimates for the corresponding parabolic problem, including inhomogeneous boundary data. As an application we derive new results for operators with VMO-coefficients.

On generalized Schoenberg theorem

O. Kushel

This talk is devoted to the generalization of the theorem, first proved by I.J. Schoenberg, concerning the variational-diminishing property of a totally positive matrix.

Let A be a linear operator, acting in the space $\mathbb{R}^n$. In this case the operator $\wedge^j A$ $(j = 1, \ldots, n)$, i.e. the j-th exterior power of the operator A, acts in the space $\wedge^j \mathbb{R}^n = \mathbb{R}^{C_n^j}$. A set $K \subset \mathbb{R}^n$ is called *a proper cone*, if it is a convex cone (i.e. for any $x, y \in K$, $\alpha \geq 0$ we have $x + y$, $\alpha x \in K$), is pointed (i.e. $K \cap (-K) = \{0\}$), closed and full (i.e. $\mathrm{int}(K) \neq \emptyset$). A linear operator A is called *generalized totally positive* if it leaves invariant a proper cone $K_1 \subset \mathbb{R}^n$, and for every j $(1 < j \leq n)$ its j-th exterior power $\wedge^j A$ leaves invariant a proper cone $K_j \subset \mathbb{R}^{C_n^j}$.

A closed subset $T \subset \mathbb{R}^n$ is called *a cone of rank* k $(0 \leq k \leq n)$, if for every $x \in T$, $\alpha \in \mathbb{R}$ the element $\alpha x \in T$ and there is at least one k-dimensional subspace and no higher dimensional subspaces in T.

Let the operator A be generalized totally positive. Then under some additional conditions we can prove the existence of n cones $T_1, \ldots, T_n$, each of which is invariant for the operator A, and the rank of the jth cone T_j is equal to j. The inverse is also true: if for every $j = 1, \ldots, n$ the operator A leaves invariant a cone T_j of rank j, then under some additional conditions we can prove the existence of n proper cones $K_1, \ldots, K_n$, such that the jth cone K_j is invariant for the jth exterior power $\wedge^j A$ of the initial operator A.

References.

1. I.J. Schoenberg, *Über variationsvermindernde lineare Transformationen.* Math. Z. **32** (1930), 321-328.

2. A. Pinkus, *Totally positive matrices.* Cambridge University Press, 2010.

On a class of J-self-adjoint operators with empty resolvent set

S. Kuzhel

In contrast to self-adjoint operators in Hilbert spaces (which necessarily have a purely real spectrum), self-adjoint operators A in Krein spaces $(\mathfrak{H}, [\cdot, \cdot]_J)$ (J-self-adjoint operators) may have spectra which are only symmetric with respect to the real axis. Moreover, the situation where $\sigma(A) = \mathbb{C}$ (i.e., A has the empty resolvent set) is also possible. It is natural to suppose that the relation $\sigma(A) = \mathbb{C}$ indicates some special structure of a J-self-adjoint operator A. We illustrate such a point by considering the set Σ_J of all J-self-adjoint extensions A of the symmetric operator S with deficiency indices $< 2, 2 >$ which commutes with J. In that case the existence of at least one $A \in \Sigma_J$ with empty resolvent set is equivalent to the existence of an additional fundamental symmetry R in $\mathfrak{H}$ such that

$$SR = RS, \qquad JR = -RJ.$$

The operators J and R can be interpreted as basis (generating) elements of the complex Clifford algebra $\mathcal{C}l_2(J, R) := \text{span}\{I, J, R, JR\}$ and they give rise to a 'rich' family of exactly solvable models of $\mathcal{PT}$-symmetric quantum mechanics (PTQM) explaining (at an abstract level) the appearance of exceptional points on the boundary of the domain of the exact $\mathcal{PT}$-symmetry in PTQM.

The talk is based on joint works with C. Trunk and with S. Albeverio and U. Günther.

Matrix polynomials: parametrization and canonical forms

P. Lancaster

We consider matrix polynomials with nonsingular leading coefficent and their linearizations. Parametrizations are obtained for strict equivalence and congruence transformations of the linearizations. The centralizer of the Jordan form plays a major role in these parametrizations. Jordan structures over either the real or complex numbers are admitted.

This talk is based on joint work with I. Zaballa.

Quadratic matrix polynomials: solvents and inverse problems

P. Lancaster

I start with the famous theorem of Langer on the factorization of Hermitian matrix functions. When applied to monic quadratic polynomials the factorization determines a right divisor and a left divisor. The strategy we follow for the inverse eigenvalue problem is to assign half the spectrum by fixing a right divisor. We then examine the class of compatible left divisors. In contrast to earlier investigations we can cope with mixed real/non-real eigenvalues and keep track of sign characteristics associated with real eigenvalues.

This is collaborative work with F. Tisseur.

Linearization of self-adjoint analytic operator functions

H. Langer

After some historical remarks, we consider a Krein space linearization of a self-adjoint analytic operator function. Of particular interest are spectral points of definite type and the local spectral function.

Variational principles for eigenvalues of operator functions and block operator matrices

M. Langer

In this talk functions T are considered which are defined on some interval I and whose values are (in general unbounded) self-adjoint operators in a Hilbert space. A point $\lambda \in I$ is called an eigenvalue of T if there exists a vector $x \in \mathrm{dom}(T(\lambda))$, $x \neq 0$, so that $T(\lambda)x = 0$. Under very mild assumptions a variational inequality for eigenvalues of T is proved, which can also be used to show that a certain subinterval of I is free of spectrum. In various situations this inequality turns out to be an equality, which is not true in general. The general variational inequality and equality are applied to the Schur complement of a block operator matrix, which yields estimates for the spectrum and eigenvalues of this block operator matrix in certain gaps of the essential spectrum.

The talk is based on joint work with M. Strauss.

Two-dimensional Hamiltonian systems with two singular endpoints

M. Langer (Part one) and H. Woracek (Part two)

Consider a two-dimensional Hamiltonian system of the form

$$y'(x) = zJH(x)y(x), \quad x \in (0, \infty), \tag{1}$$

which is in Weyl's limit point case at the endpoint ∞. If at the left endpoint 0 limit circle case prevails, the spectral theory of this system is well understood. The associated (minimal) differential operator is symmetric with defect index $(1, 1)$, and the notion of the Titchmarsh-Weyl coefficient q_H associated with (1) allows to construct a Fourier transform onto an L^2-space with respect to a scalar valued positive measure μ (appropriately including a possible point mass at infinity). The measure μ is thereby obtained from the Herglotz-integral representation of q_H; in particular, it satisfies $\int_{\mathbb{R}}(1+t2)^{-1}\,d\mu < \infty$. An inverse spectral theorem, due to L. de Branges, holds true. It states that each such measure appears as the spectral measure of a system of this kind, and that the system (1) can –at least in theory– be recovered from μ.

If also at the endpoint 0 limit point case takes place, the situation changes. The (minimal) differential operator is already selfadjoint. The classical theory leads, via the Titchmarsh-Kodaira formula, to a Fourier transform onto an $L2$-space with respect to a 2×2-matrix valued measure. In general this result cannot be improved; the minimal operator may in general have spectral points with spectral multiplicity 2.

Nevertheless, there exist classes of Hamiltonians which are in the limit point case at both endpoints, but still behave well in the sense that a Fourier transform onto a space of scalar valued functions can be constructed. One such class was already given by L.de Branges, however, no inverse results were proved. Other previous work in this direction focuses mainly on the case of Sturm-Liouville equations with singular potential (which can be regarded as a particular case of Hamiltonian systems), and also deals mainly with the direct spectral problem.

We present a class of Hamiltonians, being in the limit point case at both endpoints, for which the direct and inverse spectral problems can be solved in a complete and satisfactory way. It is defined by means of growth restrictions at the left endpoint. Concerning the direct spectral problem, we show that each Hamiltonian H of this class gives rise to a Fourier transform onto a space $L^2(\mu)$ where μ is a scalar valued positive measure such that $\int_{\mathbb{R}}(1+t2)^{-n}\,d\mu<\infty$ for some $n\in\mathbb{N}$. Probably more notable, we show that each measure subject to this growth restriction arises in this way, and that H can (in general again unconstructively) be recovered from μ.

Although the problem under consideration as well as the final solution to it is purely 'positive definite', our approach proceeds via Pontryagin space theory. We use an indefinite version of Hamiltonian systems, generalized Nevanlinna functions, and selfadjoint relations in Pontryagin spaces. We find it particularly appealing that, after a (quite significant) detour through the indefinite world, a classical solution to a classical problem is obtained.

On canonical solutions of the truncated trigonometric matrix moment problem

A. Lasarow

The main theme of the talk is the discussion of some distinguished solutions of the truncated trigonometric matrix moment problem. Roughly speaking, we discuss certain solutions which are molecular nonnegative Hermitian matrix-valued Borel measures on the unit circle with a special structure. We give some general information on this type of solutions, but we will focus on the so-called nondegenerate case. In the latter case, the measures in question form a family of solutions which can be parametrized by the set of unitary matrices. In particular, we will see that each member of this family offers an extremal property in the solution set of the moment problem concerning the weight assigned to some point of the open unit disk. In doing so, an application of the theory of orthogonal matrix polynomials on the unit circle takes a key position to get that insight.

The index formula and the spectral shift function for relatively trace class perturbations

Y. Latushkin

We compute the Fredholm index, index$(\boldsymbol{D_A})$, of the operator $\boldsymbol{D_A} = (d/dt) + \boldsymbol{A}$ on $L^2(\mathbb{R};\mathcal{H})$ associated with the operator path $\{A(t)\}_{t=-\infty}^{\infty}$, where $(\boldsymbol{A}f)(t) = A(t)f(t)$ for a.e. $t \in \mathbb{R}$, and appropriate $f \in L^2(\mathbb{R};\mathcal{H})$, via the spectral shift function $\xi(\,\cdot\,; A_+, A_-)$ associated with the pair (A_+, A_-) of asymptotic operators $A_\pm = A(\pm\infty)$ on the separable complex Hilbert space $\mathcal{H}$ in the case when $A(t)$ is generally an unbounded (relatively trace class) perturbation of the unbounded self-adjoint operator A_-.

We derive a formula (an extension of a formula due to Pushnitski) relating the spectral shift function $\xi(\,\cdot\,; A_+, A_-)$ for the pair (A_+, A_-), and the corresponding spectral shift function $\xi(\,\cdot\,; \boldsymbol{H}_2, \boldsymbol{H}_1)$ for the pair of operators $(\boldsymbol{H}_2, \boldsymbol{H}_1) = (\boldsymbol{D_A}\boldsymbol{D_A^*}, \boldsymbol{D_A^*}\boldsymbol{D_A})$ in this relative trace class context,

$$\xi(\lambda; \boldsymbol{H}_2, \boldsymbol{H}_1) = \frac{1}{\pi}\int_{-\lambda^{1/2}}^{\lambda^{1/2}} \frac{\xi(\nu; A_+, A_-)\, d\nu}{(\lambda - \nu^2)^{1/2}} \quad \text{for a.e. } \lambda > 0.$$

This formula is then used to identify the Fredholm index of $\boldsymbol{D_A}$ with $\xi(0; A_+, A_-)$. In addition, we prove that index$(\boldsymbol{D_A})$ coincides with the spectral flow $\mathrm{SpFlow}(\{A(t)\}_{t=-\infty}^{\infty})$ of the family $\{A(t)\}_{t\in\mathbb{R}}$ and also relate it to the (Fredholm) perturbation determinant for the pair (A_+, A_-):

$$\begin{aligned}\mathrm{index}(\boldsymbol{D_A}) &= \mathrm{SpFlow}(\{A(t)\}_{t=-\infty}^{\infty}) = \xi(0; A_+, A_-)\\ &= \pi^{-1}\mathrm{Im}\,\ln(\det\nolimits_{\mathcal{H}}(A_+A_-^{-1})).\end{aligned}$$

We also provide some applications in the context of supersymmetric quantum mechanics to zeta function and heat kernel regularized spectral asymmetries and the eta-invariant.

This is a joint work with F. Geztesy, A. K. Makarov, F. Sukochev, and Y. Tomilov.

Estimates for the Cartan lemma on holomorphic matrices, and generalization

J. Leiterer

We start with the well-known Cartan lemma on factorization of holomorphic matrices defined on the intersection of two rectangles. We prove a uniform estimate for the factors depending on the size of the rectangles (which is quite simple, but sharp, in some sense). Then we discuss such estimates for more general cocycles of holomorphic matrices of one complex variable. If the cocycle belongs to a covering with non-empty intersections of at least three covering sets, the problem becomes interesting. Applications to several variables appear, but also unsolved open questions remain.

Completely bounded norms of right module maps

R. Levene

Let D_n denote the algebra of diagonal $n \times n$ matrices. If T is a a Schur multiplier on the $m \times n$ matrices $M_{m,n}$ (that is, T is a D_m–D_n bimodule map), then it is well-known that $\|T\|_{cb} = \|T\|$. Using Timoney's work on elementary operators, we show that if T is merely a right D_2-module map on $M_{m,2}$, then again we have $\|T\|_{cb} = \|T\|$. However,

$$C(m,n) = \sup\left\{ \tfrac{\|T\|_{cb}}{\|T\|} : T \text{ is a right } D_n\text{-module map on } M_{m,n} \right\}$$

grows with m, n. Hence there is a bounded right ℓ^∞-module map on $B(\ell 2)$ which is not completely bounded, answering a question posed in a recent paper of Juschenko, the speaker, Todorov and Turowska.

This is joint work with R. Timoney.

Convex cones of generalized positive rational functions and Nevanlinna-Pick interpolation

I. Lewkowicz

As a motivation recall that the Nevanlinna-Pick interpolation problem is to find for given vectors $x, y \in \mathbb{C}^n$ a (complex scalar) rational function f, from a certain class so that $y_j = f(x_j)$ $j = 1, \ldots n$. All solutions may be parameterized by the corresponding Pick matrix Π. Specifically, in the framework of Schur functions, mapping the unit disk to itself, $[\Pi]_{jk} = \frac{1-y_j^* y_k}{1-x_j^* x_k}$; and in the framework of positive functions, $\mathcal{P}$, mapping the right half plane to itself, $[\Pi]_{jk} = \frac{y_j^* + y_k}{x_j^* + x_k}$.

The analogy is more intricate when one addresses Nevanlinna-Pick interpolation problem over generalized Schur functions, mapping the unit circle to the unit disk and Generalized Positive functions, $\mathcal{GP}$, mapping the imaginary axis to the right half plane.

The key to our approach is following factorization result,

$$\psi \in \mathcal{GP} \iff \psi(s) = g(s)p(s)g(s)^{\#} , \quad p \in \mathcal{P},$$

with g, g^{-1} analytic in the left half plane and $g(s)^{\#} := g(-s^*)^*$. This induces a convex partitioning of all $\mathcal{GP}$ functions into sets of function sharing the same poles and zeroes in the right half plane. Within these sets, we introduce a simple scheme for Nevanlinna-Pick interpolation.

Joint work with D. Alpay.

Decay estimates for the solutions of the system of crystal elasticity for tetragonal crystals

O. Liess

We study algebraic and geometric properties of the slowness surface of the system of crystal elasticity for tetragonal crystals in the nearly cubic case. A first group of results refers to the location and nature of the singular points of the surfaces under consideration, whereas other results relate to curvature properties of these surfaces. We will also mention results about the long time behaviour of the solutions of the system. These results are based on estimates for Fourier transforms of densities which live on surfaces with isolated singular points, respectively for situations where the surface may have poins at which the Gaussian curvature vanishes, but the mean curvature does not.

We also study the general form of quartic and sextic surfaces of "slowness type" which are quadratic in their variables.

The talk is based on joint work with C. Melotti.

Spectra of Jacobi operators: Analysis and approximation

M. Lindner

We look at bounded linear operators on vector-valued ℓ^p spaces and study their spectrum (in particular the essential spectrum) and pseudospectra. Our operators are given by infinite matrices with finitely many diagonals. We give a formula for the essential spectrum which will be discussed for concrete examples of matrices with almost periodic, slowly oscillating or random diagonals. For the case of tridiagonal matrices, we moreover give upper bounds on spectrum and pseudospectrum of the infinite matrix A in terms of pseudospectra of certain finite matrices of order n that are connected to submatrices of A. The latter sets approximate the (pseudo-)spectrum of A as n goes to infinity.

This is joint work with S. N. Chandler-Wilde and R. Chonchaiya.

Spectral asymptotic of elliptic operators with non-local boundary conditions

V. Lotoreichik

Recently the boundary triplets approach to the parametrisation of self-adjoint realisations of some elliptic expression on a smooth domain was suggested. There are two main ways. The first way is based on the ordinary triplet and allows to parametrise all self-adjoint realisations. The second way is to use the quasi boundary triplet or something similar. In such a case we parametrise only realisations with the certain regularity of the operator domain. The advantage of this approach that one can take the Dirichlet trace and the Neumann trace as the boundary mappings. The Weyl function becomes the Dirichlet-to-Neumann or the Neumann-to-Dirichlet map. These objects are very natural in the Analysis of PDE.

Using quasi boundary triples we prove various Schatten-von Neumann estimates of resolvent differences of two self-adjoint (max. dissipative or max. accumulative) realisations of some second-order elliptic expression on a bounded (or unbounded) domain with a compact smooth boundary. Among these estimates there are some of the well-known estimates due to M. Sh. Birman, M. Z. Solomyak, G. Grubb proved in a simpler way and some new. In particular the resolvent difference of the classical Robin and Neumann realisations has better asymptotic of singular numbers than the resolvent difference of Dirichlet and Neumann realisations. We also include estimates for differences of integer powers of resolvents.

These estimates together with variational principle of T. Kato allow us to compare generalised spectra of two realisations.

This is a joint work with J. Behrndt and M. Langer.

(Generalized) Nevanlinna functions - Old and new

A. Luger

The class of *Generalized Nevanlinna functions* consists of all matrix functions $Q : \mathcal{D}_Q \subset \mathbb{C} \to \mathbb{C}^{n\times n}$ which are meromorphic in $\mathbb{C}\backslash\mathbb{R}$, symmetric with respect to the real line, and for which the Nevanlinna kernel

$$K_Q(z, w) := \frac{Q(z) - Q(\overline{w})}{z - \overline{w}}$$

has finitely many negative squares.

In the talk, that will have the character of a survey, we are going to discuss different representations and analytic properties of (generalized) Nevanlinna functions. Moreover, some questions arising from applications will be considered.

Some properties of unitary operators on quaternionic Hilbert spaces

M.E. Luna-Elizarrarás

In this talk there wil be presented a refinement of some results from [1] which treats spectral theory for unitatry operators on quaternionic Hilbert spaces. In particular we consider different options for defining eigenvalues and eigenelements of quaternionic linear operators.

This work was partially supported by CONACYT projects as well as by Instituto Politécnico Nacional in the framework of its COFAA and SIP programs.

References.

[1] C.S. Sharma and T.J. Coulson, *Spectral theory for unitary operators on a quaternionic Hilbert space*, J. Math. Phys. **28** (9), 1987.

Dirichlet problems for Ornstein-Uhlenbeck operators in subsets of Hilbert spaces

A. Lunardi

We consider a family of self-adjoint Ornstein–Uhlenbeck operators $\mathcal{L}_\alpha$ in an infinite dimensional Hilbert space H having the same gaussian invariant measure $\mu = \mathcal{N}_Q$,

$$\mathcal{L}_\alpha\varphi(x) = \frac{1}{2}\,\mathrm{Tr}\,[Q^{1-\alpha}D^2\varphi(x)] - \frac{1}{2}\langle x, Q^{-\alpha}D\varphi(x)\rangle,$$

where $Q \in \mathcal{L}(H)$ is a symmetric positive operator with finite trace, and $0 \le \alpha \le 1$.

We study the Dirichlet problem for the equation $\lambda\varphi - \mathcal{L}_\alpha\varphi = f$ in a closed set $K \subset H$, with $f \in L^2(K,\mu)$. Its variational solution, trivially provided by the Lax—Milgram theorem, can be represented by means of the transition semigroup stopped to K, as in finite dimensions.

We address two problems: 1) the meaning of the Dirichlet boundary condition; 2) the regularity of the solution φ (which belongs to a Sobolev space $W^{1,2}_\alpha(K,\mu)$ by definition) of the Dirichlet problem.

Concerning the boundary condition we consider both irregular and regular boundaries. In the first case we content to have a solution whose null extension outside K belongs to $W^{1,2}_\alpha(H,\mu)$. In the second case we exploit the Malliavin's theory of surface integrals, to give a meaning to the trace of φ at ∂K and to show that it vanishes, as it is natural.

Concerning regularity, we can prove interior $W^{2,2}_\alpha$ regularity results. Regularity up to the boundary is much more complicated; however we have some partial results. For instance, we can treat the case $\alpha = 0$ for halfspaces $K = \{x \in H : \langle b, x\rangle = 1\}$ with $b \in H$, $\|b\| = 1$.

The talk is based on a joint work with G. Da Prato.

On the truncated matricial Stieltjes moment problem

C. Mädler

We discuss some aspects of the truncated matricial Stieltjes moment problem. Our approach is based on a Schur type algorithm.

The talk is based on joint work with B. Fritzsche and B. Kirstein.

Positive decompositions of selfadjoint operators

A. Maestripieri

Given a linear bounded selfadjoint operator a on a complex separable Hilbert space $(\mathcal{H}, \langle\ .\ ,\ .\ \rangle)$, we study the decompositions of a as a difference of two positive operators whose ranges satisfy an angle condition. These decompositions are related to the canonical decompositions of the indefinite metric space $(\mathcal{H}, \langle a\ .\ ,\ .\ \rangle)$, associated to a. As an application, we characterize the orbit of congruence of a, $O_a = \{gag^* : g \in GL(\mathcal{H})\}$, in terms of its positive decompositions. When a has closed range, we show that O_a with a suitable metric has a differentiable structure, in fact, it is a principal fibre bundle.

The talk is based on a joint work with G. Fongi.

Spectral theory of Schrödinger operators with point interactions on a discrete set

M. Malamud

One-dimensional Schrödinger operator $\mathrm{H}_{X,\alpha}$ with δ-interactions on a discrete set is investigated in the framework of the extension theory. Applying the technique of boundary triplets and the corresponding Weyl functions, a connection of the operators with a certain class of Jacobi matrices has been established. The discovered connection enables us to obtain conditions for the self-adjointness, lower semiboundedness, discreteness of the spectrum, and discreteness of the negative part of the spectrum of the operator $\mathrm{H}_{X,\alpha}$.

The talk is based on joint works with S. Albeverio and A. Kostenko [1,2].

[1] *S. Albeverio, A. Kostenko, M. Malamud*, Spectral theory of semibounded Schrödinger operators with point interactions on a discrete set, *submitted.*

[2] *A. Kostenko and M. Malamud*, 1–D Schrödinger operators with local point interactions on a discrete set, *J. Differential Equations* **249** (2010), 253–304.

Perturbation determinants and trace formulas for non-additive perturbations

M. Malamud

A classical notion of the perturbation determinant (PD) is naturally defined for a pair of closed operators T_0 and T with common domains of definition, $\mathrm{dom}\,(T) = \mathrm{dom}\,(T_0)$. Namely, if $V(T_0 - z)^{-1} \in \mathfrak{S}_1$ (the trace class ideal), and $V := T - T_0$, then the PD is defined by

$$D(z) = \det[I + V(T_0 - z)^{-1}].$$

We consider a pair $\{T, T_0\}$ in the case $\mathrm{dom}\,(T) \neq \mathrm{dom}\,(T_0)$ assuming only that

$$(T - z)^{-1} - (T_0 - z)^{-1} \in \mathfrak{S}_1.$$

We discuss a notion of PD in the framework of extensions theory of symmetric operators by applying boundary triplet approach. We compute the PD by means of the Weyl function and the corresponding boundary operators. We obtain different trace formulas and complete some results of M.G. Krein, V. Adamyan, B. Pavlov and others.

The talk is based on the joint work with H. Neidhardt.

Wiener-Hopf factorization for a class of matrix functions using an isomorphism with a group of scalar functions on a Riemann surface

M.T. Malheiro

An isomorphism between a group of Daniele-Khrapkov matrix functions and the group of invertible Hölder continuous scalar functions defined on a contour in an appropriate Riemann surface is presented. It is shown that this isomorphism can be used to obtain a meromorphic factorization for Daniele-Khrapkov matrices, from an appropriate factorization of scalar functions in the Riemann surface. A Wiener-Hopf factorization is obtained from the meromorphic one.

This is a joint work with C. Câmara.

A theorem of Heinz Langer on the factorization of selfadjoint operator polynomials

A. Markus

In 1973 Langer [1] proved the following theorem.

Theorem. Let $L(\lambda)$ be a selfadjoint operator polynomial of degree n and $[a, b] \subset \mathbb{R}$. If

$$L(a) \ll 0, L(b) \gg 0, L'(\lambda) \gg 0 (a \leq \lambda \leq b),$$

then $L(\lambda)$ admits a factorization

$$L(\lambda) = M(\lambda)(\lambda I - Z),$$

where $M(\lambda)$ is an operator polynomial of degree $n-1$, and Z is similar to a selfadjoint operator. Moreover, $M(\lambda)$ is invertible for all $\lambda \in [a,b]$ and $\sigma(Z) \subset [a,b]$.

I will try to explain the main ideas of the proof of this remarkable theorem, and its influence on the development of the spectral theory of selfadjoint polynomials and analytic operator functions.

[1] H. Langer, *Über eine Klasse nichtlinearer Eigenwertprobleme.* Acta Sci. Math. (Szeged), **35**(1973), 73-86.

Self-adjoint analytic operator functions: Inner linearization and factorization of the restriction to a spectral subspace

A. Markus

Let $\Delta = [a,b]$ be a real interval, D be a neighborhood of Δ and $\mathcal{H}$ be a Hilbert space. We consider a $\mathcal{L}(\mathcal{H})$-valued function $A(z)$ on D which is analytic and self-adjoint. Suppose that $A(a), A(b)$ are invertible and that $A(z)$ satisfies the Virozub-Matsaev condition on Δ. Denote

$$Q(\Delta) = (2\pi i)^{-1} \oint_{\gamma} A(z)^{-1} dz$$

where $\gamma(\subset D)$ is a smooth curve which surrounds Δ, and $\mathcal{H}(\Delta) = \text{ran}\, Q(\Delta)$.

Under these conditions $A(z)$ has a self-adjoint linearization Λ in a Hilbert space $\mathcal{F}$. We construct a special operator $S(\in \mathcal{L}(\mathcal{H}(\Delta)))$ which is similar to the linearization $\Lambda(\in \mathcal{L}(\mathcal{F}))$. Since S acts in a subspace of the originally given space $\mathcal{H}$, we call S the inner linearization of $A(z)$.

We study various properties of the inner linearization and their connection with the properties of the operator function $A(z)$. We prove, in particular, the following factorization theorem for the restriction of $A(z)$ to the subspace $\mathcal{H}(\Delta)$.

Theorem. There exists an analytic in D operator function $M(z)$ with values in $\mathcal{L}(\mathcal{H}(\Delta), \mathcal{H})$ such that

$$A(z)f = M(z)(S - z)f \ (z \in D, f \in \mathcal{H}(\Delta)).$$

For each $z \in D$ the operator $M(z)$ is injective, and its range (depending on z) is a closed subspace of $\mathcal{H}$.

The talk is based on a joint work with H. Langer and V. Matsaev.

Basisness results for some singular pencils of differential operators arising in hydrodynamics

M. Marletta

Many classical linear stability problems in hydrodynamics, MHD and other areas of physics give rise to operator pencil problems. Since the early 1990s, Trefethen has pointed out some of the dangers of this approach which arise due to pseudospectral phenomena.

However there are some more fundamental issues which afflict this methodology. Benilov, O'Brien and Sazonov proposed a model for drop formation in a rotating fluid in which the eigenfunctions form a complete set but linear stability fails even though all the eigenvalues lie in the stable half-plane. The reason for this failure is that the eigenfunctions (there are no associated functions) do not form a Riesz basis in an appropriate space.

Basisness results are rarely discussed in the fluid dynamics literature despite their central importance for linear stability, for the simple reason that they are generally rather difficult to prove. The first basisness results for the classical Orr-Sommerfeld problem, for instance, only date back to the 1980s.

In this talk I shall review some older work on pencil problems with λ-dependent boundary conditions, as well as some more recent work on singular pencil problems of a type which arise in, e.g., the analysis of pipe Poiseuille flow. For the problems with λ-dependent boundary conditions some lack-of-basisness results will be shown, while for the singular problems with λ-independent boundary conditions basisness results are obtained subject to a completeness hypothesis.

This talk is based on joint work with A. Shkalikov and Ch. Tretter.

An estimate for the number of solutions of an homogeneous generalized Riemann boundary value problem with shift and conjugation

R. Marreiros

In the real space $\widetilde{L}_2(\mathbb{R})$, we consider the generalized Riemann boundary value problem with the condition on the real line

$$\varphi_+ = a\varphi_- + a_0\overline{\varphi_-} + a_1\overline{\varphi_-(\alpha)} + a_2\overline{\varphi_-(\alpha_2)} + \cdots + a_m\overline{\varphi_-(\alpha_m)},$$

where $\alpha(t) = t+\mu$, $\mu \in \mathbb{R}$, is the shift on the real line, $\alpha_k(t) = \alpha(\alpha_{k-1}(t))$, $k \le m$, $k, m \in \mathbb{N}$, and $a, a_0, a_1, \ldots, a_m$ are continuous functions on $\overset{\circ}{\mathbb{R}} = \mathbb{R} \cup \{\infty\}$, the one point compactification of $\mathbb{R}$. Under certain conditions on the coefficients, an estimate for the number of linear independent solutions of this problem is obtained.

This is a joint work with V. Kravchenko and J. Rodriguez.

Adaptive Wavelet methods for the Hilbert transform and Riemann-Hilbert problems

F. Martin

The Hilbert transform plays an important role in signal theory and computational complex analysis. A serious problem in some of these algorithms is the *crowding* phenomenon, which occurs for example in numerical conformal mapping and, more generally, in methods for solving nonlinear Riemann-Hilbert problems.

In order to overcome these difficulties we develop an approach for computing the Hilbert transform on a non-uniform mesh using biorthogonal B-Spline wavelets. Furthermore, we propose an adaptive method for the numerical solution of Riemann-Hilbert problems involving the Hilbert transform on graded meshes and give some modifications of former algorithms which show better convergence behavior.

The talk is based on a joint work with Prof. E. Wegert.

Decay estimates for singular extensions of vector-valued Laplace transforms

M.M. Martínez

Let X be a Banach space and let $f : [0, \infty) \to X$ be a bounded measurable function. The Laplace transform of f is given by the Bochner integral

$$\hat{f}(z) := \int_0^\infty f(t) e^{-zt} dt \,, \quad \Re z > 0.$$

The use of vector-valued Laplace transform techniques has led to an important advance in the theory of linear evolution equations and semigroup theory. Among these techniques, ones of the most challenging are those concerning to the asymptotic behaviour and, in particular, *Tauberian theorems*, in which asymptotic properties of a function are deduced from properties of its transform.

We shall consider a function $f \in L^\infty([0, \infty); X)$ whose Laplace transform extends analytically to some region containing $i\mathbb{R} \setminus \{0\}$, possibly having a pole at the origin. The aim of this talk is to present estimates of the decay of certain slight modification of f in terms of the growth of $\hat{f}$ along the imaginary axis (up to the origin).

From this theorem, we will deduce some new decay estimates for C_0-semigroups of operators. Let $(T(t))_{t\geq 0}$ be a bounded C_0-semigroup on a Banach space whose infinitesimal generator A is such that $\sigma(A) \cap i\mathbb{R} \subseteq \{0\}$, where $\sigma(A)$ denotes the spectrum of A. We then estimate the decay at infinity of $\|T(t)A(1 - A)^{-2}\|$ in terms of the growth of the resolvent operator along the imaginary axis. The same technique is applied to get similar decay estimates for bounded C_0-semigroups whose infinitesimal generator has an arbitrary finite number of spectral values on the imaginary axis. These results are in the spirit of the ones recently obtained by C. J. K. Batty and T. Duckaerts and they are motivated by applications to wave equations.

Asymptotics of zeros of Heine-Stieltjes polynomials and critical measures

A. Martínez-Finkelshtein

Heine-Stieltjes polynomials can be considered as generalized eigenfunctions of certain second order linear differential operators with polynomial coefficients. They generalize the families of classical orthogonal polynomials and arise in several areas of mathematics and physics. Their zeros are saddle points of discrete energy functionals, and the description of their asymptotics is made in terms of critical measures, that generalize the standard notion of equilibrium measures in potential theory. We discuss briefly a connection of these measures with trajectories of quadratic differentials and extremal problems on the plane.

This is talk is based on a joint work with E. Rakhmanov.

Operators with specification properties

F. Martínez-Giménez

A continuous map $f : X \to X$ on a compact metric space (X, d) has the strong specification property (SSP) if for any $\delta > 0$ there is a positive integer N_δ such that for any integer $s \geq 2$, any set $\{y_1, \ldots, y_s\} \subset X$ and any integers $0 = j_1 \leq k_1 < j_2 \leq k_2 < \cdots < j_s \leq k_s$ satisfying $j_{r+1} - k_r \geq N_\delta$ for $r = 1, \ldots, s-1$, there is a point $x \in X$ such that, for each positive integer $r \leq s$ and all integers i with $j_r \leq i \leq k_r$, the following conditions hold:

$$d(f^i(x), f^i(y_r)) < \delta,$$

$$f^n(x) = x, \quad \text{where } n = N_\delta + k_s.$$

The special case $s = 2$ is called periodic specification property (PSP).

We are interested in the study of continuous and bounded operators on Banach spaces having specification properties. Based on concrete examples, it seems that the periodic specification property is stronger than chaos in the sense of Devaney but a complete study of this property for bounded operators must be done. This include the relationships with other well known dynamical properties for operators like hypercyclicity, mixing, different notions of chaos, etc.

We have made some advances in this topic and we found characterizations for different types of specification properties for backward shift operators on Banach sequence spaces. Namely, we proved that for a bounded shift operator B defined on $\ell^p(v)$, $1 \leq p < \infty$ (or $c_0(v)$) the following conditions are equivalent:

(i) $\sum_{n=1}^{\infty} v_n < \infty$ ($\lim_{n\to\infty} v_n = 0$).

(ii) B has SSP.

(iii) B has PSP.

(iv) B is Devaney chaotic.

Joint work with S. Bartoll and A. Peris.

Schur complements in Krein spaces

F. Martínez Pería

Given a (bounded) semidefinite positive operator A acting on a Hilbert space $(\mathcal{H}, \langle\ ,\ \rangle)$ and a closed subspace $\mathcal{S}$ of $\mathcal{H}$, Anderson-Trapp [1] defined the Schur complement (or shorted operator) $A_{/\mathcal{S}}$ by

$$A_{/\mathcal{S}} = \max_{\leq}\{X \in L(\mathcal{H}) : 0 \leq X \leq A,\ R(A) \subseteq \mathcal{S}^{\perp}\},$$

where $\leq$ stands for the usual order in the real vector space of selfadjoint operators. Later, Pekarev [2] proved that

$$A_{/\mathcal{S}} = A^{1/2} P_{\mathcal{M}^{\perp}} A^{1/2},$$

where $P_{\mathcal{M}^{\perp}}$ is the orthogonal projection onto the subspace $\mathcal{M}^{\perp} = A^{-1/2}(\mathcal{S}^{\perp})$.

The aim of this talk is to present a generalization of Schur complements to Krein spaces, following the ideas used by Pekarev to establish the above formula. In order to do so, recall that every J-selfadjoint operator $A \in L(\mathcal{H})$ can be factorized as $A = DD^{\#}$ where $D \in L(\mathcal{K}, \mathcal{H})$ is injective and $\mathcal{K}$ is another Krein space. However, this factorization is not unique, see [3].

Let $(\mathcal{H}, [\ ,\ ])$ be a Krein space with fundamental symmetry J. Given a (bounded) J-selfadjoint operator $A \in L(\mathcal{H})$ with the unique factorization property and a (suitable) closed subspace $\mathcal{S}$ of $\mathcal{H}$, the Schur complement $A_{/[\mathcal{S}]}$ is defined by

$$A_{/[\mathcal{S}]} = D P_{\mathcal{M}^{[\perp]}//\mathcal{M}} D^{\#},$$

where $P_{\mathcal{M}^{[\perp]}//\mathcal{M}}$ is the J-selfadjoint projection onto $\mathcal{M}^{[\perp]} = D^{-1}(\mathcal{S}^{[\perp]})$. Then, the basic properties of $A_{/[\mathcal{S}]}$ are developed and different characterizations are given, most of them resembling those of the shorted of (bounded) positive operators on a Hilbert space.

The talk is based on a joint work with A. Maestripieri [4].

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Finite Section Method for a Banach Algebra of Convolution Type Operators on $L^p(\mathbb{R})$ with Symbols Generated by PC and SO

H. Mascarenhas

We prove the applicability of the finite section method to operators belonging to the algebra generated by operators of multiplication by piecewise continuous functions and convolution operators with symbols in the algebra generated by piecewise continuous and slowly oscillating functions.

The talk is based on a joint work with A. Karlovich and P.A. Santos

Linearization and factorization of singular operator functions

V. Matsaev

Consider a function $A(z)$ with values in the space of bounded operators on a Hilbert space, analytic and bounded in a simply connected domain D in the complex plane whose boundary is a smooth closed curve. We construct a linearizator of $A(z)$ in an appropriate functional space. With its help we investigate inner linearization recently introduced by H. Langer, A. Markus, and the author (see [IEOT 63 (2009), 533-545]).

The distinction of the results in the talk is that they admit appearance of the spectrum of $A(z)$ on the boundary of the domain D and possibly non-selfadjointness of $A(z)$.

The talk is based on joint results of H. Langer, A. Markus, and the author.

Generic rank one perturbation of complex Hamiltonian matrices

C. Mehl

We study the perturbation theory of complex Hamiltonian matrices under generic rank one perturbations. If an unstructured perturbation is applied, then generically only (one of) the largest Jordan blocks for a given eigenvalue is destroyed, but all other Jordan block for that eigenvalue remain the same. The situation is different if one considers structured perturbations. There exist Hamiltonian matrices for which Hamiltonian rank one perturbations will generically increase the size of the largest Jordan block associated with the eigenvalue zero. In the talk, we explain the reasons for this surprising behaviour.

This talk is based on joint work with V. Mehrmann, A.C.M. Ran, and L. Rodman.

Self-adjoint operator polynomials and their application in optimal control

V. Mehrmann

We discuss the operators associated with discrete and continous time optimal control problems for high order linear differential-algebraic systems with variable coefficients and their discrete-time analogs. We show that the optimality boundary value problems are associated with a self-adjoint differential-algebraic operator. We discuss the properties of these operators and show that the corresponding resriction to constant coefficient system leads to odd/even or palindromic/anti-palindromic operators.

We also discuss discretization methods that approximate continuous-time self-adjoint operators with discrete-time self-adjoint operators.

This is joint work with L. Scholz.

Resonances in Quantum Chemistry: Complex Absorbing Potential Method for Systems

M. Melgaard

The Complex Absorbing Potential (CAP) method is widely used to compute resonances in Quantum Chemistry, both for scalar valued and matrix valued Hamiltonians. In the semiclassical limit $\hbar \to 0$ we consider resonances near the real axis and we establish the CAP method rigorously in an abstract matrix valued setting. The proof is based on pseudodifferential operator theory and microlocal analysis.

The talk is based on a joint work with J. Kungsman.

Finite dimensional Sturm Liouville vessels and their tau functions

A. Melnikov

In this work there is developed a theory of finite dimensional vessels, defined in [M, MV1, MVc], which originates at the work of M. Livšic [Liv1] and has many common points with [BV]. A key property of a vessel [Liv2] is that its transfer function intertwines solutions of Linear Differential Equations (LDEs) with a spectral parameter. In a special case, by choosing parameters of the vessel one obtains that its transfer function intertwines solutions of the Sturm Liouville [S, L] differential equations

$$-\frac{d^2}{dx^2}y(x) + q(x)y(x) = \lambda y(x) \tag{1}$$

with the spectral parameter λ and the *potential* $q(x)$, which is very similar to Crum transformations [Crum]. In this work it is supposed that the potential is continuously differentiable and that all its derivatives tend to zero as $x \to \infty$.

Tau function τ arises as the determinant of an invertible matrix [JMU, N] (and in general is the determinant of a Fredholm operator). As a result there arises a differential ring $\boldsymbol{\mathcal{R}}_*$ generated by $\frac{\tau'}{\tau}, e^{\int q}$, and it is proved that at the generic case ("almost always") all relevant objects (transfer function, solutions of SL equation (1)) belong to this special differential ring. For a given choice of spectral parameters, one can also analyze the Galois differential group, corresponding to the potential $q(x)$ and a choice of the spectral value λ. Solutions are Liouvillian [PS] in this case, since they are pure exponents, multiplied by polynomials.

This work can be considered as a first step toward analyzing and constructing Lax Phillips scattering theory [Pov, Fa] for Sturm Liouville differential equations on a half axis $(0, \infty)$ with singularity at 0. On the other hand, there is developed a rich and interesting theory of Vessels which has connections to the notion of τ function, arising in non linear differential equations and to the Galois differential theory for LDEs [PS, K, H].

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The inverse Problem in the quantum theory of scattering using theory of vessels

A. Melnikov

In this talk I am going to present a theory of Sturm Liouville vessels (developed in the finite-dimensional case in [M2], and originated in [Li, P, MV1, MVc, M, BV]), applied to the study of the inverse Problem in the quantum theory of scattering [F, LxPh]. This theory is a special case of a more general theory of vessels [AMV]. In the classical case, under assumption $\int_0^\infty x|q(x)|dx < \infty$ on the *potential* $q(x)$, one is interested in solutions of the Sturm Liouville (SL) differential equation [L, S]

$$-\frac{d^2}{dx^2}y(x) + q(x)y(x) = \lambda y(x), \quad \lambda \in \mathbb{C} \tag{1}$$

and compares them to the pure exponents, which can be viewed as solutions of the trivial SL equation, corresponding to $q(x) = 0$.

Transfer function of a Sturm Liouville vessel is of the form

$$S(x,\lambda) = I_2 - B^*(x)\mathbb{X}^{-1}(x)(\lambda I - A)^{-1}B(x)\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

where $A, \mathbb{X}(x)$ are bounded operators on a Hilbert space $\mathcal{H}$, $B(x) : \mathcal{H} \to \mathbb{C}^2$ is bounded. Operators depends continuously on x and satisfy vessel conditions, which are linear differential equations. For a fixed $x = x_0$, the function $S(x,\lambda)$ is a realization of a Schur class function and Schur algorithm arises in the study of this problem, which actually corresponds to finite dimensional vessels [M2]. There also arises an interesting connection to Crum transformations [Crum] in that case.

We will construct Jost solutions and study the role of the tau function $\tau(x) = \det \mathbb{X}(x)$, where $\mathbb{X}(x) = I + T(X)$, for a trace class operator $T(x)$. Moreover, functions $\Omega(x,y), K(x,y)$ satisfying Gelfan-Levitan-Marchenko equation [F, Mar] will be constructed using the vessel.

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Landau inequalities and extensions of domains for convoluted families of operators

P.J. Miana

In this talk, we present some recent results about convoluted semigroups and convoluted cosine functions defined in Banach spaces. These families of operators include C_0-semigroups and cosine functions. We show that local families of these operators may be extended in bigger domains. We also show that they satisfy Landau-Kolmogorov inequalities. These results are joint obtained with V. Keyantuo, C. Lizama and V. Poblete.

Functional perturbation results for PDE eigenvalue problems

A. Miedlar

We discuss a functional perturbation results, i.e., a functional normwise backward error for PDE eigenvalue problems. Inspired by the work of M. Arioli et al. for linear systems arising from the finite element discretization of boundary problems we will extend the ideas of functional compatibility and condition number to eigenvalue problems in their variational formulation. At the end some first ideas about stopping criteria for iterative eigenvalue solvers in the context of adaptive methods will be introduced.

This is a joint work with V. Mehrmann.

Minimal spectral functions of an ordinary differential operator

V. Mogilevskii

Let $l[y]$ be a formally selfadjoint differential expression of an even order $2n$ on the semiaxis $[0,\infty)$, let L_0 and $L(=L_0^*)$ be the corresponding minimal and maximal operator in $\mathfrak{H}=L_2[0,\infty)$ and let $\mathcal{D}$ by the domain of L.

Assume that $\hat{\mathcal{K}}$ is a subspace in $\mathbb{C}^{2n}$ and $(N_0 \quad N_1):\mathbb{C}^n\oplus\mathbb{C}^n\to\hat{\mathcal{K}}$ is a symmetric operator pair such that $\operatorname{ran}(N_0 \quad N_1)=\hat{\mathcal{K}}$. By using a decomposing boundary triplet for L [1] we consider the boundary value problem

$$l[y]-\lambda y=f \tag{1}$$

$$N_0y^{(2)}(0)+N_1y^{(1)}(0)+C'_{01}(\lambda)\Gamma'_0y-C'_{11}(\lambda)\Gamma'_1y=0 \tag{2}$$

$$C'_{02}(\lambda)\Gamma'_0y-C'_{12}(\lambda)\Gamma'_1y=0 \tag{3}$$

where $f\in\mathfrak{H}$, $y^{(1)}(0)=\{y^{[k-1]}(0)\}_{k=1}^n$ and $y^{(2)}(0)=\{y^{[2n-k]}(0)\}_{k=1}^n$ are vectors of quasi-derivatives at 0 and Γ'_0y, Γ'_1y are boundary values of a function $y\in\mathcal{D}$ at the point $b=\infty$. Moreover we suppose that the matrices

$$C_0(\lambda)=\begin{pmatrix}N_0 & C'_{01}(\lambda)\\ 0 & C'_{02}(\lambda)\end{pmatrix},\quad C_1(\lambda)=\begin{pmatrix}N_1 & C'_{11}(\lambda)\\ 0 & C'_{12}(\lambda)\end{pmatrix},\quad \lambda\in\mathbb{C}\setminus\mathbb{R} \tag{4}$$

form a Nevanlinna pair $\mathcal{P}=(C_0(\lambda):C_1(\lambda))$.

The problem (1)-(3) is a particular case of a general Nevanlinna boundary problem [2] and hence it generates a generalized resolvent and the corresponding spectral function $F_{\mathcal{P}}(t)$ of the operator L_0. In turn the problem (1)-(3) involves a decomposing boundary problem (when $C'_{01}(\lambda)=C'_{11}(\lambda)=0$).

With the boundary problem (1)-(3) we associate the m-function $m_{\mathcal{P}}(\lambda)$, which is a uniformly strict Nevanlinna function with values in $[\hat{\mathcal{K}}]$. This function is a generalization of the classical Titchmarsh-Weyl function known for decomposing boundary problems.

Let $\mathcal{K}$ be a subspace in $\mathbb{C}^{2n}$ and let $\varphi(\cdot,\lambda)(\in[\mathcal{K},\mathbb{C}])$ be an operator solution of the equation $l[y]-\lambda y=0$ with the constant initial data $\varphi^{(j)}(0,\lambda)=S_j,\ j\in\{0,1\},\ \lambda\in\mathbb{C}$ and such that $\operatorname{Ker}\varphi(t,\lambda)=\{0\}$. As is known a distribution $\Sigma_{\mathcal{P}}(\cdot):\mathbb{R}\to[\mathcal{K}]$ is called a spectral function of the boundary problem (1)-(3) corresponding to the solution φ if for each finite function $f\in\mathfrak{H}$ the Fourier transform $g_f(s):=\int\limits_0^b\varphi^*(t,s)f(t)\,dt$ satisfies the equality

$$((F_{\mathcal{P}}(b)-F_{\mathcal{P}}(a))f,f)_{\mathfrak{H}}=\int\limits_{[a,b)}(d\Sigma_{\mathcal{P}}(s)\,g_f(s),g_f(s)),\quad [a,b)\subset\mathbb{R}.$$

The natural problem is a description of spectral functions $\Sigma_{\mathcal{P}}(\cdot)=\Sigma_{\mathcal{P},\min}(\cdot)$ with the minimally possible value of $\dim\mathcal{K}$ [3] (we denote this value by $d_{\min}$). The complete solution of this problem is given in the following theorem.

Theorem 1. 1) Each selfadjoint boundary value problem admits the representation (1)-(3) with a selfadjoint pair $\mathcal{P}=(C_0:C_1)$ given by (4).

2) Let $\mathcal{P}$ be a selfadjoint pair (4) and let (1)-(3) be the corresponding boundary problem. Moreover assume that $\varphi_N(\cdot,\lambda)(\in[\hat{\mathcal{K}},\mathbb{C}])$ is an operator solution of the equation $l[y]-\lambda y=0$ with $\varphi_N^{(1)}(0,\lambda)=-N_0^*$, $\varphi_N^{(2)}(0,\lambda)=N_1^*,\ \lambda\in\mathbb{C}$. Then: (i) $n\le\dim\hat{\mathcal{K}}\le k$, where k is the defect number of L_0;

(ii) there exists a spectral function $\Sigma_{\mathcal{P},N}(\cdot):\mathbb{R}\to[\hat{\mathcal{K}}]$ of the problem (1)-(3) corresponding to φ_N;

(iii) $d_{\min}=\dim\hat{\mathcal{K}}$ and all minimal spectral functions are given by $\Sigma_{\mathcal{P},\min}(s)=X^*\Sigma_{\mathcal{P},N}(s)X$, where X is an isomorphism in $\hat{\mathcal{K}}$.

Corollary 2. Let under conditions of Theorem 1 $\widetilde{A}$ be a selfadjoint extension of L_0 defined by the boundary conditions (2), (3). Then the spectral multiplicity of the operator $\widetilde{A}$ does not exceed $\dim\hat{\mathcal{K}}(=\operatorname{rank}(N_0\ \ N_1))$.

Moreover for a fixed symmetric pair $(N_0\ \ N_1)$ we describe all spectral functions $\Sigma_{\mathcal{P},N}(s)$ in terms of the Nevanlinna boundary parameter $\mathcal{P}=(C_0(\lambda):C_1(\lambda))$. Such a description is given by means of the formula for m-functions $m_{\mathcal{P}}(\lambda)$ similar to the well known Krein formula for resolvents.

The above results can be extended to differential expressions with operator valued coefficients and arbitrary (possibly unequal) defect numbers.

The talk is based on a joint work with S. Hassi and M. Malamud.

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Linear maps on observables in von Neumann algebras preserving the maximal deviation

L. Molnár

We present results concerning the structure of certain isometries of operator algebras.

The most fundamental result of the area is due to R.V. Kadison who gave the complete description of all surjective linear isometries between (unital) C^*-algebras. He showed that any such transformation is a Jordan *-isomorphism multiplied by a unitary element.

In this talk we present the result in the paper [1] and determine the structure of all bijective linear maps between the spaces of self-adjoint elements of von Neumann algebras which preserve another numerical quantity, the so-called maximal deviation defined for quantum observables. It turns out that those transformations are also closely related to the Jordan *-isomorphisms of the underlying von Neumann algebras.

[1] L. Molnár, *Linear maps on observables in von Neumann algebras preserving the maximal deviation,* J. London Math. Soc. **81** (2010), 161–174.

Smoothness of Hill's potential and lengths of spectral gaps

V. Molyboga

Let $\{\gamma_q(n)\}_{n\in\mathbb{N}}$ be the lengths of spectral gaps in a continuous spectrum of the Hill-Schrödinger operators

$$S(q)u = -u'' + q(x)u, \quad x \in \mathbb{R},$$

with 1-periodic real-valued potentials $q \in L^2(\mathbb{T})$, $\mathbb{T} := \mathbb{R}/\mathbb{Z}$.

Let us consider the map

$$\gamma : q \mapsto \{\gamma_q(n)\}_{n\in\mathbb{N}}.$$

Then due to Garnett & Trubowitz, *Comm. Math. Helv.*, **59** (1984) we have

$$\gamma(L^2(\mathbb{T})) = l^2_+(\mathbb{N}), \quad l^2_+(\mathbb{N}) := \left\{\{a(k)\}_{k\in\mathbb{N}} \in l^2(\mathbb{N}) \mid a(k) \geq 0,\ k \in \mathbb{N}\right\}.$$

Theorem 1 ([1]). *Let weight function $\omega :\ [1,\infty) \to (0,\infty)$. We prove that under the condition*

$$\exists s \in [0,\infty) : \quad k^s \ll \omega(k) \ll k^{s+1},\ k \in \mathbb{N},$$

the map $\gamma : q \mapsto \{\gamma_q(n)\}_{n\in\mathbb{N}}$ satisfies the equalities:

$$i) \quad \gamma\left(H^\omega(\mathbb{T})\right) = h^\omega_+(\mathbb{N}), \qquad ii) \quad \gamma^{-1}\left(h^\omega_+(\mathbb{N})\right) = H^\omega(\mathbb{T}).$$

Here,

$$f(x) = \sum_{k\in\mathbb{Z}} \widehat{f}(k) e^{i\,k\pi x} \in H^\omega(\mathbb{T}) \Leftrightarrow \sum_{k\in\mathbb{Z}} \omega^2(k)|\widehat{f}(k)|^2 < \infty,$$
$$a = \{a(k)\}_{k\in\mathbb{N}} \in h^\omega_+(\mathbb{N}) \Leftrightarrow \sum_{k\in\mathbb{N}} \omega^2(k)|a(k)|^2 < \infty, \quad a(k) \geq 0,\ k \in \mathbb{N}.$$

All results were obtained jointly with V. Mikhailets.

[1] Mikhailets, V., Molyboga, V., *Smoothness of Hill's potential and lengths of spectral gaps*, arXiv: math.SP/1003.5000.

Bounds on variation of spectral subspaces under J-self-adjoint perturbations

A.K. Motovilov

Given a self-adjoint involution J on a Hilbert space $\mathfrak{H}$, we consider a J-self-adjoint operator L on $\mathfrak{H}$ of the form $L = A + V$ where A is a possibly unbounded self-adjoint operator commuting with J and V a bounded

J-self-adjoint operator anti-commuting with J. We establish optimal estimates on the position of the spectrum of L with respect to the spectrum of A and we obtain norm bounds on the operator angles between maximal uniformly definite reducing (and, in particular, spectral) subspaces of the perturbed operator L and those of the unperturbed operator A. All the bounds are given in terms of the norm of V and the distances between pairs of disjoint spectral sets associated with the operator A or/and the operator L. As an example, the quantum harmonic oscillator under a $\mathcal{PT}$-symmetric perturbation is discussed. The a priori and a posteriori sharp norm bounds obtained for the operator angles generalize the celebrated Davis-Kahan trigonometric theorems to the case of J-self-adjoint perturbations.

The talk is based on joint works with S. Albeverio, A.A. Shkalikov, and C. Tretter.

Parabolic equations on graphs with nonlinear boundary conditions

D. Mugnolo

We consider a linear heat equation on a network equipped with a general nonlinear boundary condition. After transforming it into a vector-valued partial differential equation on an interval with a coupled boundary condition, it is possible to discuss the original problem by means of gradient system techniques. We obtain global well-posedness and relevant qualitative properties of the solution to the system in dependence of properties of the nonlinear functions appearing in the boundary conditions. Our results generalize the case of a diffusion equation with nonlinear Robin boundary condition for an individual interval.

This is joint work with R. Pröpper.

Orbits of operators and operator semigroups

V. Müller

Let T be a bounded linear operator acting on a Banach space X. Typically the behaviour of the orbit $\{T^n x : n = 0, 1, \dots\}$ depends essentially on the choice of the initial vector $x \in X$.

The existence of very irregular orbits has been studied intensely in the theory of hypercyclic and supercyclic vectors.

The lecture will be a survey of results concerning the existence of very regular orbits, for example orbits satisfying $\|T^n x\| \to \infty$, or orbits with $\|T^n x\|$ "large" for all n.

Similar questions can also be studies for weak orbits $\langle T^n x, x^* \rangle$, where $x \in X, x^* \in X^*$.

We will also discuss analogous questions for strongly continuous semigroups $(T(t))_{t \geq 0}$ of operators.

From an abstract second-order differential system to a semigroup

I. Nakić

We study second-order differential systems involving sesquilinear forms as coefficients, which arise in the study of vibrational systems. We show how to translate these systems into linear Cauchy problems, and we solve them in terms of the semigroup theory. In particular, we show that, in the general case, the infinitesimal generator of the corresponding semigroup cannot be written as a block operator matrix, but its inverse can.

The talk is based on a joint work with A. Suhadolc and K. Veselić.

Norm estimate of operators related to the harmonic Bergman projection

K. Nam

It is well known that the weighted harmonic Bergman projection on the ball is bounded on the weighted L^p space for $1 < p < \infty$, but not for $p = 1$. Our first result is an optimal norm estimate for one-parameter family of operators associated with the weighted harmonic Bergman projections on the ball. We then use this result and derive an optimal norm estimate for the weighted harmonic Berman projections.

The talk is based on a joint work with B.R. Choe and H. Koo.

Maximal regularity of cylindrical parameter-elliptic boundary value problems

T. Nau

For parameter-elliptic boundary value problems in domains $V \subset \mathbb{R}^k$ with compact boundary, $\mathcal{R}$-sectoriality of the related L^p-realizations is known. We make use of this result to show $\mathcal{R}$-sectoriality for the L^p-realizations of a class of boundary value problems in cylindrical domains $W \times V$, where W is either given as the fullspace $\mathbb{R}^n$ or the cube $[0, 2\pi]^n$. Due to a result of L. Weis, this gives maximal regularity for the corresponding Cauchy problem. The differential operators A under consideration are assumed to resolve into two parts $A = A_1 + A_2$, both parameter-elliptic, such that A_1 acts merely on W and A_2 acts merely on V. As a strong tool to treat model problems of this kind, continuous and discrete operator-valued Fourier multiplier theorems are used. In this context, $\mathcal{R}$-boundedness of operator families, namely the range of $(\lambda + a_1(\cdot) + A_2)^{-1}$ plays a major role. This indicates how spectral properties of A_2 can be used to derive according properties for A. To some extent this approach allows to treat vector-valued boundary value problems in the above domains. In the case where W is given as a cube, periodic and antiperiodic boundary conditions are treated.

The talk includes a joint work with J. Saal.

On the unitary equivalence of absolutely continuous parts of self-adjoint extensions

H. Neidhardt

The classical Weyl-von Neumann theorem states that for any self-adjoint operator A_0 in a separable Hilbert space $\mathfrak{H}$ there exists a (non-unique) Hilbert-Schmidt operator $C = C^*$ such that the perturbed operator $A_0 + C$ has purely point spectrum. We are interesting whether this result remains valid for non-additive perturbations by considering the set Ext_A of self-adjoint extensions of a given densely defined symmetric operator A in $\mathfrak{H}$ and some fixed $A_0 = A_0^* \in \mathrm{Ext}_A$. We show that for some A

and A_0 the absolutely continuous spectrum $\sigma_{ac}(A_0)$ of A_0 remains stable under compact non-additive perturbations, that is, $\sigma_{ac}(A_0) = \sigma_{ac}(\widetilde{A})$ for any $\widetilde{A} \in \mathrm{Ext}_A$ provided that the resolvent difference $(\widetilde{A}-i)^{-1}-(A_0-i)^{-1}$ is compact and $\sigma_{ac}(A_0)$ might only increase if not. We investigate the same property for the absolutely continuous part of A_0 and show that for a wide class of symmetric operators A the absolutely continuous parts of $\widetilde{A} \in \mathrm{Ext}_A$ and A_0 are unitarily equivalent whenever their resolvent difference is compact. Namely, it is true whenever the Weyl function $M(\cdot)$ of a pair $\{A, A_0\}$ has bounded limits $M(t) := w - \lim_{y \to +0} M(t+iy)$ for a.e. $t \in \mathbb{R}$. This result is applied to direct sums of symmetric operators and Sturm-Liouville operators with operator potentials.

The talk is based on a joint work with M.M. Malamud

Spectral properties of an Oseen-type operator, appearing in mathematical models of fluid flow past a rotating body

J. Neustupa

We describe the spectrum of a linear Oseen–type operator which arises from equations of motion of a viscous incompressible fluid in the exterior of a rotating compact body. The operator is considered in the L^q–space. We show that the essential spectrum consists of an infinite set of overlapping parabolic regions in the left half–plane of the complex plane. The full spectrum coincides with the essential and continuous spectrum if the operator is considered in the whole 3D space. The proofs are based on the Fourier transform in the whole space and the transfer of the results to the exterior domain.

Wedge diffraction problems with angle $2\pi/n$ and Dirichlet and Neumann conditions

A.P. Nolasco

We consider harmonic wave diffraction problems in which the field does not depend on the third variable and the wave incidence is perpendicular. These problems are formulated as two-dimensional, mixed elliptic boundary value problems in a non-rectangular wedge. Using analytical methods for boundary integral operators (more precisely, pseudodifferential operators) together with symmetry arguments, we solve explicitly a number of reference problems for the Helmholtz equation regarding particular wedge angles, boundary conditions, and space settings, which can be modified and generalized in various ways. The solution of these problems in Sobolev spaces was open for some fifty years.

The talk is based on a joint work with F.-O. Speck and T. Ehrhardt.

Recovering matching conditions for star graphs

M. Nowaczyk

I will talk about the inverse problem for the Schrödinger operator on a star graph. It is proven that such Schrödinger operator, i.e. the graph, the real potential on it and the matching conditions at the central vertex, can be reconstructed from the Titchmarsh-Weyl matrix function associated with the graph boundary. I will show that the reconstruction is also unique if the spectral data include not the whole Titchmarsh-Weyl function but its principal block.

The talk is based on a joint work with S. Avdonin and P. Kurasov.

An iteration procedure for a class of difference equations

M.A. Nowak

We consider approximation methods for operator equations of the form

$$Au + Bu = f\,;$$

where A is a discrete Wiener-Hopf operator on l_p $(1 \leq p < \infty)$ which symbol has roots on the unit circle. Conditions on perturbation B and f are given in order to guarantee the applicability of projection-iterative methods. Effective error estimates, and simultaneously, decaying properties for solutions are obtained in terms of some smooth spaces. The talk is based of the work with P. A. Cojuhari [1], and also [2].

References

[1] Cojuhari P. A., Nowak M. A., *Projection-iterative methods for a class of difference equations*, Integral Equations and Operator Theory, 64 (2009), 155–175.

[2] Nowak M. A., *Approximation methods for a class of discrete Wiener-Hopf equations*, Opuscula Math., 29/3 (2009), 271–288.

On the shift semigroup on the Hardy space of Dirichlet series

A. Olofsson

We consider the Hardy space $\mathcal{H}^2$ of Dirichlet series

$$f(s) = \sum_{n=1}^{+\infty} a_n n^{-s}, \quad \Re(s) > 1/2,$$

with finite norm

$$\|f\|^2_{\mathcal{H}^2} = \sum_{n=1}^{+\infty} |a_n|^2 < +\infty.$$

The space $\mathcal{H}^2$ was introduced by Hedenmalm, Lindqvist and Seip in their in 1997 paper as a Dirichlet series counterpart of the standard Hardy space of the unit disc.

For every positive integer $n \in \mathbb{Z}^+$ we have a natural operator $S(n)$ acting on $\mathcal{H}2$ given by multiplication by the Dirichlet monomial n^{-s}, that is,

$$S(n)f(s) = n^{-s} f(s), \quad \Re(s) > 1/2,$$

for $f \in \mathcal{H}^2$. This provides us with a function $S : \mathbb{Z}^+ \ni n \mapsto S(n)$ which is easily seen to be a multiplicative semigroup of isometries. We characterize this shift semigroup $S : \mathbb{Z}^+ \to \mathcal{L}(\mathcal{H}^2)$ up to unitary equivalence by means of a Wold decomposition. As an application we have that a shift invariant subspace of $\mathcal{H}^2$ is unitarily equivalent to $\mathcal{H}^2$ if and only if it has the form $\varphi\mathcal{H}^2$ for some $\mathcal{H}^2$-inner function φ.

Balanced realizations

M.R. Opmeer

Various kinds of balanced realizations have been shown to be useful to solve certain problems in operator theory and control theory. We will give an overview of some of these applications with emphasis on the use of balanced realizations for model reduction in control theory. The decay rate of the singular values of a Hankel operator plays an important role in these model reduction applications of balanced realizations and we will therefore also consider this issue in a control of partial differential equations context.

Exploiting the natural block-structure of hierarchically-decomposed variational discretizations of elliptic boundary value problems

J. Ovall

Finite element discretizations of elliptic boundary value problems lead to large, sparse, but very ill-conditioned linear systems, which must be solved to within a certain tolerance to attain the optimal approximation quality of the computed finite element solution. Given a simplicial partition of the domain, the standard Lagrange space $\bar{V}$, consisting of globally-continuous functions which are polynomials of degree p when restricted to any simplex, is readily decomposed as $\bar{V} = \hat{V} \oplus \tilde{V}$, where $\hat{V}$ consists of piecewise-linear functions, and $\tilde{V}$ consists of the piecewise-degree-p-polynomials which vanish on the simplex vertices. This decomposition gives rise to a natural block-structure, with properties which can be exploited computationally by recognizing that:

1. the ill-conditioning of the matrix is the "fault" of the (relatively small) $(\hat{V}, \hat{V})$ block,
2. the (much larger) $(\tilde{V}, \tilde{V})$ block is spectrally equivalent to its diagonal,
3. a strong Cauchy inequality exists between the spaces $\hat{V}$ and $\tilde{V}$.

These considerations suggest a block Gauß-Seidel iteration which can be used either as a static iteration or as a preconditioner for a Krylov iteration (e.g. GMRES). After revisiting some of the known, but perhaps not well-known-enough results, we will provide a (hopefully new) argument that the full system is nearly solved to the required accuracy in the first Gauß-Seidel sweep. In the spirit of the aims of the minisymposium, and because the arguments seem most natural in this way, we will shift back and forth between finite element/variational and linear algebra notation and terminology.

θ-Antieigenvalues and θ-Antieigenvectors

K. Paul

We introduce the notion of θ-antieigenvalue and θ-antieigenvector of a bounded linear operator on complex Hilbert space and study the realtion between θ-antieigenvalue and centre of mass of a bounded linear operator. We show that the notion of real antieigenvalue, imaginary antieigenvalue and symmetric antieigenvalue follows as a special case of θ-antieigenvalue. We also show how the notion of total antieigenvalue is related to the θ-antieigenvalue. In fact, we show that all the notions of antieigenvalues studied so far follows from the concept of θ- antieigenvalue. We illustrate with example how to calculate the θ-antieigenvalue for an operator acting on a finite dimensional Hilbert space.

The talk is based on a joint work with G. Das.

The infinite-dimensional Sylvester differential equation and periodic output regulation

L. Paunonen

In this presentation we study the infinite-dimensional Sylvester differential equation

$$\dot{\Sigma}(t) + \Sigma(t)B(t) = A(t)\Sigma(t) + C(t), \qquad \Sigma(t_0) = \Sigma_0$$

where $(A(t), \mathcal{D}(A))$ and $(B(t), \mathcal{D}(B))$ are families of unbounded operators on Banach spaces X and Y, respectively, and $C(\cdot) \in C(\mathbb{R}, \mathcal{L}_s(Y, X))$. Our main interests are the results on the solvability of the equation and in particular the conditions for the existence of a unique periodic solution when the families $(A(t), \mathcal{D}(A))$ and $(B(t), \mathcal{D}(B))$ and the operator-valued function $C(\cdot)$ are periodic.

These results have an application in the output regulation of a distributed parameter system with a nonautonomous signal generator

$$\dot{v}(t) = S(t)v(t), \qquad v(0) = v_0,$$

where $S(\cdot) \in C^1(\mathbb{R}, \mathbb{C}^{q\times q})$ is a periodic function. We show that the controllers solving the periodic output regulation problem can be characterized using the properties of the periodic solution of a Sylvester differential equation. This generalizes the results related to the control of distributed parameter systems with autonomous exosystems where the controllers solving the output regulation problem can be characterized by the solvability of certain constrained Sylvester equations.

The talk is based on joint work with S. Pohjolainen.

Hypercyclic dynamics of translation operators and universal harmonic functions of slow growth

A. Peris

A harmonic function H on $\mathbb{R}^N$ is said to be universal with respect to translations if the set of translates $\{H(\cdot + a)\ ;\ a \in \mathbb{R}^N\}$ is dense in the space of all harmonic functions on $\mathbb{R}^N$ with the topology of local uniform convergence, that is, the topology of uniform convergence on compact subsets of $\mathbb{R}^N$, also called compact-open topology. Dzagnidze (1964) showed that there are universal harmonic functions on $\mathbb{R}^N$. Recently, Armitage (2005) proved that universal harmonic functions can also have slow growth. More precisely, given any $\phi : [0, +\infty[\to]0, +\infty[$, a continuous increasing function such that

$$\lim_{t\to\infty} \frac{\log \phi(t)}{(\log t)^2} = +\infty, \qquad (1)$$

then there is a universal harmonic function H on $\mathbb{R}^N$ that satisfies $|H(x)| \leq \phi(||x||)$ for all $x \in \mathbb{R}^N$. Armitage asked whether the condition (1) can be relaxed: Is it true that, for every $N > 2$, one can find universal harmonic functions on $\mathbb{R}^N$ with arbitrarily slow transcendental growth? In other words, can the exponent 2 of $\log t$ be reduced to 1? We answer positively this question.

Our techniques depend on the analysis of the hypercyclic behaviour of the translation operator $T_a : X \to X$, defined on certain Banach spaces X consisting of harmonic functions of slow growth.

This is a joint work with M. C. Gómez-Collado, F. Martínez-Giménez and F. Rodenas.

Poly-Bergman spaces on domains Möbius equivalent to a disk

L.V. Pessoa

Let U be a complex domain Möbius equivalent to a disk and let j be in $\mathbb{Z}_{\pm}$. The talk will focusses on explicit representation of the poly-Bergman projection $B_{U,j}$ in terms of the singular integral operators $S_{U,j}$. One also show how the Lebesgue space $L^2(U, \mathrm{d}A)$ decompose on the true poly-Bergman spaces. The poly-Bergman kernels of U are explicitly calculated.

Locally definite normal operators in Krein spaces

F. Philipp

Let N be a bounded normal operator in the Krein space $(\mathcal{H}, [\cdot,\cdot])$, i.e. $NN^+ = N^+N$, where N^+ denotes the Krein space adjoint of N. We say that a number $\lambda \in \sigma_{ap}(N)$ is a *spectral point of positive type* of N if for every sequence $(x_n) \subset \mathcal{H}$ with $\|x_n\| = 1$ for all $n \in \mathbb{N}$ and $(N - \lambda)x_n \to 0$ as $n \to \infty$ we have

$$\liminf_{n\to\infty} [x_n, x_n] > 0.$$

In the paper [1] the authors showed that there exists a local spectral function for the *selfadjoint* operator N on an interval Δ if every spectral point of N in Δ is of positive type. For normal operators we prove the following theorem.

Theorem. *Assume that $\sigma(\operatorname{Re} N) \subset \mathbb{R}$, $\sigma(\operatorname{Im} N) \subset \mathbb{R}$ and that there exist $M > 0$, $n \in \mathbb{N}$ and an open neighborhood $\mathcal{U}$ of $\sigma(\operatorname{Im} N)$ in $\mathbb{C}$ such that*

$$\|(\operatorname{Im} N - \lambda)^{-1}\| \leq M\,|\operatorname{Im} \lambda|^{-n} \quad \text{for all } \lambda \in \mathcal{U} \setminus \mathbb{R}.$$

If $R \subset \mathbb{C}$ is a closed rectangle such that every spectral point of N in R is of positive type with respect to N, then N has a local spectral function on R.

This talk is based on a joint work with C. Trunk and V. Strauss.

References

[1] H. Langer, A. Markus, V. Matsaev: Locally definite operators in indefinite inner product spaces. Math. Ann. **308** (1997), 405–424.

An evolutionary problem with an impedance type boundary condition

R. Picard

A well-posedness result for a time-shift invariant class of evolutionary operator equations is considered and exemplified by an application to an impedance type initial boundary value problem for the system of linear acoustics. The problem class allows for memory effects in the domain as well as on the domain boundary.

Examples and Applications of Test Functions

J. Pickering

Test functions can provide a useful framework for function theory. In particular, they make it easy to use various existing tools, such as realisation, modelling, transfer functions and functional calculus. We will introduce the framework, and give some examples of problems that can (and can't) be solved in this framework.

Dixmier traces of operators on Banach and Hilbert spaces

A. Pietsch

Traces of operators on a Hilbert space were first considered by J. von Neumann in 1932. In collaboration with F. J. Murray, he extended this concept to $W^{\star}$-algebras. Another direction of development led to a theory of traces of operators on Banach spaces (R. Schatten, A. Grothendieck, and A. Pietsch).

Originally, the definition of a trace was designed as a generalization of the classical trace of a square matrix. In a next step, traces were considered as unitarily invariant positive linear functionals. Taking this point of view, Dixmier (1966) constructed exotic traces by using a modification of Banach limits. Surprisingly, these strange objects became a powerful tool in Connes's *"Noncommutative Geometry."*

The lecture gives a new approach to the theory of Dixmier traces that is based on results obtained about 20 years ago. I would like to show that Banach space techniques are useful also in the setting of Hilbert spaces.

References

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[2] A. Pietsch, *Operator ideals with a trace*, Math. Nachr. 100 (1981), 61–91.

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[4] A. L. Carey and F. A. Sukochev, *Dixmier traces and some applications to non-commutative geometry*, Russian Math. Surveys 6 (2006), 1039–1099.

Existence of a tree of Stieltjes strings corresponding to given spectra of Dirichlet and Neumann problems

V. Pivovarchik

Transversal vibrations of a plane tree of Stieltjes strings rooted at an interior vertex is considered with Dirichlet boundary conditions at each pendant vertex. Continuity and Kirchhoff conditions are imposed at each interior vertex except of the root. Dirichlet problem is the one with Dirichlet conditions at the root and Neumann problem is the one with continuity and Kirchhoff conditions at the root. It is shown that strict interlacing is a sufficient condition for two sequences of real numbers to be the spectra of Neumann and Dirichlet problems generated by Stieltjes string recurrent relations on any prescribed tree. The corresponding necessary conditions are compared with the obtained sufficient strict interlacing conditions.

The talk is based mainly on results in [V. Pivovarchik, Existence of a tree of Stieltjes strings corresponding to two given spectra. J. Phys. A: Math. Theor. **42** (2009) no. 37, 375213].

One-dimensional and multidimensional spectral order

A. Płaneta

As shown by Kadison, the set $\mathcal{S}$ of all bounded selfadjoint operators on a complex Hilbert space $\mathcal{H}$ is an anti-lattice, which means that for $A, B \in \mathcal{S}$, a greatest lower bound for A and B exists with respect to the usual ordering "$\leqslant$" in $\mathcal{S}$ if and only if A and B are comparable (cf. [1]). A little bit earlier, Sherman proved that if the set of all selfadjoint elements of a C^*-algebra $\mathcal{A}$ of bounded linear operators on $\mathcal{H}$ is lattice ordered by "$\leqslant$", then $\mathcal{A}$ is commutative (cf. [3]). To overcome these disadvantages of the partial order "$\leqslant$", Olson introduced in 1971 the so called *spectral order*, which is denoted by "$\preccurlyeq$". He proved, among other things, that the set of all selfadjoint elements of a von Neumann algebra of bounded linear operators on $\mathcal{H}$ is a conditionally complete lattice with respect to the spectral order (cf. [2]).

We extend the spectral order to the case of n-tuples of spectrally commuting selfadjoint operators and investigate its properties.

References

[1] R. V. Kadison, Order properties of bounded self-adjoint operators, *Proc. Amer. Math. Soc.* **2** (1951), 505-510.

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The talk is based on a joint work with J. Stochel.

This work is supported by MNiSzW Grant N N201 526638.

Laplacians on non-Convex polygons, Quantum graphs and Dirichlet network squeezing

A. Posilicano

By Birman and Skvortsov it is known that the Laplace operator with domain $C_0^\infty(\Omega)$ is a symmetric operator with deficiency indices (n, n), where n denotes the number of non-convex corners of the plane curvilinear polygon Ω. Here we provide, by Krein's resolvent formula, all the self-adjoint extensions of such a symmetric operator. Then, by a resolvent correspondence between self-adjoint Laplacians on a graph G and self-adjoint Dirichlet Laplacians on the non-convex curvilinear polygon G^ϵ given by a suitable fattening of G, we propose a recipe for approximating a quantum graph by squeezing a Dirichlet network.

Convergence of the Dirichlet-to-Neumann map on thin branched manifolds

O. Post

We consider a family of manifolds $(X_\varepsilon)_\varepsilon$ that shrinks to a metric graph X_0 as $\varepsilon \to 0$, i.e., a topological graph where each edge is assigned a length. A simple example is given by the (smoothed) surface of the ε-tubular neighbourhood of X_0. Let Y_0 be the set of vertices of degree 1, and Y_ε the corresponding boundary of X_ε. Using boundary triples based on first order Sobolev spaces, we can define the Dirichlet-to-Neumann map $\Lambda_\varepsilon(z)$ associated to the boundary Y_ε in X_ε and a Laplace-type operator $\Delta_\varepsilon \geq 0$ for $\varepsilon \geq 0$. In particular, for suitable φ on Y_ε, we let $\Lambda_\varepsilon(z)\varphi$ be the "normal" derivative of the Dirichlet solution h of $(\Delta_\varepsilon - z)h = 0$ with boundary data φ.

Our main result is the convergence of the Dirichlet-to-Neumann map $\Lambda_\varepsilon(z)$ to $\Lambda_0(z)$ in a suitable sense, since the operators act in different spaces. In this way, we can approximately calculate the Dirichlet-to-Neumann operator on the more complicated space X_ε in terms of the simpler one on X_0. The talk is based on a joint work with J. Behrndt.

Operator-Lipschitz functions and spectral shift

D. Potapov

The talk will show that every Lipschitz function is operator-Lipschitz with respect to the Schatten classes S_p of compact operators with $1 < p < \infty$. The talk will also discuss a connection with integrability of the spectral shift function of higher order.

The talk is based on the joint work with F. Sukochev and A. Skripka.

On existence of maximal semidefinite invariant subspaces

S.G. Pyatkov

We consider the class of J-dissipative operators in some Krein space E. Recall that a Krein space is a Hilbert space H endowed with the usual inner product (x, y) as well as an indefinite inner product $[x, y] = (Jx, y)$, where J is a self-adjoint operator such that $J^2 = 1$. A subspace $L \subset H$ is said to be nonnegative (positive, uniformly positive) if the inequality $[x, x] \geq 0$ ($[x, x] > 0$, $[x, x] \geq \delta_0 \|x\|^2$ (δ_0 is a positive constant)) holds for all $x \in L$. If a nonnegative subspace L admits no nontrivial nonpositive extensions then it is said to be maximal nonpositive. By analogy, we can define nonpositive, negative, and uniformly negative subspaces as well as maximal positive, uniformly positive subspaces, etc. Let A be a linear operator in H with domain $D(A)$. A densely defined operator A is said to be dissipative in H if $-\mathrm{Re}\,(Ax,x) \geq 0$ for all $x \in D(A)$. A dissipative operator is said to be maximal dissipative or m-dissipative if it admits no nontrivial dissipative extensions. An operator A is said to be J-dissipative (J-maximal dissipative) in the Krein space $(H, [\cdot,\cdot])$ if JA is a dissipative (m-dissipative) operator in H. Given a J-maximal dissipative operator $A : H \to H$, we discuss a problem of finding maximal nonnegative (nonpositive) subspaces invariant under the operator A. The problem on the existence of invariant maximal semidefinite subspaces turned out to be a focus of attention in the theory of operators in Pontryagin and Krein spaces. The first results were obtained in the celebrated article by Pontryagin M.S. in 1944. Later his results were generalized by many authors, in particular, by Krein M.G., Langer H., Jonas P., Dritschel M.A., Azizov T.Ya., and Shkalikov A.A. The latest results and the bibliography can be found in [1]-[3].

We refine some results on existence of invariant maximal semidefinite subspaces for J-dissipative operators and present also new necessary condition for the existence of these subspaces. The results are applied to the particular case when the operator A admits the representation in the form of an operator matrix in the canonical decomposition $H = H^+ + H^-$, ($H^\pm = P^\pm H$, $J = P^+ - P^-$, $P^\pm$ – othoprojections) and to some differential operators. The main conditions ensuring the existence of maximal semidefinite invariant subspaces are stated in the terms of the interpolation theory of Banach spaces (see, for instance, [4]). For example, assume that the imaginary axis belong to the resolvent set of A and put $H_1 = D(A)$ and H_{-1} is a completion of H in the norm $\|u\|_{-1} = \|A^{-1}u\|$ ($\|\cdot\|$ is the norm in H). Assume also that we have the estimate $(A - i\omega)^{-1}\| \leq c/(1 + |\omega|)$ for all $\omega \in \mathbb{R}$ (c is a constant). In this case sufficient and necessary conditions of existence of semidefinite invariant subspaces are connected with the condition $(H_1, H_{-1})_{1/2,2} = H$.

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An interpolation problem for Potapov functions

U. Raabe

We will discuss briefly the structure of the Taylor coefficient sequences of matrix-valued functions which belong to the Potapov class (with respect to some signature matrix J) in the open unit disk and which are holomorphic at 0. Furthermore, an interpolation problem for such functions will be considered. Using an access which is based on central Potapov functions, we will present a complete description of the solution set of this problem. Moreover, the corresponding Weyl matrix balls will be discussed. The talk is based on joint work with B. Fritzsche and B. Kirstein.

Interpolation and Fuchsian groups

M. Raghupathi

We will present results on Nevanlinna-Pick interpolation and Töplitz corona problems for fixed-point algebras of Fuchsian groups. These results allow us to generalize earlier work of Abrahamse, and Ball.

We will outline two approaches to these problems. The first uses a distance formula due, in various forms, to Sarason, Ball, Arveson, and McCullough. The second considers the extremal case of the problem and is based on work of McCullough for planar domains.

Part of this talk is based on joint work with B.D. Wick.

Eigenvalues of rank one perturbations of matrices

A.C.M. Ran

The talk is concerned with the effects that rank one perturbations have on the eigenvalues of matrices, including Jordan structure. Many results concerning this problem are known, and in the talk we shall give an overview of existing results for matrices that have no special structure. Several new results will be mentioned as well.

The topic will be motivated by a practical problem, originating from systems and control theory. We shall show that the new results mentioned above might have impact on the design of a controller that stabilizes the system based on dynamic output feedback.

The talk is closely related to the lecture of Chr. Mehl in the session on matrices in indefinite inner product spaces, in which surprising results are presented for the case where the matrix has some structure.

The talk is based on joint work with Chr. Mehl, V. Mehrmann, L. Rodman, and on joint work with M. Wojtylak.

The discrete algebraic Riccati equation and Hermitian block Toeplitz matrices

A.C.M. Ran

In the lecture we shall discuss the representation of the full set of solutions of the discrete algebraic Riccati equation in terms of two solutions, the difference of which is invertible. It turns out that if two such solutions exist, then all solutions can be described in terms of one of these solutions and the solution of a Stein equation. A complete parametrization is available in that case, which relies on the theory of indefinite inner product spaces. Under some additional hypotheses we show that there are two solutions which differ by an invertible matrix. If time permits we shall discuss a special case connected to invertible Hermitian block Toeplitz matrices.

The talk is based on joint work with L. Lerer.

Asymptotics of the discrete spectrum of a model operator associated with a system of three particles on a lattice

T.H. Rasulov

In this talk we consider a bounded self-adjoint model operator H of the form
$$H = H_0 - V_1 - V_2,$$
associated with a system of three identical particles on a three-dimensional lattice. Here H_0 is the multiplication operator with the parameter function w and V_i, $i = 1, 2$, are partial integral operators. We discuss the case where the parameter function w has a special form with non-degenerate minimum at n points of the six-dimensional torus ($n > 1$).

If we denote by m the bottom of the essential spectrum $\sigma_{ess}(H)$ of H and by $N(z)$ the number of eigenvalues of H lying below z, $z < m$, then the main result of this talk is the following relation between the number n and the function $N(\cdot)$:
$$\lim_{z \to m-0} |log|z - m||^{-1} N(z) = n\mathcal{U}_0, \ (0 < \mathcal{U}_0 < \infty),$$
in the case where corresponding Friedrichs model has a threshold energy resonance.

The talk is based on results of the paper [1].

[1]. *T.Kh. Rasulov.* Asymptotics of the Discrete Spectrum of a Model Operator Associated with the System of Three-Particles on a Lattice. Theor. Math. Phys. **163** (2010), No. 1, 429–437.

L^∞ estimates for fractional resolvent powers

J. Rehberg

It is well known in the analysis of partial differential equations that L^∞-estimates are crucial. We will show that they are satisfied for elliptic divergence operators, even if the coefficients are nonsmooth and the boundary conditions are mixed. In particular, no regularity condition has to be demanded on the Dirichlet boundary part, not even continuity. We emphasize the point that these estimates are obtained even for fractional powers of such operators, which gives good perspectives for perturbation theory.

Einstein transforms

G. Ren

The Einstein gyrovetor space algebraically regulates the Beltrami ball model of hyperbolic geometry. Introduced by Einstein and founded the special theory of relativity, Einstein addition is the standard velocity addition of relativistically admissible velocities. The algebraic and geometric structure of Einstein addition is sufficiently studied by Ungar. By Ungar's theory, commutativity and associativity in associative algebra and Euclidean geometry are extended to gyrocommutativity and gyroassociativity in nonassociative algebra and hyperbolic geometry.

In this talk, we shall consider the analysis theory related to Einstein addition, or its variant, Einstein transform. The Einstein transform is a bijection of the real unit ball. Its complexification turns out to be exactly the Möbius transformation in the complex unit ball. This observation makes us to give an investigation of Einstein transform through holomorphic approach, including the Einstein invariant Laplacian, Poisson kernel, Bergman kernel, Schwarz Lemma, and Heisenberg group in $\mathbb{R}^n$.

Non-autonomous Ornstein-Uhlenbeck operators in exterior domains

A. Rhandi

We consider non-autonomous Ornstein-Uhlenbeck operators in smooth exterior domains $\Omega \subset \mathbb{R}^d$ subject to Dirichlet boundary conditions. Under suitable assumptions on the coefficients, the solution of the corresponding non-autonomous parabolic Cauchy problem is governed by an evolution system $\{P_\Omega(t,s)\}_{0\le s\le t}$ on $L_p(\Omega)$ for $1 < p < \infty$. Furthermore, L^p-estimates for higher order spatial derivatives and L^p-L^q smoothing properties of $P_\Omega(t,s)$, $0 \le s \le t$ are obtained.

Higher rank numerical ranges

L. Rodman

The rank k numerical range of a linear bounded operator A on a complex Hilbert space $\mathcal{H}$ is defined as follows:

$$\Lambda_k(A) = \{\lambda \in \mathbf{C} : PAP = \lambda P \text{ for some rank } k \text{ orthogonal projection } P \text{ on } \mathcal{H}\}.$$

Here k is a positive integer smaller than the dimension of $\mathcal{H}$. If $k = 1$, then the standard numerical range of A is obtained. Besides mathematical interest, the study of higher rank numerical ranges is motivated by applications in quantum error correction.

Basic properties of $\Lambda_k(A)$ will be discussed, such as convexity. In particular, under appropriate hypotheses, a description will be given of linear preservers of rank k numerical ranges, i.e., linear maps ϕ such that $\Lambda_k(A) = \Lambda_k(\phi(A))$ for all operators A (this result was obtained jointly with S. Clark, C.-K. Li, J. Mahle).

Perturbation analysis of canonical forms

L. Rodman

Consider selfadjoint and unitary linear transformations with respect to indefinite inner products in finite dimensional complex vector spaces. Such transformations have well known canonical forms. The talk will focus on the behavior of these forms under several types of small perturbations in the transformations. Of particular interest are structure preserving changes. Several recent results concerning structure preserving perturbations are presented, including persistence of the sign characteristig and the Lipschitz property of canonical bases. As an application, Lipschitz properties of invariant subspaces with various definiteness conditions are established.

Spectral theory for elliptic differential operators and Dirichlet-to-Neumann maps

J. Rohleder

We consider a formally symmetric second order elliptic differential expression of the form

$$\mathcal{L}u = -\sum_{j,k=1}^{n} (\partial_j a_{jk})\, \partial_k u + au$$

with variable coefficients on a bounded domain Ω in $\mathbb{R}^n$ and a class of selfadjoint realizations of $\mathcal{L}$ in $L^2(\Omega)$ with certain nonlocal boundary conditions, such that the spectrum of these operators is discrete. The main objective of this talk is to show how the eigenvalues and associated eigenspaces of these operators can be characterized with the help of a Dirichlet-to-Neumann map associated with $\mathcal{L}$ on the boundary of Ω.

The talk is based on a joint work with J. Behrndt.

On the invertibility of paired operators within ordered groups

E.M. Rojas

Using the Fourier coefficient representation of functions defined on L^p spaces over a compact connected abelian group, we will present an invertibility study for a class of paired operators acting between these L^p spaces. The main goal is to obtain sufficient conditions to the invertibility of this type of operators, and their inverses representation. This is done for the particular case of paired operators with coefficients in the corresponding Wiener algebra upon the consideration of consequent factorizations. As a consequence, conditions for the one-sided invertibility and two-sided invertibility of the paired operators are identified and explicit formulas for their inverses are also presented.

The talk is based on a joint work with L.P. Castro.

The operator Fejér-Riesz theorem

J. Rovnyak

This will be an expository lecture, accessible to nonexperts, that surveys a selection of topics centering around Rosenblum's operator generalization of the classical Fejér-Riesz theorem on the factorization of nonnegative trigonometric polynomials. The topics include a proof via Schur complements, an update on spectral factorization, and a glimpse at multivariable problems and the noncommutative theory. The lecture is based on joint work with M. A. Dritschel.

Passive impedance systems and stochastic realizations of stationary processes

N. Rozhenko

We state the connection between the theory of stochastic realizations ([1]) and the theory of passive impedance systems ([2], [3]). This connection leads us to a new approach to study stochastic processes, linear stochastic systems and corresponding prediction, filtering, stability problems, both in discrete and continuous time cases. We study stationary regular vector processes with spectral densities which are the boundary values of matrix-functions with bounded Nevanlinna characteristic. Such types of stochastic processes give rise of natural and widely used class of realization models. These models lead to simple recursion estimate algorithms called filters. Specifically the processes with rational spectral densities admit minimal realizations with finite-dimensional state spaces which take on the role of such filters.

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The talk is based on a joint work with D.Z. Arov.

On Rarita-Schwinger type operators

J. Ryan

Rarita-Schwinger operators, also known as Stein-Weiss operators, are first order differential operators that are natural generalizations of the Dirac operator which in turn is a natural generalization of the Cauchy-Riemann operator. Here we will review the basic facts of Rarita-Schwinger operators including their fundamental solutions, basic integral formulas and their conformal invariance and conformal covariance of their solutions. We will also look at their second order analogues, which are natural analogues of the Laplace operator. We will describe their basic integral formulas and conformal invariance.

The talk is based on a joint work with J. Li

On well-posedness and meromorphic solutions of the Korteweg-de Vries equation with non-decaying initial data supported on a left half-line

A. Rybkin

The talk is concerned with the initial value problem (Cauchy problem) for the Korteweg-de Vries (KdV) equation on the domain $-\infty < x < \infty, t \geq 0$:

$$\begin{cases} \partial_t V - 6V\partial_x V + \partial_x^3 V = 0 \\ V(x,0) = V_0(x) \end{cases} \tag{1}$$

with real initial data V_0 vanishing on $(0,\infty)$ and essentially arbitrary on $(-\infty,0)$. We show that if the spectrum of the half line Schrodinger operator

$$H = -\partial_x^2 + V_0(x), \quad u(-0) = 0 \tag{2}$$

is bounded below and has a non-trivial absolutely continuous component then the problem (1) is well-posed. Moreover, under the KdV flow any such initial profile $V_0(x)$ (no matter how rough and without any decay assumption) instantaneously evolves into a meromorphic function in x on the whole complex plane with no real poles. Our treatment is based on a suitable modification of the inverse scattering transform and a detailed investigation of the Titchmarsh-Weyl $m-$function associated with (2). As by-product, we improve some related results of others.

A functional calculus based on axially monogenic functions

I. Sabadini

We will recall the notion of slice monogenic functions and some of their properties. In particular, we will discuss the main features of the analogue of the Cauchy kernel. Then we will state an integral form of the Fueter mapping theorem, which allows to associate to any slice monogenic function an axially monogenic function. We will use this integral transform to define a functional calculus for n-tuples of linear operators.

The talk is based on a joint work with F. Colombo and F. Sommen.

Equiconvergence theorems for Sturm–Liouville operators with singular potentials

I.V. Sadovnichaya

We deal with the Sturm–Liouville operator

$$Ly = l(y) = -\frac{d^2y}{dx^2} + q(x)y,$$

with Dirichlet boundary conditions $y(0) = y(\pi) = 0$ in the space $L_2[0, \pi]$. We assume that the potential q is complex-valued and has the form $q(x) = u'(x)$, where $u \in W_2^{\theta}[0, \pi]$ with $0 < \theta < 1/2$. Here the derivative is treated in the distributional sense, and $W_2^{\theta}[0, \pi] = [L_2, W_2^1]_{\theta}$ is the Sobolev space with fractional order of smoothness defined by interpolation. We consider the problem of equiconvergence in $W_2^\theta[0,\pi]$ and $C^\theta[0,\pi]$–norm of two expansion of a function $f \in L_2[0,\pi]$. The first one is constructed using the system of the eigenfunctions and associated functions of the operator L, while the second one is the Fourier expansion in the series of sines.

Theorem. *Let $R > 0$, $0 < \theta < 1/2$. Consider operator L acting in the space $L_2[0,\pi]$ with the Dirichlet boundary conditions. Suppose that the complex-valued potential $q(x) = u'(x)$, where $u(x) \in B_{\theta,R}$.*

Let $\{y_n(x)\}_{n=1}^\infty$ be the system of the eigenfunctions and associated functions of the operator L and $\{w_n(x)\}_{n=1}^\infty$ be the biorthogonal system.

For an arbitrary function $f \in L_2[0,\pi]$ denote

$$c_n := (f(x), w_n(x)), \quad c_{n,0} := \sqrt{2/\pi}(f(x), \sin nx).$$

Then

1) there exist a natural number $M = M_{\theta,R}$ and a positive number $C = C_{\theta,R}$ such that for all $m \geq M$ and for all $f \in L_2[0,\pi]$

$$\left\| \sum_{n=1}^m c_n y_n(x) - \sum_{n=1}^m \sqrt{\frac{2}{\pi}} c_{n,0} \sin nx \right\|_{W_2^\theta} \leq C \left(\sqrt{\sum_{n \geq m^{1-\theta}} |c_{n,0}|^2} + \frac{\|f\|_{L_2}}{m^{\theta(1-\theta)}} \right).$$

2)

$$\left\| \sum_{n=1}^m c_n y_n(x) - \sum_{n=1}^m \sqrt{\frac{2}{\pi}} c_{n,0} \sin nx \right\|_{C^\theta} \to 0, \quad m \to +\infty.$$

Factorisation of the wave function and sine-Gordon theory in a semi-strip

A. Sakhnovich

Factorization formula for wave functions of integrable systems is studied. Results on uniqueness and existence in sine-Gordon theory are given as applications. Inverse spectral problem for skew-self-adjoint Dirac-type system is discussed too.

Some examples for radical Banach algebrae defined by fractional derivation

L. Sánchez-Lajusticia

We present a family of radical convolution Banach algebras of Sobolev type (defined in terms of fractional derivation). The particular case of (usual) derivatives on the interval (0,1) is treated in detail.

Galerkin method with graded meshes for Wiener-Hopf operators with PC symbols in L^p spaces

P.A. Santos

This paper is concerned with the applicability of maximum defect polynomial Galerkin spline approximation methods with graded meshes to systems of Wiener-Hopf operators with piecewise continuous generating function in L^p spaces. For this, an algebra of sequences is introduced, which contains the approximating sequences we are interested in. There is a direct relationship between the stability of the approximation method for a given operator and invertibility of the corresponding sequence in this algebra. Exploring this relationship, we apply techniques to make possible the use of local principles and identification of the local algebras in order to derive stability criteria for the approximation sequences.

Projective free Banach algebras and the stabilization problem in control theory

A. Sasane

he stabilization problem in control theory is, roughly speaking, the following: given an unstable plant, find a controller, such that the overall transfer function of their feedback interconnection is stable. It is known that plants possessing a coprime factorization are stabilizable. This raises the natural question of whether also stabilizable plants admit a coprime

factorization. It is also known that if the ring of stable transfer functions under consideration is projective free, then the answer to the question is yes. In this talk we will show that common classes of stable transfer functions are indeed projective free by first showing that all Banach algebras R with a contractible maximal ideal space are projective free.

The talk is based on joint work with Alexander Brudnyi.

Extension of the ν-metric

A. Sasane

The stabilization problem in control theory is, roughly speaking, the following: given an unstable plant P, find a controller C, such that the overall transfer function of their feedback interconnection is stable. In the *robust* stabilization problem, one goes a step further. One knows that the plant P is just an approximation of reality, and so one would really like the controller C to not only stabilize the *nominal* plant P, but also all sufficiently close plants P' to P. The question of what one means by "closeness" of plants thus arises naturally. So one needs a function d defined on pairs of stabilizable plants such that

1. d is a metric on the set of all stabilizable plants,
2. d is amenable to computation, and
3. d has "good"properties in the robust stabilization problem.

Such a desirable metric, was introduced by Glenn Vinnicombe in 1993, and is called the *ν-metric*. There essentially the ring R of stable transfer functions was taken to be the set of the rational functions without poles in the closed unit disk or, more generally, the disk algebra. The problem of what happens when R is some other ring of stable transfer functions of infinite-dimensional systems was left open. In this talk, we address this issue, and give an extension of the ν-metric.

The talk is based on joint work with Joseph A. Ball.

Selected topics on the extension problem for positive definite functions

Z. Sasvári

Heinz Langer and Mark Grigorievich Krein devoted numerous papers to the extension problem for positive definite functions. In the present talk we will speak about some selected topics from this field and about some recent applications.

Spectral properties of the Dirac operators on finite interval with potentials from L_p and Sobolev spaces

A.M. Savchuk

We consider the Dirac operator L generated in the space $(L_2[0,1])^2$ by the differential expression

$$B\frac{d}{dx} + Q, \qquad B = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \qquad Q = \begin{pmatrix} q_1 & q_2 \\ q_3 & q_4 \end{pmatrix},$$

and some regular boundary conditions. We study the cases when the entries of Q belong to the space $L_p[0,1]$, with some $p \in [1,\infty)$ or to the Sobolev space $W_2^\theta[0,1]$, $\theta \in [0,1/2)$. We find asymptotic formulae for the eigenvalues and eigenfunctions of the operator L. We find also conditions when the eigen and associated functions of this operator form a Riesz basis.

Noncommutative Positivstellensätze for matrix algebras

Y. Savchuk

Positivstellensätze express (some) positive elements of (some) $*$-algebras in terms of weighted sums of squares. Here positive elements can be defined by $*$-representations, by point evaluations or by general $*$-orderings. In the talk various types of Positivstellensätze for noncommutative $*$-algebras are proposed and discussed. New Positivstellensätze for matrix $*$-algebras are presented. Some open problems are formulated.

This is a joint work with K. Schmüdgen.

Hierarchies of noncommutative KdV-type equations

C. Schiebold

We start out from a recursive method to construct simultaneous solutions to all equations in the noncommutative counterparts to the potential KdV, KdV and mKdV hierarchies. Applications concern both the classical scalar hierarchies (countable nonlinear superposition) and matrix hierarchies (generalized multisoliton solutions). Finally we turn to structural properties of the recursion operator of the noncommutative KdV.

Large parts of the talk are based on joint work with S. Carillo.

Linear operators with two invariant subspaces: Swiss Cheese Theorem and an application to control theory

M. Schmidmeier

Through recent developments in the representation theory of finite dimensional algebras, refined invariants, new classification results, and detailed descriptions have been obtained for systems consisting of vector spaces, linear operators, and invariant subspaces, see in particular [RS, Section (0.1)].

In this talk we focus on the case of pairs of invariant subspaces, more precisely, we consider systems (V, T, U_1, U_2) where V is a finite dimensional vector space, T a linear operator acting on V nilpotently, and where $U_1 \subset U_2 \subset V$ are two T-invariant subspaces.

The complexity of the problem of classifying such systems depends on the upper bound n on the nilpotency index: For $n \leq 3$, there are only finitely many indecomposable systems, while the problem is considered infeasible for $n \geq 5$. The case where $n = 4$ is controlled by a finite dimensional algebra of tubular type $\mathbb{E}_7$. We determine the set of dimension triples

$$\{(\dim U_1, \dim U_2/U_1, \dim V/U_2) : (V, T, U_1, U_2) \text{ indecomposable}\} \subset \mathbb{R}^3,$$

which is contained in a cylinder with axis $(1,1,1)\mathbb{R}$, and show that the corresponding dual complex is unbounded, connected and simply connected but surprisingly contains holes.

In control theory, invariant subspaces arise as the controllable and the unobservable states in a linear time-invariant control system. Their quotient is the minimal realization in the Kalman decomposition, which we discuss for typical systems arising in this talk.

This is a report on joint work with A. Moore.

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Spectral properties of rotationally symmetric massless Dirac operators

K.M. Schmidt

In its original form, the Dirac operator is the fundamental Hamiltonian for relativistic quantum mechanics of a massive particle of spin 1/2. Without the mass term, it can be taken as a model for the neutrino particle; however, as it does not carry electrical charge, the addition of an electrostatic-type potential will not then be of physical interest. Nevertheless, the zero eigenspace of a massless Dirac-type operator with magnetic potential has been shown to be of fundamental importance in the question of the stability of matter. More recently, the massless Dirac operator with a plain electric potential, particularly in two dimensions, has gained interest as a physically relevant description of effective electron movement in graphene.

The results reported here show that the essential spectrum of massless Dirac operators with a rotationally symmetric potential in two and three spatial dimensions covers the whole real line. Moreover, limit values of the potential at infinity can be eigenvalues of the operator, but outside the limit range of the potential the spectrum is purely absolutely continuous under a mild variation condition on the radial potential.

A new estimate in the subspace perturbation problem

A. Seelmann

We study the problem of variation of spectral subspaces for bounded linear self-adjoint operators. We obtain a new estimate on the norm of the difference of two spectral projections associated with isolated parts of the spectrum of the perturbed and unperturbed operators. The result is discussed for off-diagonal perturbations. In this case this improves a result obtained earlier by Kostrykin, Makarov and Motovilov in [Trans. Amer. Math. Soc. (2007)].

This talk is based on a joint work with K. A. Makarov.

From Toeplitz matrix-sequences to generalized locally Toeplitz sequences

S. Serra-Capizzano

Recently, the class of Generalized Locally Toeplitz (GLT) sequences has been introduced as a generalization both of classical Toeplitz sequences and of variable coefficient differential operators. For every sequence of the class, a rigorous description of the asymptotic spectrum has been given in terms of a function (the symbol) that can be easily identified. The latter generalizes the notion of a symbol for differential operators (discrete and continuous) and for Toeplitz sequences where it is identified through the Fourier coefficients and is related to the classical Fourier Analysis. The GLT class has nice algebraic properties and indeed it has been proven that it is stable under linear combinations, products, and inversion when the sequence which is inverted shows a sparsely vanishing symbol (sparsely vanishing symbol = a symbol which vanishes at most in a set of zero Lebesgue measure). The GLT class is quite rich and virtually includes any Finite Difference or Finite Element approximation of PDEs, Toeplitz sequences with Lebesgue integrable symbols, and the algebra generated by such sequences. We will also present recent results in the block non-Hermitian setting, including the (local) approximation of systems of PDEs.

THE RIEMANN-HILBERT BOUNDARY VALUE PROBLEM WITH A COUNTABLE SET OF COEFFICIENT AND TWO-SIDE CURLING AT INFINITY OF ORDER LESS THEN $1/2$

P. Shabalin

Let L be the real axis at the plane of the complex variable z, $D = \{z, \mathrm{Im} z > 0\}$. We need to find an analytic function $F(z)$ in the domain D by the boundary condition $a(t)\mathrm{Re}F(t) - b(t)\mathrm{Im}F(t) = c(t), \quad t \in L$, where $a(t)$, $b(t)$, $c(t)$ are given real functions on L, continuous everywhere except of ordinary discontinuity points t_k, $k = \pm 1, \pm 2, \cdots$, $\lim\limits_{k\to\pm\infty} t_k = \pm\infty$.

Let us denote $G(t) = a(t) - ib(t)$ and let $\nu(t) = \arg G(t)$ be the branch defined on every interval of continuity of coefficients, so that the numbers $\delta_k = \nu(t_k + 0) - \nu(t_k - 0)$ satisfy conditions $0 \le \delta_k < 2\pi$, $k = \pm 1, \pm 2, \cdots$. Now let us denote $\kappa_k = \delta_k/\pi - [\delta_k/\pi]$, $k = \pm 1, \pm 2, \cdots$, where $0 \le \kappa_k < 1$. Let's consider that $\varphi_1(t)$ - the continuous component of function $\nu(t)$, satisfies to a condition $\varphi_1(t) = \nu^- t^\rho + \tilde{\nu}(t), t > 0, \quad \varphi_1(t) = \nu^+ |t|^\rho + \tilde{\nu}(t), t < 0$, where ν^-, ν^+, ρ are constant, $0 < \rho < 1/2$, $\tilde{\nu}(t) \in H_L(\mu)$, $0 < \mu \le 1$. We denote $n^*_-(x) = \sum\limits_{j=1}^{k-1} \kappa_{-j}, -t_{-k+1} \le x < -t_{-k}$, $n^*_+(x) = \sum\limits_{j=1}^{k-1} \kappa_j, t_{k-1} \le x < t_k$. Assume that the numbers t_j, κ_j are satisfy the conditions $n^*_+(x) = \Delta_+ x^{\kappa_+} + o(x^{\kappa_+}), n^*_-(x) = \Delta_- x^{\kappa_-} + o(x^{\kappa_-}), x \to +\infty$, where κ_+, κ_-, Δ_+, Δ_- are positive constants $\kappa_- < 1/2$, $\kappa_+ < 1/2$. We set that $n^*_-(-t_{-k}) - \Delta_-(-t_{-k})^{\kappa_-} = p_{-k}, \quad n^*_+(t_k) - \Delta_+(t_k)^{\kappa_+} = p_k$ where we choose the numbers p_{-k}, p_k, t_{-k}, t_k so that the inequalities $p_{-k} = -n^*_-(-t_{-k}) + \Delta_-(-t_{-(k+1)})^{\kappa_-}, p_k = -n^*_+(t_k) + \Delta_+(t_{k+1})^{\kappa_+}$ are fulfilled. At the given restrictions full research of resolvability of a homogeneous problem in one class is conducted. The formula of the common decision of an unhomogeneous problem is proved.

The talk is based on a joint work with R. Salimov.

On some new estimates related to Paley's problem for plurisubharmonic functions in C^n

R.F. Shamoyan

In [1, 2] the author introduces a very simple approach to obtain estimates related to PAley"s problem in case of several complex variables, which is based on onedimensional Essen"s inequality,restriction operator and a chain of elementar estimates connected with so- called slice function which usually allows to get results in higher dimension from one dimensional results.This approach will be discussed and developed.

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On classes of derivable functions generated by PDE's with constant coefficients

M. Shapiro

The aim of this talk is to show that given a PDE with real or complex partial derivatives and with constant coefficients it is possible, in certain cases, to assign to it an algebra (of hypercomplex numbers) and a set of functions with values in the algebra, in such a way that the components of functions are solutions of the PDE. It is a far-reaching generalization of the well known fact that solutions to the two-dimensional (real) Laplace equation can be provided by assigning to it the algebra of complex numbers and holomorphic functions with values in this algebra.

The talk is based on a joint work with A. Pogorui and R.M. Rodríguez-Dagnino, which was partially supported by CONACYT projects as well as by Instituto Politécnico Nacional in the framework of its COFAA and SIP programs.

On invariant subspaces of an operator, some exterior power of which is positive

P. Sharma

Let a linear operator A act in the finite dimensional space $\mathbb{R}^n$. In this case we can define its jth ($j = 2, \ldots, n$) exterior power $\wedge^j A$, which acts in the finite-dimensional space $\mathbb{R}^{C_n^j}$. Let for some j_0 ($1 \leq j_0 \leq n$) the operator $\wedge^{j_0} A$ leave invariant a pointed, closed and full convex cone $K_{j_0} \subset \mathbb{R}^{C_n^{j_0}}$. Let, in addition, the only nonempty subset of $\partial(K_{j_0})$, which is left invariant by $\wedge^{j_0} A$, be $\{0\}$. Then under some additional conditions we can prove, that the initial operator A leaves invariant a cone $\mathcal{T}(K_{j_0})$ of rank j_0, i.e. a closed subset $\mathcal{T}(K_{j_0}) \subset \mathbb{R}^n$, which satisfy the following conditions:

1) For every $x \in \mathcal{T}(K_{j_0})$, $\alpha \in \mathbb{R}$ the element $\alpha x \in \mathcal{T}(K_{j_0})$.

2) There is at least one j_0-dimensional subspace and no higher dimensional subspaces in $\mathcal{T}(K_{j_0})$.

Moreover, the operator A has a unique invariant j_0-dimensional subspace M, lying in $A\mathcal{T}(K_{j_0})$. The restriction of the operator A to the subspace M is invertible. If another finite-dimensional subspace $L \subset A\mathcal{T}(K_{j_0})$ is also invariant for the operator A, then $L \subset M$.

Some estimates of the spectral gap of the operator A are also obtained.

The talk is based on a joint work with O. Kushel.

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On the spectrum of some class of Jacobi operators in Krein space

I.A. Sheipak

The class of three-diagonal Jacobi matrix with exponentially increasing elements is considered. Under some assumptions the matrix corresponds to unbounded self-adjoint operator L in the weighted space $l_2(\omega)$

with the scalar product $(x,y) = \sum_{k=1}^{\infty} \omega_k x_k \overline{y_k}$. The weight can arise indefinite metric in some cases.

We proved that the eigenvalue problem for this operator is equivalent to the eigenvalue problem of Sturm–Liouville operator with discrete self-similar weight. The asymptotic formulas for eigenvalues are obtained. These formulas differ for cases of definite and indefinite metrics.

Theorem 1) Spectrum of operator L is discrete. If L is self-adjoint in Hilbert space then all eigenvalues are positive and simple. There exists a positive number c such that eigenvalues λ_n enumerated in increasing order satisfy the following asymptotic formula

$$\lambda_n = cq^n(1 + o(1)) \quad (n \to +\infty).$$

2) If L is self-adjoint in Krein space then all eigenvalues are simple. There exists a positive number c such that positive eigenvalues λ_n enumerated in increasing order satisfy the asymptotic formula

$$\lambda_{n+1} = cq^{2n}(1 + o(1)) \quad (n \to +\infty)$$

and negative eigenvalues λ_{-n} enumerated in increasing order by absolute values satisfy the following asymptotic formula

$$\lambda_{-(n+2)} = -cq^{2n+1}(1 + o(1)) \quad (n \to +\infty).$$

The work is supported by the Russian Fund for Basic Research (grant No. 10-01-00423).

On the L_p-L_q maximal regularity of the Stokes equations with first order boundary condition in a general domain

Y. Shibata

I would like to talk about some L_p-L_q maximal regularity result for the Stokes system with first order boundary condition in a general domain under the assumption of the unique solvability of weak Neumann or weak Dirichlet-Neumann equations, which arises in a study of the one or two phase Navier-Stokes flow with free surface in a general domain.

On perturbations of self-adjoint or normal operators

A. Shkalikov

In the first part of the talk we shall deal with perturbations of a self-adjoint or normal operator T with discrete spectrum. There are results which allow to compare the eigenvalue counting functions $n(r, T)$ and $n(r, T + B)$, provided that the perturbation B is relatively compact or p-subordinated to T, i.e.

$$\|Bx\| \leqslant \text{const}\|Tx\|^p \, \|x\|^{1-p}, \quad \text{for } x \in \mathcal{D}(T) \tag{1}$$

with some $p < 1$. There are results (obtained by M.Keldysh, F.Brauder, S.Agmon, V.Lidskii, I.Gohberg and M.Krein, A.Markus and V.Matsaev, V.Kaznelson, M.Agranovich and others) which allow to assert that the eigen- and associated vectors of the perturbed operator $T + B$ form a basis for Abel summubility method or an uncoditional basis, provided that some relations between p and the order of growth of $n(r, T)$ hold. We shall present similar results replacing the condition (1) by a weaker assumption

$$\|B\varphi_k\| \leqslant \text{const}|\mu_k|^p,$$

where $\{\varphi_k\}_{k=1}^{\infty}$ is an orthonormal system of the eigenvectors of T corresponding to the eigenvalues $\{\mu_k\}_{k=1}^{\infty}$. The advantage of the last condition will be demonstrated by concrete examples.

In the second part we will discuss perturbations of a self-adjoint operator T whose spectrum consists of infinitely many components $\{\sigma_k\}_{k=1}^{\infty}$ separated by gaps: $\text{dist}(\sigma_k, \sigma_{k+1}) \geqslant \text{const}$.

Invariant subspaces, definitizable operators and operators matrices

A.A. Shkalikov

We shall discuss results of H. Langer and their developments connected with invariant subspace problem, theory of definitizable operators and theory of operator matrices.

Boundary singularities of holomorphic generators

D. Shoikhet

We study a problem of separation of boundary singularities for generators of continuous semigroups of holomorphic self-mappings. It enables us to establish some quantitative algebraic and geometric characteristics related to the linearization models for semigroups of holomorphic mappings and the geometry of backward flow invariant domains. Yet another look at the problem in question leads to an infinitesimal version of boundary interpolation theorem for holomorphic generators in the spirit of Pick and Nevanlinna.

Approximation of singular Sturm-Liouville problems at regular singularity in indefinite metric framework

Yu. Shondin

We consider the singular expressions

$$\ell = -\frac{d^2}{dx^2} + v(x), \quad v(x) = \frac{\nu^2 - 1/4}{x^2} + \frac{v_{-1}}{x} + v_{reg}(x)$$

on $(0, b), b \leq \infty$ and assume $\nu \geq 1$. In this case the 'limit point case' prevails at $x = 0$ in L^2-setting of the boundary problem.However, in this case, there is a Pontryagin space realization S of the expression ℓ in a Pontryagin space Π_κ with $\kappa = [\frac{\nu+1}{2}]$, which is symmetric and has defect indices $(1, 1)$, and the 'limit circle' case at $x = 0$ is reproduced in Π_κ.

We disscuss the approximation of self-adjoint extensions of S by realizations of the following regular boundary problems

$$\ell y = zy, \quad y'(\varepsilon) = \beta(\varepsilon, z)y(\varepsilon),$$

on extending intervals (ε, b), when $\varepsilon \to 0$. Here $\beta(\varepsilon, z)$, for each $\varepsilon > 0$ is a polynomial of z of degree $[\nu]$. The coefficients of these polynomials are determined by asymptotic expansion in ε of the Titchmarsh-Weyl coefficients $m(\varepsilon, z)$ associated with family of the regular boundary problems for ℓ on (ε, b).

The talk is based on joint papers with A. Dijksma and A. Luger in [1].

References

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$\mathcal{PT}$-symmetric Robin boundary conditions

P. Siegl

We summarize the results on Hamiltonians with $\mathcal{PT}$-symmetric Robin boundary conditions: one-dimensional toy model, $\mathcal{PT}$-symmetric waveguide, and $\mathcal{PT}$-symmetric models in curved manifolds. The similarity to self-adjoint and normal operators is discussed. New interpretation of the $\mathcal{PT}$-symmetric Robin boundary conditions in quantum mechanics will be presented.

New results are based on a joint work with D. Krejčiřík and H. Hernandez-Coronado.

Asymptotic behaviour of flows in infinite networks

E. Sikolya

We consider a transport process on an infinite network and, using the corresponding flow semigroup, investigate its long term behavior. Combining methods from functional analysis, graph theory and stochastics, we are able to characterize the networks for which the flow semigroup converges strongly to a periodic group.

The talk is based on a joint work with B. Dorn and V. Keicher.

Banach algebras generated by Toeplitz and Hankel operators with piecewise continuous generating functions

B. Silbermann

The talk is devoted to the study of a symbol calculus for operators belonging to algebras generated by Toeplitz and Hankel operators with piecewise continuous generating functions. It is supposed for instance that these operators act on classical Banach spaces such as $l^p, L^p(\mathbb{R}), 1 < p < \infty$.

Square-integrable solutions and Weyl functions for singular canonical systems

H. de Snoo

Boundary value problems for the singular canonical system of differential equations $Jf'(t) - H(t)f(t) = \lambda\Delta(t)f(t)$ are studied in the Hilbert space $L^2_\Delta(\imath)$. With the help of a general monotonicity principle for non-decreasing matrix functions the square-integrable solutions are specified. The main purpose is to determine boundary triplets and Weyl functions for the maximal relation associated to the canonical differential equation in $L^2_\Delta(\imath)$ with possibly unequal defect numbers. It will be shown that the Weyl function M singles out the square-integrable solutions of the corresponding homogeneous system of canonical differential equations, so that M can be regarded as the natural generalization of the Titchmarsh-Weyl coefficient for singular Sturm-Liouville operators in the limit-point case.

This is joint work with J. Behrndt, S. Hassi, and R. Wietsma.

Micro-localization from Clifford Analysis

F. Sommen

The theory of formal boundary values of holomorphic functions in the upper and lower half plane leads to the definition of the spaces of hyperfunctions and microfunctions on the real line. This can be thought of as singular analytic signals. In the higher-dimensional case there are two approaches to the theory of hyperfunctions and of microfunctions: the classical one using functions of several complex variables and an approach from Clifford analysis, which is related to the Riesz transform and makes use of the monogenic Cauchy kernel, so that it is related to monogenic signals. In the process of micro-localization one starts from the Radon transform which gives the delta function as an integral over the sphere of one-dimensional delta functions on hyperplanes. The microlocal decomposition of the delta function is obtained as a deformation of the Radon inversion formula, where the kernel is singular in one point and in one direction. In our presentation we derive this formula from the Radon decomposition of the Cauchy kernel restricted on a parabolic surface. In this way complicated techniques from several complex variables can be bypassed.

Fourier-Borel transforms in Clifford Analysis

F. Sommen

In this presentation we introduce several generalizations to Clifford analysis of the classical Fourier-Borel transform for analytic or holomorphic functionals. We also prove that every analytic or holomorphic functional admits a unique decomposition as a series of Dirac derivatives of monogenic functionals, a result which is dual to the Fischer decomposition.

Progress in wedge diffraction

F.-O. Speck

Erhard Meister developed with me in the 1980's an operator theoretical approach for so-called canonical problems in diffraction theory. Starting from a weak formulation of Sommerfeld type (boundary value or transmission) problems we defined the associated operator (mapping the solution space into the data space) and constructed relations with singular operators, e.g., with Wiener-Hopf operators in Sobolev spaces, for which (generalized) inverses could be explicitly obtained, in closed analytical form, by matrix factorization methods. This research gave an impact to an intensive study of (1) the factorization of non-rational matrix functions (such as Daniele-Khrapkov matrices), (2) operator relations in general (such as equivalent after extension relations and its generalizations) and (3) new techniques for the normalization of singular operators (associated with ill-posed problems). In 1990-94 our cooperation with Francisco Teixeira and Frank Penzel resulted in a series of papers on rectangular wedge diffraction problems, in which Wiener-Hopf-Hankel operators had to be inverted. So far, this was possible only in particular cases with rather sophisticated methods. In 2002-05 I came back to this class of problems when preparing the Erhard Meister Memorial Volume (in the Birkhäuser OT series, Vol. 147). Together with Lus Castro, Francisco Teixeira and (finally) Roland Duduchava, we developed a rigorous approach to solve boundary value problems for the Helmholtz equation in a quadrant with rather arbitrary boundary conditions (including impedance and oblique derivative data). The results were based upon new techniques for an asymmetric factorization of scalar and matrix functions due to the Wiener-Hopf plus Hankel operators in view. The present method consists of a combination of our knowledge about the analytical solution of Sommerfeld and rectangular wedge diffraction problems with new symmetry arguments that relate the present to previously solved problems and yield the explicit analytical solution in a great number of cases. For this purpose we introduce here so-called "Sommerfeld potentials" (explicit solutions to special Sommerfeld problems) whose use turns out to be most efficient. It is surprising that the case where the angle is an integer part of 2π can be solved completely whilst the case of "rational" angles $\alpha = 2\pi m/n$ for $m > 1$ appears much harder and remains, in general, unsolved at present. As an interesting and very direct conclusion we obtain the result that, for the angles under consideration, the behavior of the field shows the same singularity in the corner as the singular behavior in the corresponding half-plane or Sommerfeld potential cases. This work is based on joint research with Torsten Ehrhardt and Ana Paula Nolasco. Some key references are given below.

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Factorizations, Riemann-Hilbert problems and the corona theorem

I.M. Spitkovsky

The solvability of the Riemann-Hilbert boundary value problem on the real line is described in the case when its matrix coefficient admits a Wiener-Hopf type factorization with bounded outer factors but rather general diagonal elements of its middle factor. This covers, in particular, the almost periodic setting, when the factorization multiples belong to the algebra generated by the functions $e_\lambda(x) := e^{i\lambda x}$, $\lambda \in \mathbb{R}$. Connections with the corona problem are discussed. Based on those, constructive factorization criteria are derived for several types of triangular 2×2 matrices with diagonal entries $e_{\pm\lambda}$ and non-zero off diagonal entry of the form $a_- e_{-\beta} + a_+ e_\nu$ with $\nu, \beta \geq 0$, $\nu + \beta > 0$ and $a_\pm$ analytic and bounded in the upper/lower half plane.

The talk is partially based on joint work with M.C. Câmara, C. Diogo and Yu.I. Karlovich.

Non-linearities and quaternionic holomorphic functions

W. Sprößig

Let G be a bounded domain in $\mathbb{R}^3$ with a smooth boundary. Classes of strong non-linear initial-boundary value problems are formulated and discussed. Solutions can be described with the help of quaternionic holomorphic functions.

Electrodynamics with quaternions

W. Sprößig

E. Meister published in 2001 a quaternionic approach on initial boundary value problems for electromagnetic fields. In this talk we will show advandages of the quaternionic formulation of Maxwell equations and their treatment using a quaternionic operator calculus. Some aspects in E. Meister's paper are integrated in this talk.

Passive and conservative state/signal systems in continuous time

O.J. Staffans

In this talk we discuss passive and conservative state/signal systems in continuous time. Such a system can be used to model, e.g., a passive linear electrical circuit containing lumped and/or distributed resistances, capacitors, inductors, and wave guides, etc. Most of the standard partial differential equations appearing in physics on can be written in state/signal form.

A passive state/signal system consists of three components:

A) an internal Hilbert *state space* $\mathcal{X}$,

B) a Kreĭn *signal space* $\mathcal{W}$ through which the system interacts with the external world, and

C) a *generating subspace* V of the product space $\mathcal{X} \times \mathcal{X} \times \mathcal{W}$.

The generating subspace is required to be maximally nonnegative with respect to a certain "energy" inner product and to satisfy an extra non-degeneracy condition. We denote this system by $\Sigma = (V; \mathcal{X}, \mathcal{W})$.

The set of all *classical trajectories* of Σ on some interval I consists of a continuously differentiable $\mathcal{X}$-valued state component x and a continuous $\mathcal{W}$-valued signal component w satisfying

$$(\dot{x}(t), x(t), w(t)) \in V, \qquad t \in I.$$

The set of all generalized trajectories of Σ is obtained from the family of all classical trajectories by a standard approxiation procedure.

By the *future behavior* of Σ we mean the set of all signal parts w of all stable trajectores (x, w) of Σ on $[0, \infty)$ satisfying the extra condition $x(0) = 0$. This set is a right-shift invariant subspace of $L^2([0, \infty); \mathcal{W})$ and it is maximal nonnegative with respect to the Kreĭn space inner product in $L^2([0, \infty); \mathcal{W})$ inherited from $\mathcal{W}$. Such a subspace is called a *passive future behavior.* Each passive future behavior can be realized as the future behavior of a passive state/signal system Σ, and it is possible to require Σ to have, for example, one of the following three sets of properties: a) Σ is observable and co-energy preserving; b) Σ is controllable and energy preserving; c) Σ is simple and conservative. Realizations satisfying of the sets of conditions a), b), or c) are determined uniquely by the given future behavior. It is even possible to construct *canonical* realizations, i.e., realizations with satisfy a), b), or c), and which are *uniquely determined by the given data.*

The talk is based on joint work with D.Z. Arov and M. Kurula.

AAK approximants to functions with branch points

H. Stahl

We are concerned with the asymptotic behavior of AAK (Adamjan-Arov-Krein) approximants to functions f that are holomorphic outside the unit disk $\mathbb{D}$ and a little bit beyond, and have all their singularities in a compact set of capacity zero in $\mathbb{D}$. Among these singularities there should be also branch points. Our main object is the investigation of the asymptotic distribution of the convergence behavior of the AAK approximants.

AAK approximants are meromorphic functions in $\mathbb{D}$ with a controlled, finite number of poles, and they have a minimal deviation in the uniform norm on $\mathbb{T}$ from the function f to be approximated. There is an important and very interesting connection with Hankel operators and their singular values, which is central for their understanding. In our talk this topic will play a minor role, instead we shall concentrate on questions that are important for understanding of the analytic background of the arcs in $\mathbb{D}$ that form the support of the asymptotic distribution. Methodologically, the main tools of the investigation belong to potential theory and geometric theory of functions.

The talk is based on a joint work with L. Baratchart and M. Yattselev.

Solution procedures for frictional contact

E.P. Stephan

The first part of the talk deals with dual formulations for unilateral contact problems with Coulomb friction. Starting from the complementary energy minimization problem, Lagrangian multipliers are introduced to include the governing equation, the symmetry of the stress tensor as well as the boundary conditions on the Neumann and contact boundary. Since the functional arising from the friction part is nondifferentiable an additional Lagrangian multiplier is introduced. This procedure yields a dual-dual formulation of a two-fold saddle point structure. Two different Inf-Sup conditions are introduced to ensure existence of a solution. The system is solved with a nested Uzawa algorithm.

In the second part of the talk a mixed hp-time discontinuous Galerkin method for elasto-dynamic contact problem with friction is considered. The contact conditions are resolved by a biorthogonal Lagrange multiplier and are component-wise decoupled. On the one hand the arising problem can be solved by an Uzawa algorithm in conjunction with a block-diagonalization of the global system matrix. On the other hand the decoupled contact conditions can be represented by the problem of finding the root of a non-linear complementary function. This non-linear problem can in turn be solved efficiently by a semi-smooth Newton method. In all cases numerical experiments are given demonstrating the strengths and limitations of the approaches.

The talk is based on a joint work with M. Andres and L. Banz.

Completion problem for subnormal and completely hyperexpansive weighted shifts

J. Stochel

Truncations of completely alternating sequences are entirely characterized. The completely hyperexpansive completion problem is solved for finite sequences of (positive) numbers in terms of positivity of attached matrices. Solutions to the problem are written explicitly for sequences of two, three, four, five and six numbers. As an application, an explicit solution of the subnormal completion problem for five numbers is given.

The talk is based on a joint work with Z.J. Jabłoński, I.B. Jung and J.A. Kwak.

Compactness of the complex Green operator

E. Straube

We indicate a new proof for, and a slight improvent of, a recent result of A. Raich on compactness of the complex Green operator on a CR-submanifold of $\mathbb{C}^n$ of hypersurface type. Let M be a smooth compact pseudoconvex CR-submanifold of $\mathbb{C}^n$ of hypersurface type, let $dim_{\mathbb{C}} T_z^{1,0}(M) = m-1$, $z \in M$, and let $1 \leq q \leq m-2$. If M satisfies property(P_q) and property(P_{m-1-q}), then the complex Green operator on $(0,q)$-forms is compact. Our proof is based on the fact that locally, M is CR-equivalent to an actual hypersurface, and the corresponding compactness result of Raich and the author for boundaries of domains.

On the unicity of the integro-polynomial representation for definitizable moment problems

V. Strauss

A real sequence $\{c_k\}_{k=0}^{\infty}$ is called *Hamburger-definitizable* if there are real numbers $\gamma_0, \gamma_1, \ldots, \gamma_n$ and a Hamburger moment sequence (i.e., a positive sequence) $\{d_k\}_{k=0}^{\infty}$ such that

$$d_k = \sum_{i=0}^{n} \gamma_i c_{i+k} \quad \forall\, k = 0, 1, \ldots .$$

In other words, $\{c_k\}_{k=0}^{\infty}$ is called Hamburger-definitizable if it can be mapped, by some finite difference operator, to a Hamburger moment sequence.

In the same way, a real sequence $\{c_k\}_{k=0}^{\infty}$ is called *Hausdorff-definitizable* if it can be mapped, by some finite difference operator, to a Hausdorff moment sequence.

Hamburger-definitizable and Hausdorff-definitizable sequences have an integro-polynomial representation constructed by the same way as the integro-polynomial representation for their trigonometric analog (see [1]). Here we study the problem of unicity for elements of this representation.

The talk is based on a joint work with L. Navarro.

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On a smoothness of a linear pencil and its factor

L.I. Sukhocheva

Let $L(t) = M(t)(Z - tI)$, where $L(t)$, $M(t)$ are matrix valued functions, Z is a quadratic matrix. If $M = M(t)$ is continuous and $L = L(t)$ is $n + 1 > p$ times differential at t_0 then $M = M(t)$ is $n + 1 - p$ times differentiable at t_0. Here p is the maximal size of a Jordan chains of Z at t_0 .

The work is supported by the Russian Foundation for Basic Researches, grant 08-01-00566-a.

Looking for the target space of a would-be Hankel operator

F.H. Szafraniec

Bearing complexity of Hankel operators in typical unbounded domains in mind I intend to propose an axiomatic in a sense approach to their definitions. It is somehow in spirit of V. Pták and P. Vrbovaá, Operators of Toeplitz and Hankel type, *Acta Math. Sci. (Szeged)* 52(1988), 117-140 though more analytical in the nature.

On some problems related with some special operator classes

Mubariz Tapdıgoğlu

We will consider the following operators:

$$\mathcal{K}_{\varphi,\theta,\Omega} := [T_{\overline{\varphi}\Omega}, T_\theta]\, \varphi\,(M_\theta)\,,$$

$$\mathcal{L}_{\varphi,\theta,\Omega} := [T_{\overline{\theta}\Omega}, T_\varphi]\, \varphi\,(M_\theta)\,,$$

where $\Omega \in (\sum) \cup \{1\}$, $\varphi \in H^\infty(\mathbb{D})$, $\theta \in (\sum)$ (the set of all inner functions), T_f $(f \in L^\infty(\partial\mathbb{D}))$ denotes the Toeplitz operator on the Hardy space $H^2(\mathbb{D})$ over the unit disc $\mathbb{D} = \{z \in \mathbb{C} : |z| < 1\}$ and $\varphi\,(M_\theta)\, h := P_\theta\,(\varphi h)\,,\, h \in \mathcal{K}_\theta := H^2 \Theta \theta H^2$, is the function of model operator of the Sz.-Nagy and Foias. In terms of Berezin symbols we study some properties of these operators. We also describe the invariant subspaces of some isometric multiplication operators on the Hilbert spaces with reproducing kernels in terms of so-called distance function of N.K. Nikolski. Some other problems also are considered.

This is a joint work with M. Gürdal.

This work is supported by the Scientific and Technological Research Council of Turkey (TÜBİTAK) with Project 109T590.

The Dirichlet-to-Neumann operator on rough domains

T. ter Elst

We consider a bounded connected open set $\Omega \subset \mathbb{R}^d$ whose boundary Γ has a finite $(d-1)$-dimensional Hausdorff measure. Then we define the Dirichlet-to-Neumann operator D_0 on $L_2(\Gamma)$ by form methods. The operator $-D_0$ is self-adjoint and generates a contractive C_0-semigroup $S = (S_t)_{t>0}$ on $L_2(\Gamma)$. We show that the asymptotic behaviour of S_t as $t \to \infty$ is related to properties of the trace of functions in $H^1(\Omega)$ which Ω may or may not have.

The talk is based on a joint work with W. Arendt.

Interpolation in de Branges-Rovnyak spaces

S. ter Horst

Branges-Rovnyak spaces play a prominent role in Hilbert space approaches to H^∞-interpolation, but a systematic study of interpolation for functions in a Branges-Rovnyak space is missing. In this talk we consider a general form of Nevanlinna-Pick interpolation for such functions. As Branges-Rovnyak spaces are Hilbert spaces, the solution set has, as one may expect, the form of the sum of a unique minimal norm solution and the solution to a metric constrained homogeneous interpolation problem. However, to obtain an explicit description of the solutions, requires a study of Redheffer transformations that come up in the description of a related Nevanlinna-Pick interpolation problem for H^∞-functions. This study leads to some observations on Redheffer transformations that parallel results on transfer-function realizations. The talk is based on joint work with J.A. Ball and V. Bolotnikov.

Weyl–Titchmarsh theory for Schrödinger operators with strongly singular potentials

G. Teschl

We develop Weyl–Titchmarsh theory for Schrödinger operators with strongly singular potentials such as perturbed spherical Schrödinger operators (also known as Bessel operators). It is known that in such situations one can still define a corresponding singular Weyl m-function and it was recently shown that there is also an associated spectral transformation. Here we will give a general criterion when the singular Weyl function can be analytically extended to the upper half plane and prove a local Borg–Marchenko type uniqueness result. Our criteria will in particular cover the aforementioned case of perturbed spherical Schrödinger operators.

The talk is based on a joint work with A. Kostenko and A. Sakhnovich.

When is a non-self-adjoint Hill operator a spectral operator of scalar type?

V. Tkachenko

We consider a Hill operator

$$H = -\frac{d^2}{dx^2} + V(x), \qquad x \in \mathbb{R},$$

with a complex-valued π-periodic potential $V(x)$ such that $V \in \mathcal{L}^2([0,\pi])$ and prove a criterion for it to be a spectral operator of scalar type in the sense of Dunford [1]. This criterion is stated in two versions.

The first version is given in terms of three entire functions which are independent parameters uniquely determining the potential V, *cf.*, [2], and the second one is formulated in terms of algebraic and geometric properties of spectra of periodic/antiperiodic and Dirichlet boundary problems generated by H in the space $\mathcal{L}^2([0,\pi])$.

The problem of deciding which Hill operators are spectral operators of scalar type appears to have been open for about 50 years.

This is a joint work with F. Gesztesy published in [3].

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On operator analogue of Jackson inequality

S. Torba

Classical Jackson inequality establishes the relationship between the degree of smoothness of a function and the rate of convergence to zero of the best approximation of this function by some simpler functions.

An operator analogue of Jackson inequality is considered. Let A is the closed unbounded operator on the complex Banach space such that A generates bounded C_0 group $U(t)$. We call the quantity $\inf \|x-y\|$ as the best approximation $\mathcal{E}_r(x)$ of vector x, where infimum is taken over all exponential type entire vectors of operator A with type, not exceeding r.

Recently, Y. Kryakin obtained almost optimal values of constants in the classical Jackson inequality. We shows that similar results are true for the operator analogue of this inequality.

Operators on partial inner product spaces

C. Trapani

Many families of function spaces, such as L^p spaces, Besov spaces, amalgam spaces or modulation spaces, exhibit the common feature of being indexed by one parameter (or more) which measures the behavior (regularity, decay properties) of particular functions. All these families of spaces are, or contain, scales or lattices of Banach spaces and constitute special cases of the so-called *partial inner product spaces (PIP-space s)* that play a central role in analysis, in mathematical physics and in signal processing (e.g. wavelet or Gabor analysis). The basic idea for this structure is that such families should be taken as a whole and operators, bases, frames on them should be defined globally, for the whole family, instead of individual spaces.

In this talk, we shall give an overview of PIP-space s and operators on them, illustrating the results by families of spaces of interest in mathematical physics and signal analysis. In particular, an operator on a PIP-space is a *coherent* collection of linear maps, each one of them acting on one space of the family: they are often regular objects when considered on the global structure of a PIP-space but possibly singular when considered in an individual space. Various classes of operators will be considered and the link between (partial) *-algebras of operators on a PIP-space and (partial) *-algebras of unbounded operators acting in Hilbert spaces will be briefly discussed.

The talk is based on the joint research monograph with J.-P. Antoine, *Partial inner product spaces: Theory and applications*, (Lecture Notes in Mathematics #1989, 2009)

Corona theorems and 1-positive square

T. Trent

We will relate Bezout equations and multiplier algebras on reproducing kernel Hilbert spaces, whose reproducing kernels have 1-positive square. This leads to the following theorems, which will be discussed:

(1) Best estimates in the matrix corona theorem

(2) Corona theorem for Dirichlet space

(3) Corona theorem for weighted Dirichlet spaces

(4) Corona theorem for Drury-Arveson space.

Unbounded block operator matrices and recent applications

C. Tretter

Spectral problems for systems of ordinary or partial linear differential equations occur frequently in mathematical physics. The spectral theory of block operator matrices is a powerful tool to study their spectral properties. Various methods to enclose the spectrum and to investigate its structure are presented. Examples from several applications including magnetohydrodynamics and quantum mechanics are given.

The talk is based on joint work with various coauthors, in particular, including H. Langer and U. Günther.

On Weyl's commutation relations in infinitesimal form

E. Tsekanovskii

We discuss new developments in the representation theory for Weyl's commutation relations. The talk is based on joint work with K.A. Makarov.

Atoms in a thin layer

M. Tusek

A hydrogen-like atom in a plane-parallel slab is considered. The energy spectrum of such atom is investigated as the width of the slab a tends to zero. It turns out that it is well approximated by the spectrum of a two-dimensional Hamiltonian that we call the effective Hamiltonian. The spectrum of the effective Hamiltonian can be still barely found by analytic methods. Nevertheless, it may be proved that the norm resolvent limit of the effective Hamiltonian as $a \to 0$ is nothing but the two-dimensional hydrogenic Hamiltonian. The latter model is exactly solvable. Consequently, one may use the exact knowledge of the eigenvalues of the two-dimensional hydrogenic Hamiltonian to approximate the eigenvalues of the initial Hamiltonian. The talk is based on a joint work with P. Duclos and P. Stovicek.

Spectral problem for a class of non-self-adjoint Jacobi matrices

M. Tyaglov

We study the distribution of eigenvalues of tridiagonal matrices of the form

$$J_n^{(k)} = \begin{pmatrix} a & b_1 & 0 & \dots & 0 & 0 \\ c_1 & 0 & b_2 & \dots & 0 & 0 \\ 0 & c_2 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 0 & b_{n-1} \\ 0 & 0 & 0 & \dots & c_{n-1} & 0 \end{pmatrix}, \tag{1}$$

where $a \in \mathbb{R}$, $c_j > 0$ and, for some number k, $0 \leqslant k \leqslant n-1$,

$$b_j < 0 \qquad \text{whenever} \quad j \leqslant k,$$

$$b_j > 0 \qquad \text{whenever} \quad j > k.$$

It is established that for any k, $0 \leqslant k \leqslant n-1$, the characteristic polynomial of the matrix $J_n^{(k)}$ is a generalized Hurwitz polynomial [1]. Conversely, for any generalized Hurwitz polynomial p of degree n, there exists a unique matrix of the form (1) whose characteristic polynomial is p. Special cases of $k = 0$ and $k = n-1$ are particularly discussed.

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On the location of roots of Hermite-Biehler polynomials

M. Tyaglov

The well-known Hermite-Biehler theorem claims that a univariate monic polynomial s of degree k has all roots in the open upper half-plane if and only if $s = p + iq$ where p and q are real polynomials of degree k and $k-1$ resp. with all real, simple and interlacing roots, and q has a negative leading coefficient. Considering roots of p as cyclically ordered on $\mathbb{R}P^1$ we show that the open disk D in $\mathbb{C}P^1$ having a pair of consecutive roots of p as its diameter is the maximal univalent disk for the function $R = \frac{q}{p}$. In particular, each disk D contains at most one root of the polynomial s. This solves a special case of the so-called Hermite-Biehler problem.

The talk is based on a joint work with B. Shapiro and V. Kostov.

Generalized Triebel-Lizorkin spaces and bounded H^∞-calculus for $\mathcal{R}_q$-sectorial operators

A. Ullmann

A set $\mathcal{T}$ of bounded operators on a Banach function space X is called $\mathcal{R}_q$-bounded if an estimate

$$\Big\|\Big(\sum_{j=1}^{n} |T_j x_j|^q\Big)^{1/q}\Big\|_X \lesssim \Big\|\Big(\sum_{j=1}^{n} |x_j|^q\Big)^{1/q}\Big\|_X$$

holds uniformly for $T_j \in \mathcal{T}, x_j \in X, n \in \mathbb{N}$. A sectorial operator A in X is called $\mathcal{R}_q$-sectorial, if the set $\{\lambda R(\lambda, A) \,|\, \lambda \notin \overline{\Sigma}_\omega\}$ is $\mathcal{R}_q$-bounded outside some closed sector $\overline{\Sigma}_\omega$. We will associate certain homogeneous and inhomogeneous intermediate spaces $\dot{X}^\theta_{q,A}, X^\theta_{q,A}$ to A which correspond to the classical Triebel-Lizorkin spaces if $X = L^p$ and $A = -\Delta$. We will show that the part of A has always a bounded H^∞-calculus in the homogeneous space $\dot{X}^\theta_{q,A}$ and that these spaces are stable under certain kinds of perturbation. As an application we will obtain a bounded H^∞-calculus for uniformly elliptic operators with Hölder-continuous coefficients in the classical Triebel-Lizorkin spaces.

Asymptotic expansions for symmetric spaces

H. Upmeier

In geometric quantization of symplectic manifolds, asymptotic expansions of star products and other operators depending on "Planck's constant" play an important role. In this talk we present multi-variable asymptotic expansions for the Berezin transform and the star product (of Wick and anti-Wick type) in the complex-analytic setting of hermitian symmetric spaces. The compact type (Riemann sphere, projective space, Grassmannians) and the non-compact type (unit disk, unit ball, matrix balls) are treated in a uniform way.

The talk is based on joint work with M. Englis.

Vector-valued Bergman spaces and intertwining operators

H. Upmeier

It is well-known that Bergman spaces of holomorphic functions on hermitian symmetric domains, generalizing the unit disk, play an important role in harmonic analysis of (semi-simple) Lie groups G. We consider Hilbert spaces of holomorphic functions $f(z, \zeta)$, where z belongs to a symmetric domain D and ζ belongs to a "Grassmann type" compact dual space of D. In this setting the Lie group G does not act irreducibly but the C^*-algebra of Toeplitz operators is still irreducible. The explicit construction of intertwining operators (due to Korányi and Misra for the unit disk) leads to multi-variable hypergeometric functions.

Dolbeault complexes and hyperfunction theory in the bicomplex setting

A. Vajiac

In this talk I will discuss the study of bicomplex holomorphic functions of several variables in $\mathbb{BC}^n$. In particular I will show how to construct an abstract Dolbeault complex which provides a fine resolution for sheaves of bicomplex holomorphic functions. As a corollary, I will show that the bicomplex hyperfunctions can be represented as classes of differential forms of degree $3n-1$.

The talk is based on a joint work with F. Colombo, I. Sabadini, D.C. Struppa, and M. Vajiac.

Commutative algebras of Toeplitz operators, hyperbolic geometry, and Berezin quantization

N. Vasilevski

We will discuss a quite unexpected phenomenon in the theory of Toeplitz operators on the Bergman space: the existence of a reach family of commutative C*-algebras generated by Toeplitz operators with non-trivial symbols. As it tuns out the smoothness properties of symbols do not play any role in the commutativity, the symbols can be merely measurable. Everything is governed here by the geometry of the underlying manifold, the hyperbolic geometry of the unit disk. At the same time the complete classification of such commutative algebras involves the Berezin quantization procedure.

These commutative algebras come with a powerful research tool, the spectral type representation for the operators under study. This permit us to answer to many important questions in the area. As an example we consider a kind of the semi-classical analysis of the spectra of Toeplitz operators.

Calculating Hausdorff dimensions of invariant sets using spectral theory

S. Verduyn Lunel

The dimension of an invariant set of a dynamical system is one of the most important characteristics. In this talk we present a new approach to compute the Hausdorff dimension of conformally self-similar invariant sets. The approach is based on a direct spectral analysis of the transfer operator associated with the dynamical system. This operator theoretic approach relies on the theory of trace class operators and their determinants and is a tribute to one of the fundamental contributions of Israel Gohberg to operator theory. In the case that the maps defining the dynamical system are analytic, our method yields a sequence of successive approximations that converge to the Hausdorff dimension of the invariant set at a super-exponential rate. This allows us to estimate the dimension very precisely. We illustrate our approach with examples from dynamical systems and from number theory via Diophantine approximations.

On the Neumann problem for the Helmholtz equation in a plane angle

T.J. Villalba-Vega

We consider the Neumann boundary value problem for the Helmholtz equation in a plane angle $\beta < \pi$ and boundary data from the space $H^{-\frac{1}{2}+\varepsilon}(\Gamma)$, $0 \leq \varepsilon < 1/2$, where $\Gamma = \partial\Omega$. The case $\varepsilon = 0$ was analyzed by A. Merzon and P. Zhevandrov in 2000. Here we extend their results for $\varepsilon \in (0, 1/2)$. Namely, we prove that for these boundary conditions the solution of the Helmholtz equation in Ω exists in the Sobolev space $H^{1+\varepsilon}(\Omega)$, is unique and depends continuously on the boundary data. Moreover we give the Sommerfeld representation for these solutions. This can be used to formulate explicit compatibility conditions on the data for regularity properties of the corresponding solution. We use the method of the complex characteristics, which consists of the representation of solutions in the form of the inverse Fourier-Laplace transform of some combination of the Fourier transform of the boundary data divided by the symbol of the Helmholtz operator and using the "connection equation".

The talk is based on a joint work with A. Merzon and F.-O. Speck.

Realization of noncommutative functions

V. Vinnikov

A theorem of Ball–Groenewald–Malakorn (BGM) describes various incarnations of the d-variable noncommutative Schur class as characteristic functions of unitary colligations associated to appropriate noncommutative multidimensional systems. BGM view the characteristic (transfer) function as a formal power series in d noncommuting indeterminates that can be then evaluated on appropriate d tuples of operators on a Hilbert space (e.g., forming a row contraction). However it can be also viewed as a *noncommutative function* as introduced by Kaliuzhnyi-Verbovetskyi–Vinnikov; i.e., it is a function defined on appropriate d-tuples of complex matrices of all sizes which satisfies certain compatibility conditions as we vary the size of matrices — it respects direct sums and simultaneous similarities. The d-tuples of matrices where this function is defined form in fact the noncommutative unit ball over $\mathbb{C}^d$ endowed with an appropriate operator space structure. This indicates a possible generalization towards realization theorems for contractive noncommutative functions on the noncommutative unit ball over a general operator space.

Noncommutative free Levy–Hincin formula

V. Vinnikov

There is a version of the classical Levy–Hincin formula for the free convolution: a compactly supported probability measure on $\mathbb{R}$ is free infinitely divisible if and only if the so called R-transform of the measure is a Pick function. After reviewing the necessary background from free probability, I will discuss a similar result in the operator-valued case. It turns out that the correct analogue of the R-transform is now a noncommutative function over a C^*-algebra.

This talk is based on a joint work with M. Popa.

On sufficient conditions for the total positivity of matrices and applications

A. Vishnyakova

We will discuss the following sufficient condition of the total positivity of matrices.

Theorem (O. Katkova, A. Vishnyakova). *Suppose $M = (a_{i,j})$ be a $k \times k$ matrix with positive entries and $a_{i,j}a_{i+1,j+1} \geq 4\cos^2\frac{\pi}{k+1}\, a_{i,j+1}a_{i+1,j}$ $(1 \leq i \leq k-1,\ 1 \leq j \leq k-1)$. Then the matrix M is totally positive.*

The constant $4\cos^2\frac{\pi}{k+1}$ in this Theorem is the smallest possible not only in the class of $k \times k$ matrices with positive entries but in the classes of $k \times k$ Toeplitz matrices and of $k \times k$ Hankel matrices. Some applications and generalizations of the above result to almost strictly totally positive matrices, Pólya frequency sequences, moment problem, hyperbolic polynomials, Hurwitz stable polynomials and positive polynomials will be presented.

The talk is based on a joint work with O. Katkova.

Spectral analysis for hyperbolic integrodifferential equations in Hilbert space

V. Vlasov

We study the solvability for abstract hyperbolic equations with variable delay and integral Volterra terms. We consider several spectral problems in autonomous cases by considering the operator-valued functions as the symbols of the equations under investigations. We analyse the structure of the spectra for the symbols of the above mentioned integrodifferential equations. We also present some applications of our results to integrodifferential equations of Gurtin-Pipkin type arising from the theory of heat propagation with memory, to the integrodifferential equations arising in the theory of visco-elasticity, acoustic problems in the theory of porous media.

The talk is based on a joint work with N. Rautian.

L_p-theory for second order elliptic operators with complex coefficients

H. Vogt

The aim of the talk is to show how one can associate the (minus) generator of a C_0-semigroup on $L_p(\Omega)$ with the formal differential expression

$$\mathcal{L} = -\nabla \cdot (a\nabla) + b_1 \cdot \nabla + \nabla \cdot b_2 + q$$

on an open set $\Omega \subseteq \mathbb{R}^n$, with *complex* measurable coefficients $a\colon \Omega \to \mathbb{C}^{N\times N}$, $b_1, b_2\colon \Omega \to \mathbb{C}^N$ and $q\colon \Omega \to \mathbb{C}$.

Under suitable conditions on the coefficients, one can use Kato's representation theorem for sectorial forms to associate an m-sectorial operator A_2 in $L_2(\Omega)$ with the expression $\mathcal{L}$. The question then is to what L_p-spaces the C_0-semigroup e^{-tA_2} extrapolates. We also discuss the case that the sesquilinear form associated with $\mathcal{L}$ is not sectorial, in which one does not necessarily obtain a C_0-semigroup on $L_2(\Omega)$.

The talk is based on joint work with A. F. M. ter Elst, Z. Sobol and V. Liskevich.

Positive extensions of matrices indexed by a homogeneous tree

D. Volok

Let T be a homogeneous tree of order q – that is, an acyclic, undirected, connected graph such that every node belongs to exactly $q+1$ edges. Let T_n be a maximal subgraph of T with the property that the distance between any two nodes of T_n does not exceed n, and let $A = [a_{t,s}]_{t,s\in T_n}$ be a square positive definite matrix indexed by the nodes of T_n. The matrix A is said to be *isotropic* if $a_{t,s}$ depends only on the distance between the nodes t and s (in the case $q = 1$ this means that A is a real symmetric Toeplitz matrix). The positive extension problem for the matrix A consists in finding all such isotropic positive definite matrices B indexed by the nodes of $T_{n+1} \supset T_n$ that the diagonal block of B corresponding to T_n coincides with A.

In this talk we shall discuss a solution of the positive extension problem based on the canonical form of isotropic matrices, obtained in a joint work with D. Alpay.

Rational discrete analytic functions

D. Volok

A function $f : \mathbb{Z}^2 \longrightarrow \mathbb{C}$ is said to be discrete analytic if

$$\frac{f(x+1,y+1)-f(x,y)}{1+i} = \frac{f(x,y+1)-f(x+1,y)}{i-1} \quad \forall x,y \in \mathbb{Z}.$$

This notion of discrete analyticity, developed by J. Ferrand (Lelong), R.J. Duffin and others, has many analogies with the classical continuous theory. However, there is also a notable difference: the pointwise product of two discrete analytic functions is not necessarily discrete analytic. For example, the functions $z = x + yi$ and $z^2 = x^2 - y^2 + 2xyi$ are discrete analytic but the function $z^3 = x^3 - 3xy^2 + 3x^2yi - y^3i$ is not. Nevertheless, one can show that for every polynomial $p(x)$ there is a unique discrete analytic polynomial $P(x,y)$ such that $P(x,0) = p(x)$ $\forall x \in \mathbb{Z}$.

We shall discuss a generalization of this result for rational functions.

The talk is based on a joint work with D. Alpay.

Harmonic analysis on $GF(p^{p^{\infty}})$ and the corresponding Heisenberg-Weyl group

A. Vourdas

The Pontryagin dual group $\widehat{G}$ of $GF(p^{p^{\infty}})$ is introduced as the inverse limit of an inverse system comprised of the $GF(p^{p^{\ell}})$ ($\ell \in \mathbb{Z}_0^+$), with homomorphisms between them. The properties of the profinite group $\widehat{G}$ are discussed.

Harmonic analysis on $GF(p^{p^{\infty}})$ is then studied. The Heisenberg-Weyl $HW[\widehat{G}, GF(p^{p^{\infty}}), \widehat{G}]$ group of displacements in the $GF(p^{p^{\infty}}) \times \hat{G}$ phase space, is shown to be a locally compact and totally disconnected topological group. The formalism introduces algebraic concepts from the theory of Galois fields into harmonic analysis. For example, transformations analogous to Frobenius transformations in Galois theory, are introduced into harmonic analysis.

Multicomponent transmission problems with spectral parameter in equations and boundary conditions

V. Voytitsky

We consider linear multicomponent boundary value transmission problems where we have several unknown functions defined on jointed boundary domains and connected with each other only by boundary transmission conditions. These functions satisfy elliptic equations in domains with Lipshitz boundaries and boundary conditions contained the spectral parameter. We consider two main situations when we have equal values of functions or normal derivatives on joint boundaries. Such problems arise in different applied problems from theory of diffraction, theory of elasticity, mechanics, hydrodynamics and others (see e.g. [1], [2]).

To obtain qualitative properties of these problems, i.e. discreteness of spectrum, basis properties of eigenfunctions, we research the abstract auxiliary boundary value problems. We use such name for generalized problems formulated in terms of operators from abstract Green's formula. Last one can be constructed by given triple of Hilbert spaces and abstract trace operator (see [3] and [4], [5]). For problems of transmission we use abstract Green's formula for mixed boundary value problems proved by N. Kopachevsky. It can be constructed by given triple of Hilbert spaces and set of abstract traces operators acting to special boundary spaces (to parts of boundary) with additional imbedding properties (see [6]). Finding weak solutions of abstract boundary value problems as elements satisfying variational identities we can introduce and research properties of corresponding linear operators acting in some Hilbert spaces. As a result we find operator statement of initial boundary value problem that is eigenvalue problem for an operator-matrix or eigenvalue problem for a pencil of operators.

The talk is based on a joint work of N. Kopachevsky, P. Starkov and V. Voytitsky (see [6]).

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On invariant subspaces of absolutely (p, q)-summing operators

G. Wanjala

Let $1 \leq p, q < \infty$ and let T be a bounded linear operator acting on a Krein space $\mathcal{K}$. We say that the operator T is *absolutely* (p, q)*-summing* if there exists a constant $c > 0$ for which

$$\left(\sum_{i=1}^{n} \|Tk_i\|^p \right)^{1/p} \leq c \cdot \sup \left\{ \left(\sum_{i=1}^{n} |\langle k_i, k \rangle|^q \right)^{1/q} : k \in \mathcal{K}, \|k\| \leq 1 \right\}$$

irrespective of how we choose a finite collection $\{k_1, k_2, \ldots, k_n\}$ of vectors in $\mathcal{K}$. These operators form a linear subspace of $B(\mathcal{K})$, the class of all bounded linear operators acting on $\mathcal{K}$, which we denote by $\Pi_{p,q}(\mathcal{K})$. The smallest constant c for which the above inequality holds is denoted by $\pi_{p,q}(T)$. Existence of a maximal negative invariant subspace for $T \in \Pi_{p,q}(\mathcal{K})$ with $\pi_{p,q}(T) \leq 1$ will be discussed.

Operator factorization and the Darboux Crum transformation

B.A. Watson

The Darboux-Crum transformation will be considered in an operator theoretic setting for Sturm-Liouville problems with eigenvalue-dependent boundary conditions. When posed as operator factorization some new features of the transformation become apparent.

Calculating adjoints of operators on infinite graphs

M. Waurick

The notion of systems with integration by parts is introduced. With this the spatial operator of the transport equation and the spatial operator of the wave or heat equation on graphs can be defined. The considered graphs can consist of arbitrarily many edges and vertices if the lengths of the edges have strictly positive lower bounds. The respective adjoints of the operators on those graphs can be calculated and skew-selfadjoint operators can be classified via boundary values. With the work of R. Picard (Math. Meth. app. Sci. 32: 1768-1803 [2009]) we can therefore show well-posedness results for the respective evolutionary problems.

On the stability of inverse resonance problems

R. Weikard

Inverse spectral and scattering problems are a classical subject in mathematical physics. In this talk a particular variant, the inverse resonance problem is presented. Since, in practical settings, one can typically not expect to obtain all the necessary data and since, in any case, recovery algorithms cannot make use of all data even if they were available, we investigate which information may be contained from finite noisy data.

Results were obtained jointly with M. Brown, I. Knowles, M. Marletta, S. Naboko and R. Shterenberg.

Spectral theoretic methods for stochastic partial differential equations

L. Weis

Strong regularity estimates are necessary to solve evolution equations and stochastic evolution equations via fixed point methods. We show, in the case of parabolic equations, that maximal regularity is (almost) equivalent to the boundedness of the holomorphic functional calculus of the defining partial differential operators. We present a short proof of this fact, which reduces the problem essentially to operator-theoretic arguments.

This talk is based on joint work with M. Veraar and J. Van Neerven.

On Levi functions

W.L. Wendland

Fundamental solutions to elliptic partial differential operators are explicitly known only in particular cases whereas Levi functions can always be constructed.

In this lecture, the simple case of a second order operator with variable coeffients will be considered and with Levi Functions a system of domain–boundary integral equations for the Dirichlet problem will be obtained. The mapping properties of the corresponding operators will provide the opportunity of employing efficient solution techniques.

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Products of Nevanlinna functions with symmetric rational functions

H.L. Wietsma

To a symmetric rational function r associate the class $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ of generalized Nevanlinna functions by the formula

$$\mathcal{N}_\kappa^{\tilde{\kappa}}(r) = \{\, Q \in \mathcal{N}_\kappa : rQ \in \mathcal{N}_{\tilde{\kappa}} \,\}.$$

Here $\mathcal{N}_\kappa$, $\kappa \in \mathbb{N}$, denotes the class of generalized Nevanlinna function with κ negative squares.

With $\widetilde{\kappa} = 0$ the classes $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ extend the Kreĭn-Langer classes $\mathcal{N}_\kappa^+$ of generalized Nevanlinna functions, see [4], and with $\kappa = 0$ the classes $\mathcal{N}_\kappa^{\tilde{\kappa}}(r)$ extend the classes $\mathcal{S}^{\pm\widetilde{\kappa}}(\alpha, \beta)$ $(-\infty < \alpha < \beta < \infty)$ of Nevanlinna functions introduced by V.A. Derkach and M.M. Malamud in [2], as a generalization of the Stieltjes and inverse Stieltjes classes $\mathcal{S}$ and $\mathcal{S}^{-1}$, which were originally introduced by M.G. Kreĭn, see e.g. [3]. Furthermore, the classes $\mathcal{N}_{\kappa}^{\tilde{\kappa}}(r)$ contain the classes $\widetilde{\mathcal{N}}_{k}^{\pm\kappa}$ introduced by V.A. Derkach in [1].

By means of a characterization of Nevanlinna functions with gaps by their behavior at the endpoints of the gaps, the classes $\mathcal{N}_{\kappa}^{\tilde{\kappa}}(r)$ can be completely described. Furthermore, for a function $Q \in \mathcal{N}_{\kappa}^{\tilde{\kappa}}(r)$ the realization of rQ is connected to the realization of Q.

This talk is based on joint work with S. Hassi.

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Factorizations of maximal sectorial relations

H. Winkler

Sectorial operators or relations have maximal sectorial extensions. It is shown that some special extensions, namely the Kreĭn-von Neumann and Friedrichs extensions, can be characterized in terms of factorizations. Furthermore, all extremal maximal sectorial extensions of a sectorial relation are characterized in terms of analogous factorizations. As in the case of nonnegative relations, the factorizations of the Kreĭn-von Neumann and

Friedrichs extensions lead to a novel approach to the transversality and equality of the extreme extensions and to the notion of positive closability (the Kreĭn-von Neumann extension being an operator). In particular, all extremal maximal sectorial extensions of a bounded sectorial operator are characterized.

The talk is based on a joint work with S. Hassi, A. Sandovici and H. de Snoo.

The pair of operators $T^{[*]}T$ and $TT^{[*]}$; J–dilations and canonical forms

M. Wojtylak

We will describe a procedure of dilating an operator in an infinite dimensional Krein space. Namely, let $\mathcal{H}_0$, $\mathcal{K}_0$, $\mathcal{H}$, $\mathcal{K}$ be Krein spaces. We say that an operator $T \in \mathbf{B}(\mathcal{H}, \mathcal{K})$ is an *J–dilation of* $T_0 \in \mathbf{B}(\mathcal{H}_0, \mathcal{K}_0)$ if the following three conditions are satisfied:

(i) $\mathcal{H}_0$ is a subspace of $\mathcal{H}$, $\mathcal{K}_0$ is a subspace of $\mathcal{K}$.

(ii) There exist subspaces $\mathcal{H}_i$ of $\mathcal{H}$ and $\mathcal{K}_i$ of $\mathcal{K}$ ($i = 1, 2, 3$) such that

$$\mathcal{H} = \mathcal{H}_0 \boxplus \mathcal{H}_1 \boxplus (\mathcal{H}_2 \dot{+} \mathcal{H}_3), \qquad \mathcal{K} = \mathcal{K}_0 \boxplus \mathcal{K}_1 \boxplus (\mathcal{K}_2 \dot{+} \mathcal{K}_3),$$

where $\mathcal{H}_1$ and $\mathcal{K}_1$ are Krein spaces, $\mathcal{H}_2$ and $\mathcal{H}_3$ ($\mathcal{K}_2$ and $\mathcal{K}_3$) are skewly linked neutral spaces such that $\mathcal{H}_2 + \mathcal{H}_3$ ($\mathcal{K}_2 + \mathcal{K}_3$) is a Krein space.

(iii) The operator T has a following representation with respect to the above decomposition

$$T = \begin{pmatrix} T_0 & 0 & T_{02} & 0 \\ 0 & 0 & 0 & 0 \\ T_{20} & 0 & T_2 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{1}$$

It appears that the dilated operators $T^{[*]}T$ and $TT^{[*]}$ inherit many spectral properties from the operators $T_0T_0^{[*]}$ and $T_0^{[*]}T_0$, respectively. We use the J-dilation procedure to study and compare the canonical forms of matrices $T^{[*]}T$ and $TT^{[*]}$ in a finite dimensional Krein space.

The talk is based on a joint work

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Zeros of nonpositive type of Nevanlinna functions with one negative square

M. Wojtylak

A generalized Nevanlinna function $Q(z)$ with one negative square has precisely one generalized zero of nonpositive type in the closed extended upper halfplane. The fractional linear transformation

$$Q_\tau(z) = \frac{Q(z) - \tau}{1 + \tau Q(z)}, \qquad \tau \in \mathbb{R} \cup \{\infty\},$$

of $Q(z)$ is a Nevanlinna function with one negative square as well. Let $\alpha(\tau)$ define the generalized zero of nonpositive type of $Q(\tau)$. We investigate the properties of $\alpha(\tau)$, seen as a function of the parameter τ.

The talk is based on a joint work with H.S.V. de Snoo and H. Winkler.

The abstract Titchmarsh-Weyl M-function and its relation to the spectrum

I. Wood

In the setting of adjoint pairs of operators we consider the question: to what extent does the Weyl M-function see the same singularities as the resolvent of a certain restriction A_B of the maximal operator? We obtain results showing that it is possible to describe explicitly certain spaces $\mathcal{S}$ and $\tilde{\mathcal{S}}$ such that the resolvent bordered by projections onto these subspaces is analytic everywhere that the M-function is analytic.

We then look at specific examples where we determine the space $\mathcal{S}$ explicitly and which together indicate that the abstract results are probably best possible.

Two-dimensional Hamiltonian systems with two singular endpoints

M. Langer (Part one) and H. Woracek (Part two)

see M. Langer and H. Woracek

Hamiltonians with Riesz bases of eigenvectors and Riccati equations

C. Wyss

We consider the algebraic Riccati equation

$$A^*X + XA + XQ_1X - Q_2 = 0,$$

which appears in the problem of optimal control of a linear system, for the case that A is a normal operator with compact resolvent and Q_1, Q_2 are unbounded, selfadjoint and nonnegative. Using the well-known correspondence of solutions X to invariant graph subspaces of the Hamiltonian

$$T = \begin{pmatrix} A & Q_1 \\ Q_2 & -A^* \end{pmatrix},$$

we prove the existence of infinitely many selfadjoint solutions. Our main tools are a Riesz basis with parentheses of generalised eigenvectors of T and two indefinite inner products associated with T. We also obtain conditions which yield nonnegative, nonpositive, and bounded solutions.

Spectral properties of nonselfadjoint one-dimensional singular perturbations of unbounded selfadjoint operators

D. Yakubovich

A (linear unbounded) operator A is called a finite-dimensional singular perturbation of an operator A_0 if their graphs differ in a finite-dimensional space. We study spectral properties of a one-dimensional singular perturbation A of an unbounded selfadjoint operator A_0 with compact resolvent. Our approach is based on a functional model of this operator, similar to a model by V. Kapustin. We assume that the spectrum of A is real. We show that for any operator A of our class there exist an inner function $\Theta(z)$ and an outer function $\varphi(z)$ in the upper half plane $\mathbb{C}_+$ with $\frac{\varphi}{z+i} \in H^2$ and

$$\Theta = \frac{\varphi}{\overline{\varphi}} \qquad \text{a.e. on } \mathbb{R}$$

such that A is unitarily equivalent to the operator $T = T_{\Theta,\varphi}$ which acts on the model space $K_\Theta = H^2(\mathbb{C}_+) \ominus \Theta H^2(\mathbb{C}_+)$, with the domain defined as

$$\mathcal{D}(T) = \{f \in K_\Theta : \qquad \exists\, c = c(f) \in \mathbb{C} : zf - c\varphi \in K_\Theta\},$$

and

$$Tf = zf - c\varphi, \qquad f \in \mathcal{D}(T).$$

We give criteria for completeness of eigenvectors and for the possibility to remove the whole spectrum by an adequate perturbation of the type considered in terms of the sparsity of the spectrum of the unperturbed operator. The proofs use entire functions of Hermite–Biehler and Cartwright classes.

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The talk is based on a joint work with A. Baranov.

The cosine of a linear operator revisited

N. Yannakakis

We show that the cosine of a large class of strictly positive linear operators from a Banach space, not isomorphic to a Hilbert space, into its dual is zero. Using this fact we are able to obtain a result concerning evolution triples.

Convergent interpolation to Cauchy integrals over analytic arcs with Jacobi-type weights

M. Yattselev

We design uniformly convergent sequences of rational interpolants to Cauchy integrals of the form $f_\mu(z) = \int (z-t)^{-1} d\mu(t)$, where

$$d\mu(t) = (1-t)^\alpha (1+t)^\beta h(t) dt, \quad \alpha, \beta > -1,$$

and h is a non-vanishing smooth function on an analytic arc Δ with endpoints ± 1. Namely, we show that for any analytic arc there exist probability Borel measures that make it symmetric in the sense of Stahl [1] (that is, the normal derivatives, taken from the left and right-hand sides of Δ, of the Green potential of each such measure coincide). Proper discretization of these measures produces sought interpolation schemes. The proof of convergence of the corresponding rational interpolants proceeds via $\bar{\partial}$-extension [2] of the Riemann-Hilbert approach [3] applied to the underlying boundary value problem.

This talk is based on a joint work with L. Baratchart.

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Localization of the numerical range for quasi-sectorial contractions and semigroup approximations

V. Zagrebnov

We study the numerical range of *quasi-sectorial* contractions and obtain three results. Our *first* theorem gives characterization of the maximal sectorial generator A in terms of the corresponding quasi-sectorial contraction semigroup $\{\exp(-tA)\}_{t\geq 0}$. The *second* result establishes for these quasi-sectorial contractions a quite accurate localization of their numerical range. We give for this class of semigroups a new proof of the Euler operator-norm approximation: $\exp(-tA) = \lim\limits_{n\to\infty}(I + tA/n)^{-n}$, $t \geq 0$, with the optimal estimate: $O(1/n)$, of the convergence rate, which takes into account the value of the sectorial generator angle (the *third* result).

The talk is based on a joint work with Yury Arlinskiĭ.

Inverse problem for ill-posed linear differential-algebraic equations with variable coefficients

S. Zhuk

The talk describes a solution of the following inverse problem: given measurements y of some vector φ in the form $y = H\varphi + \eta$, to estimate φ, provided $L\varphi = f$, (f, η) are uncertain, that is $(f, \eta) \in G$ and G is a convex bounded closed set, L is a closed linear mapping in some Hilbert space, H is a bounded linear mapping. The estimation $\hat{\varphi}$ is chosen from the following minimax criterion: to minimize the worst-case error $\sup_{f,\eta}\{\|\varphi - \hat{\varphi}\|^2, L\varphi = f, y = H\varphi + \eta\} \to \inf_{\varphi}$. The main result is a sub-optimal minimax estimation of φ, valid for L with non-closed range, obtained through Tikhonov regularization approach. This result is then applied to the construction of the state estimation algorithm for uncertain linear differential-algebraic equation with variable coefficients.

Estimates of large eigenvalues for some Jacobi matrices with unbouded entries

L. Zielinski

In this talk we are interested in a self-adjoint operator defined by an infinite Jacobi matrix with off-diagonal entries dominated by the diagonal entries. The spectrum is discrete under the assumption that the diagonal entries are growing to infinity and we consider additional assumptions on the entries allowing us to describe the asymptotic behaviour of large eigenvalues.

Self-adjoint fourth order differential operators with eigenvalue parameter dependent boundary conditions

B. Zinsou

We consider the eigenvalue problem

$$y^{(4)}(\lambda, x) - (gy')'(\lambda, x) = \lambda^2 y(\lambda, x)$$

with separated boundary conditions $B_j(\lambda)y = 0$ for $j = 1, \ldots, 4$, where $g \in C^1[0, a]$ is a real valued function, $B_j(\lambda)y = y^{[p_j]}(a_j)$ or $B_j(\lambda)y = y^{[p_j]}(a_j) + i\varepsilon_j \alpha \lambda y^{[q_j]}(a_j)$, $a_j = 0$ for $j = 1, 2$ and $a_j = a >$ for $j = 3, 4$, $\alpha > 0$, $\varepsilon_j \in \{-1, 1\}$. We will associate to the above eigenvalue problem a quadratic operator pencil $L(\lambda) = \lambda^2 M - i\alpha\lambda K - A$ in the space $L_2(0, a) \oplus \mathbb{C}^k$, where

$$M = \begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix} \text{ and } K = \begin{pmatrix} 0 & 0 \\ 0 & I \end{pmatrix}$$

are bounded self-adjoint operators and k is the number of boundary conditions which depend on λ. We give necessary and sufficient conditions for the operator A to be self-adjoint.

Complete sets of metrics in solvable PT-symmetric models

M. Znojil

The concept of the observable coordinate of a point particle moving in a 1D potential $V(q)$ is often generalized to a not necessarily measurable real argument x entering a concrete representation $\psi(x) \in L^2(R)$ of an abstract ket-vector $|\psi\rangle$. As a consequence, the "friendly" space $L^2(R) := \mathcal{H}^{(F)}$ may (and, for the so called PT-symmetric Hamiltonians H, does) prove unphysical. Fortunately, a return to textbook theory can be mediated by the replacement of linear functionals in $\mathcal{H}^{(F)}$ [i.e., dual vectors called, usually,"Dirac's" bra-vectors $\langle\psi|]$ by "brabras" $\langle\!\langle\psi| = \langle\psi|\Theta$. They are defined in terms of a suitable "metric" $\Theta \neq I$ and form the linear functionals in a unitarily inequivalent and potentially physical Hilbert space $\mathcal{H}^{(P)}$. The nature and elimination of the ambiguity of the assignment of metric Θ to a given Hamiltonian H will be studied. We shall take advantage of the possibility of construction of all of the admissible, $H-$compatible metrics $\Theta = \Theta(H)$ in a few simplified, schematic models.

Port-Hamiltonian systems and boundary triplets

H. Zwart

Port-Hamiltonian systems have been introduced in systems theory. When a port-Hamiltonian system has is no interaction with its environment, then it has a conserved quantity, known as the Hamiltonian. Typical examples are hyperbolic partial differential equations, like the vibrating string. When there is an interaction with the environment, then the change of the Hamiltonian is determined by a (power) flow through the boundary.

Boundary triplets are normally used for parabolic partial differential equations, such as the diffusion equation. Central for boundary triplets is the (generalized) Greens identity.

Hence the class of partial differential equations for these topics is different. Furthermore, in port-Hamiltonian systems, the time places an important role, whereas this is totally absent in the theory of boundary triplets.

Despite all these differences, we show we can identity a common underlying structure, known as a Dirac structure for port-Hamiltonian systems, and as a hyper-maximal neutral subspace in Kreĭn spaces. This implies that results in one field carry over to the other field. However, since these fields have developed independently of each other, similar results and concepts were introduced, but under different names. We will clarify the links between both fields. Special emphasis will be laid on the interconnection of port-Hamiltonian systems and thus of boundary triplet.

As may be concluded from the above summary, the talk will have a strong tutorial nature.

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