Adaptive Reduced Model Generation for Parametrized Systems

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Projection-based reduction methods for linear or nonlinear systems require the careful choice of a projection basis, which can approximate all possible solution trajectories. Several of such basis generation procedures require a repeated choice of frequency points, parameter values, or time instants for basis enrichment. This is the case for rational interpolation methods, reduced basis (RB) methods, frequency-weighted Gramian-based reduction, proper orthogonal decomposition, etc. A first common problem here is the mostly unknown optimal choice of parameter values. A further problem is, that parameter-dependent problems frequently reveal high solution variability, and therefore high accuracy in reduced models must be paid by high, and possibly inpractical, reduced dimension.

We present an approach for adaptive reduced model generation addressing these two issues [2]. The method aims at a simultaneous control of accuracy and dimension of the reduced model by specifying independently both a maximum error tolerance and maximal reduced model dimension. These goals can be obtained by an adaptive parameter domain partition procedure, inspired by [1]. On each adaptively determined subdomain, we construct a local model, i.e. a reduced basis by a greedy search algorithm. The reduced simulation then only requires a selection of the correct local model. The generation of the local models additionally can make use of an adaptive training set extension procedure [3], which adaptively determines the correct location of parameter points.

We demonstrate the applicability of the approach to an RB-method for an instationary advection problem. Compared to a single-reduced-model strategy, we obtain an increased offline computation time and data-storage requirements, due to the multiple local models. However, this is rewarded with shortened online computation time and improved online accuracy.

The method seems promising for other stationary or instationary, linear or nonlinear problems, as long as an error indicator (estimator or true error) is available in the offline phase. The role of the parameter can likewise be taken by frequency point or time-instant in the reduction methods mentioned above.

References

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