Balanced model reduction of gradient systems

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Joint work with

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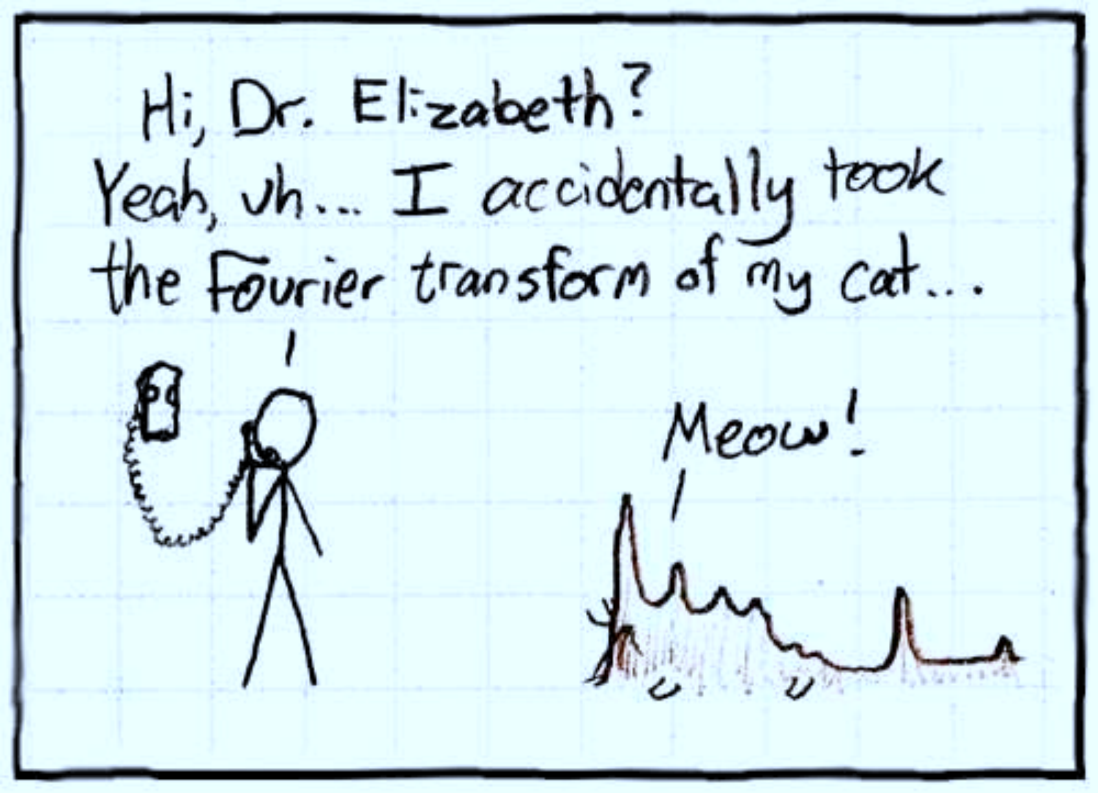
Modred2010, Berlin

Change of topic from abstract



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Balancing based structure preserving order reduction for port-Hamiltonian (pH) systems

- Based on work of Polyuga and Van der Schaft for linear pH systems, to reduce non-minimal pH system to a minimal pH systems (precise reduction).

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- Extension to nonlinear case done by Scherpen and Van der Schaft.
- Based on **either** observability **or** controllability, resulting in different models.
- Combination should result in approximate model order reduction. However, unfinished, equations are still "ugly".

Outline

For gradient system we do have nice results!

Outline:

- **Introduction**
  - gradient systems
  - linear balanced realizations
- Linear gradient systems and balancing
- Nonlinear gradient systems and balancing
- Concluding remarks

FWN, ITM

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Introduction

- Gradient systems large class of systems, i.e., linear and nonlinear RLC circuits, Brayton-Moser systems, etc.

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- Popular in circuits / systems / control literature of 70's
- Gradient form of drift vector field, along with pseudo-Riemannian metric.
- System theoretic relation with symmetric systems, i.e., input vector field and output map such that overall system is symmetric. For linear systems: $H(s) = H^T(s)$, with H transfer matrix.

Introduction: gradient systems

Nonlinear gradient system:

$$\begin{aligned} G(x)\dot{x} &= -\frac{\partial P}{\partial x}(x) + \frac{\partial h}{\partial x}(x)u \\ y &= h(x) \end{aligned}$$

$G(x) = G^T(x)$ invertible pseudo-Riemannian metric, P mixed potential function.

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Linear gradient system with mixed potential function $\frac{1}{2}x^T P x$:

$$\begin{aligned} G\dot{x} &= -Px + C^T u \\ y &= Cx \end{aligned}$$

$P = P^T$, $G = G^T \neq 0$, $H(s) = C(sI - G^{-1}P)^{-1}G^{-1}C^T = H(s)^T$.

Introduction: gradient systems

Linear gradient system:

$$\begin{aligned} G\dot{x} &= -Px + C^T u \\ y &= Cx \end{aligned}$$

Example: x currents/voltages through inductors/over condensators,
$G = blockdiag\{L, -C\}$,
P matrix containing resistors, conductors and interconnection structure
u sources, and y corresponding currents or voltages (power outputs).

Introduction: gradient systems

Model order reduction for large scale gradient systems should "preserve" as much as possible:

- Input/output structure (interconnection structure).
- Gradient structure.

Focus presentation: combine

- balanced realization based model reduction
- preserve gradient structure.

Introduction: Linear balancing review

Continuous-time, causal linear input-output system
$S : u \to y$ with impulse response $H(t)$.
If S is also BIBO stable then the system **Hankel operator**:

$$\begin{aligned} \mathcal{H} \quad &: \quad L_2^m[0, +\infty) \to L_2^p[0, +\infty) \\ &: \quad \hat{u} \to \hat{y}(t) = \int_0^\infty H(t + \tau)\hat{u}(\tau)\, d\tau. \end{aligned}$$

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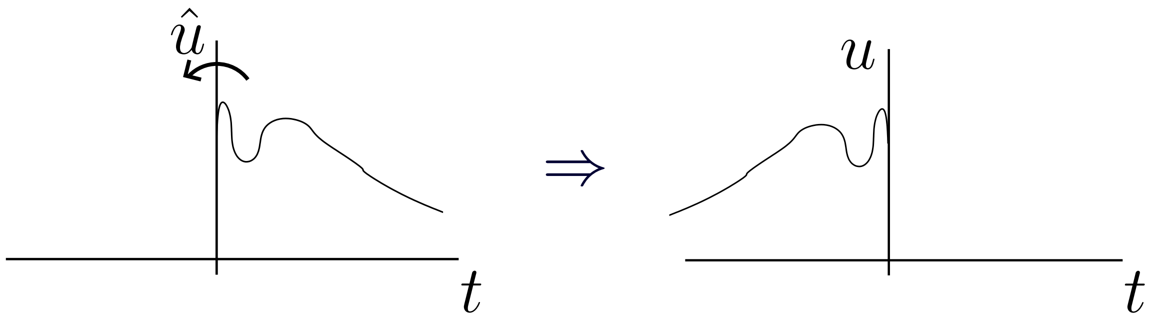
Time flipping operator $\mathcal{F} : L_2^m[0, +\infty) \to L_2^m(-\infty, 0]$



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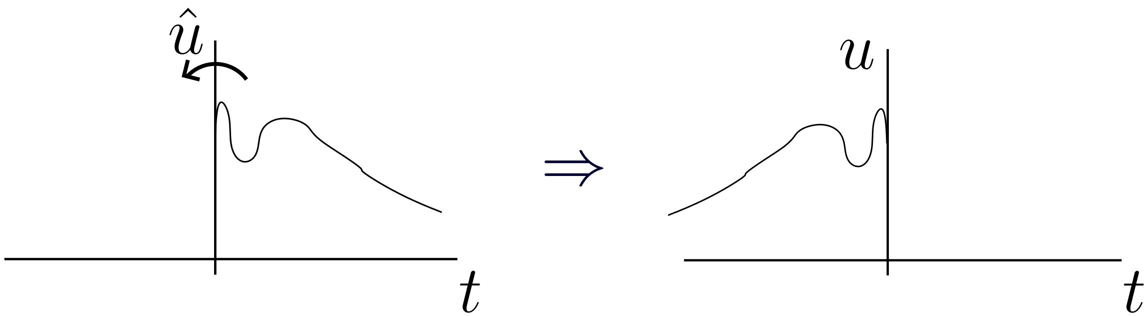

$$\mathcal{H}(\hat{u}) = S \circ \mathcal{F}(\hat{u})$$

Introduction: Linear balancing *(continued)*

$\mathcal{H} = \mathcal{O}\mathcal{C}$, with the **controllability** and **observability operators** $\mathcal{C}$ and $\mathcal{O}$.

with σ_i are **Hankel singular values**, i.e., σ_i^2 are eigenvalues of $\mathcal{H}^*\mathcal{H}$.

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(A, B, C) as. stable state space realization of S of order n.

- σ_i^2 are eigenvalues of MW, where $W \geq 0$ and $M \geq 0$ are the usual **controllability** and **observability Gramians** fulfilling

$$\begin{aligned} AW + WA^T &= -BB^T \\ A^TM + MA &= -C^TC \end{aligned}$$

Introduction: Linear balancing *(continued)*

(A, B, C) is minimal $\Leftrightarrow M > 0$ and $W > 0$.

If (A, B, C) is minimal and as. stable, then there exists a state space representation where

$$\Sigma := M = W = \begin{pmatrix} \sigma_1 & & 0 \\ & \ddots & \\ 0 & & \sigma_n \end{pmatrix}$$

$\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0$ Hankel singular values. Then system is in **balanced form**.

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  - gradient systems
  - linear balanced realizations
- **Linear gradient systems and balancing**
- Nonlinear gradient systems and balancing
- Concluding remarks

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Linear gradient systems and balancing

Consider linear system $\dot{x} = Ax + Bu$, $y = Cx$
$x \in \mathbb{R}^n, u \in \mathbb{R}^m, y \in \mathbb{R}^m$.

Gradient system if there exists $G = G^T \neq 0$ satisfying

$$A^T G = GA\,, \quad B^T G = C \qquad (*)$$

Since $H(s) = H^T(s)$, G satisfying (*) is **unique** for controllable and observable system.
With $P = -GA = P^T$:

$$\begin{aligned} G\dot{x} &= -Px + C^T u \\ y &= Cx \end{aligned}$$

Linear gradient systems and balancing

The controllability and obervability Gramian W and M of (A, B, C) are unique solutions of

$$AW + WA^T = BB^T\,, \quad A^TM + MA = C^TC$$

Pre- and postmultiplying first eq. by G, and using gradient cond.

$$GAWG + GWA^TG = GBB^TG \Leftrightarrow$$

$$A^T(GWG) + (GWG)A = C^TC$$

implying $GWG = M$.

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implying $GWG = M$.

In **balanced coordinates** with distinct Hankel s.v.'s:

$$G\Sigma G = \Sigma \Rightarrow G = diag\{\pm 1\} \qquad (G > 0 \Rightarrow G = I)$$

Linear gradient systems and balanced trunc.

Remark: If some Hankel s.v.'s are equal, coordinates can be chosen such that $G = diag\{\pm 1\}$

Remark: Passivity conditions are easily obtained.

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Proposition: Consider gradient system with $\sigma_1 \geq \cdots \sigma_k >> \sigma_{k+1} \geq \cdots \geq \sigma_n$ in balanced form with $G = diag\{\pm 1\}$. Truncate $x_{k+1}, \ldots, x_n$. Then reduced order model is gradient system

$$\begin{aligned} \hat{G}\dot{\hat{x}} &= \hat{P}\hat{x} + \hat{C}^T u\,, \quad \hat{x} \in \mathbb{R}^k \\ \hat{y} &= \hat{C}\hat{x} \end{aligned}$$

$\hat{G} = diag\{\pm 1\} = G_{11}$, $\hat{P} = P_{11}$, $\hat{C} = C_1$.

Linear gradient systems and singular pert.

Proposition: Consider gradient system with $\sigma_1 \geq \cdots \sigma_k >> \sigma_{k+1} \geq \cdots \geq \sigma_n$ in balanced form with $G = diag\{\pm 1\}$. Apply $\dot{x}_{k+1} = \ldots = \dot{x}_n = 0$, then again reduced order gradient system with $\hat{G} = G_{11}$, and $\hat{P} = \hat{P}^T$

$$\hat{P} = P_{11} - P_{12} P_{22}^{-1} P_{21}$$

and with output equation

$$\hat{y} = \hat{C}\hat{x} + \hat{D}u$$

where $\hat{C} := C_1 - C_2 P_{22}^{-1} P_{21}$ and $\hat{D} := C_2 P_{22}^{-1} C_2^T$.

Linear gradient systems: electrical circuits

In balanced coordinates, inductors and condensators all transformed to ± 1. **Mixed potential** function normally given as

$$\frac{1}{2} i^T \mathcal{R} i - \frac{1}{2} v^T \mathcal{G} v + i^T \Lambda v$$

with $\mathcal{R}$ resistors, $\mathcal{G}$ conductors, Λ interconnection matrix (topology of circuit), and i and v inductor currents and capacitor voltages.

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In balanced coordinates, structure mixed potential may be lost. After truncation, circuit realization may also require additional **transformers.**

Linear gradient system: the cross Gramian

Consider so-called **cross Gramian** X, with

$$WM = X^2\,,$$

and X unique solution of **Sylvester** equation $AX + XA = BC$.
In fact, $X = WG = G^{-1}M$. Thus, in balanced coordinates

$$X = \Sigma G \Rightarrow X = diag\{\pm\sigma_i\}$$

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Corollary: Assume $\sigma_i \neq \sigma_j$, $\forall i, j$, Let $\bar{x} = Sx$ be s.t. $SXS^{-1} = \mathrm{diag}(\pm\sigma_1, \pm\sigma_2, \cdots, \pm\sigma_n)$. Then there exists a diagonal matrix D such that DS is a balancing transformation.

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- **Nonlinear gradient systems and balancing**
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Nonlinear gradient systems and balancing

Nonlinear gradient system with $G(x) = G^T(x)$ invertible:

$$\begin{aligned} G(x)\dot{x} &= -\frac{\partial P}{\partial x}(x) + \frac{\partial h}{\partial x}(x)u \\ y &= h(x) \end{aligned}$$

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- Recall BM nonlinear circuits are gradient systems.
- External characterization given in Cortes et. al. (2005), entailing two different **prolongations** of system.
- Main idea: symmetry obtained for prolongations in observability and accessibility.
- Linear case special case, nonlinear needs prolongations, **complicating balancing procedure!**

Nonlinear systems: balancing

Smooth system

$$\begin{aligned} \dot{x} &= f(x) + g(x)u \\ y &= h(x) \end{aligned}$$

where $u \in \mathbb{R}^m$, $y \in \mathbb{R}^p$, and $x \in M$ (manifold of dim n).

Assumptions:

- $f(0) = 0$, 0 as. stable eq. point for $u = 0$, $x \in X$.
- $h(0) = 0$.
- Controllability function L_c and observability function L_o smooth and exist.

Energy functions: Gramian extensions

$$L_c(x_0) = \min_{\substack{u \in L_2(-\infty,0) \\ x(-\infty)=0,\, x(0)=x_0}} \frac{1}{2}\int_{-\infty}^{0} \| u(t) \|^2 \, dt$$

Minimum amount of control energy necessary to reach state x_0. L_c is the so-called **controllability function**.

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$$L_o(x_0) = \frac{1}{2}\int_{0}^{\infty} \| y(t) \|^2 \, dt, \qquad \substack{x(0)=x_0 \\ u(\tau)=0,\ 0\le\tau<\infty}$$

Output energy generated by state x_0.

L_o is the so-called **observability function**

Nonlinear systems: balancing

- In linear case: $L_o(x) = \frac{1}{2}x^T M x$ and $L_c(x) = \frac{1}{2}x^T W^{-1} x$.
- Lyapunov and Hamilton-Jacobi-Bellmann equations characterize L_o and L_c.
- Role of observability and controllability for linear systems is replaced by **zero-state observability** and **asymptotic reachability** (or anti-stabilizability).

Nonlinear systems: balancing

Lots of research efforts later (Fujimoto, Gray, Scherpen):

- Under appropriate conditions, there exists neighborhood X of 0 and $x = \Phi(z)$ s.t.

$$L_c(\Phi(z)) = \frac{1}{2}\sum_{i=1}^{n} \frac{z_i^2}{\bar{\sigma}_i(z_i)} \quad L_o(\Phi(z)) = \frac{1}{2}\sum_{i=1}^{n} z_i^2 \bar{\sigma}_i(z_i).$$

In particular, on X, $\quad \|\Sigma\|_H = \sup_{\substack{z_1 \\ \Phi(z_1,0,\ldots,0)\in X}} \bar{\sigma}_1(z_1).$

- Singular value functions unique at coordinate axes.
- Tool for balanced structure preserving model reduction.

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- Singular value functions unique at coordinate axes.
- Tool for balanced structure preserving model reduction.
- Discrete time version similar! Fujimoto, Scherpen 2010.

Nonlinear gradient systems: balanced trunc.

Take $T(x) = G(x)^{-1}$, then balancing can be applied. Assume that $\sigma_k(x_k) >> \sigma_{k+1}(x_{k+1})$, and split state (and matrices and functions) accordingly, i.e. ,$x^a = (x_1, \ldots, x_k)^T$, $x^b = (x_{k+1}, \ldots, x_n)^T$. Balanced trunction, $x^b = 0$, then

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Proposition: If

$$T^{ab}(x^a, 0)\frac{\partial P}{\partial x^b}(x^a, 0) = 0, \quad \text{and} \quad T^{ab}(x^a, 0)\frac{\partial h}{\partial x^b}(\bar{x}^a, 0) = 0,$$

then reduced order system is gradient system with pseudo-Riemannian metric $G^a(x^a) = T^{aa}(x^a, 0)^{-1}$.

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Main problem compared with linear case: no specific structure for $G(x)$ is obtained!

Nonlinear gradient systems: sing. pert.

Also singular perturbation reduction different in nonlinear case.

Restrictive assumption: in bal. form x^b part of system linear.

Proposition: Under appropriate assumptions, reduced order system via singular perturbations is gradient again, i.e.,

$$\dot{x}^a = \hat{T}(x^a)\frac{\partial \hat{P}}{\partial x^a}(x^a) + \hat{T}(x^a)\frac{\partial \hat{h}}{\partial x^a}(x^a)u, \quad y = \hat{h}(x^a, u)$$

with $\hat{P}(x^a)$, $\hat{h}(x^a, u)$ follow from solving x^b from

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Claim: Via nonlinear Schur complement linearity assumption can be removed.

Nonlinear gradient systems: cross Gramian

Cross-Gramian for linear gradient systems uses symmetry property. How about nonlinear case?

- In Ionescu, Scherpen 2009 extension is given for prolongation and gradient extension $\longrightarrow$ a **Sylvester** like equation is difficult to obtain.

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- Again due to lack of structure in $G(x)$, no direct link with balancing is yet obtained.

Nonlinear gradient systems: cross Gramian

Result based on cross-Gramian definition for prolongations of gradient system:

Corollary: Take $f(x) = G(x)^{-1} \frac{\partial P}{\partial x}(x)$. Nonlinear cross Gramian $L(x)$ fulfills the following Sylvester like equation

$$p^T T(x) L(x) \frac{\partial f}{\partial x}(x) v + \frac{1}{2} p^T \frac{\partial h}{\partial x}(x) \frac{\partial^T h}{\partial x}(x) v =$$

$$-v^T \frac{\partial^2 L_o^p}{\partial v \partial x}(x, v) f(x) + \frac{\partial^T L_o^p}{\partial x}(x, v) f(x) - \bar{\mathcal{F}}^T L(x) v,$$

with $\bar{\mathcal{F}} = \mathcal{F} - T(x)\dot{G}(x)v$, and p and v states of prolongations.

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Remark: In linear case Sylvester equation obtained!

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- How to deal with DAE systems?