Variational formulation and integration of electric circuits

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Variational integrators [1] are geometric integrators that are symplectic, momentum-preserving and have an excellent long-time energy behavior. This means that energy (for conservative systems) or the energy rate (for systems with external forces) is on average preserved in the discrete solution even for large step sizes, i.e., there is no numerical energy dissipation or generation. Variational integrators have been mainly developed and used for a wide variety of mechanical systems. However, considering real-life systems, these are in general not of purely mechanical character. In fact, more and more systems become multidisciplinary in the sense, that not only mechanical parts, but also electric and software subsystems are involved, resulting into mechatronic systems. Since the integration of these systems with a unified simulation tool is desirable, the aim is to extend the applicability of variational integrators to mechatronical systems.
As a first step towards a unified simulation, in this talk, a variational framework for nonlinear electric circuits consisting of inductors, capacitors, resistors, and voltage sources is presented [2, 3]. The Lagrangian formulation is based a degenerate Lagrangian and takes the basic circuit laws into account. The resulting differential-algebraic system can be reduced by performing the variational principle on a reduced space. It is shown under which conditions the corresponding reduced Lagrangian is regular and thus the equations of motion transform to an ordinary differential equation system. Based on a discrete variational formulation, a variational integrator for the structure-preserving simulation of nonlinear electric circuits is derived and demonstrated by numerical examples. It can be shown that for the developed integrators the energy rate as well as the current frequencies are better preserved in comparison to standard circuit simulations with e.g. Runge-Kutta or BDF methods.
The approach is extended to hybrid systems that allows for the inclusion of transistors. Transistors naturally introduce a hybrid behavior to the circuit, that means an interaction between continuous dynamics and discrete events. As a first step, we consider a simple diode model that makes it possible to study a basic state-controlled switching feature for the electric circuit. For the variational formulation and simulation of such a hybrid electric systems, we adapt strategies for the construction of nonsmooth variational collision integrators [4] .
In the future, we want to analyze further properties of the resulting equations of motion based on the circuit topology and relate the results to classical ones from the well established Modified Nodal Analysis. Furthermore, we will investigate in which way the variational techniques approve for the simulation of more complex circuits including further circuits elements as well as for the simulation and optimization of large mechatronic systems.

References

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