On a model hierarchy for gas flow simulation

Jeroen J. Stolwijk

Technische Universität Berlin

joint work with Volker Mehrmann

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- Both the energy transition and the (gas) crisis in Eastern Europe are important topics today in Europe.
- These topics force Europe to use and transport natural gas efficiently.

		2020	2030	2040	2050
Climate	Greenhouse gases (vs. 1990)	- 40%	- 55%	- 70%	- 80 to - 95%
Renewable energies	Share of electricity	35%	50%	65%	80%
	Overall share (Gross final energy consumption)	18%	30%	45%	60%

Figure: Goals of the energy transition in Germany [1].

[1] http://energytransition.de. Retrieved: September 16, 2014.

Pipeline Network

Sources: gas-roads.eu, mobilexag.de, and nordkurier.de.

Euler equations: continuity equation, impulse equation, and energy equation. In one dimension:

$$\begin{aligned} \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) &= 0, \\ \frac{\partial}{\partial t}(\rho v) + \frac{\partial}{\partial x}(p + \rho v^2) &= -\frac{\lambda}{2D}\rho v|v| - g\rho h', \\ \frac{\partial}{\partial t}\left(\rho(\frac{1}{2}v^2 + e)\right) + \frac{\partial}{\partial x}\left(\rho v(\frac{1}{2}v^2 + e) + pv\right) &= -\frac{k_w}{D}(T - T_w), \end{aligned} \tag{TA1}$$

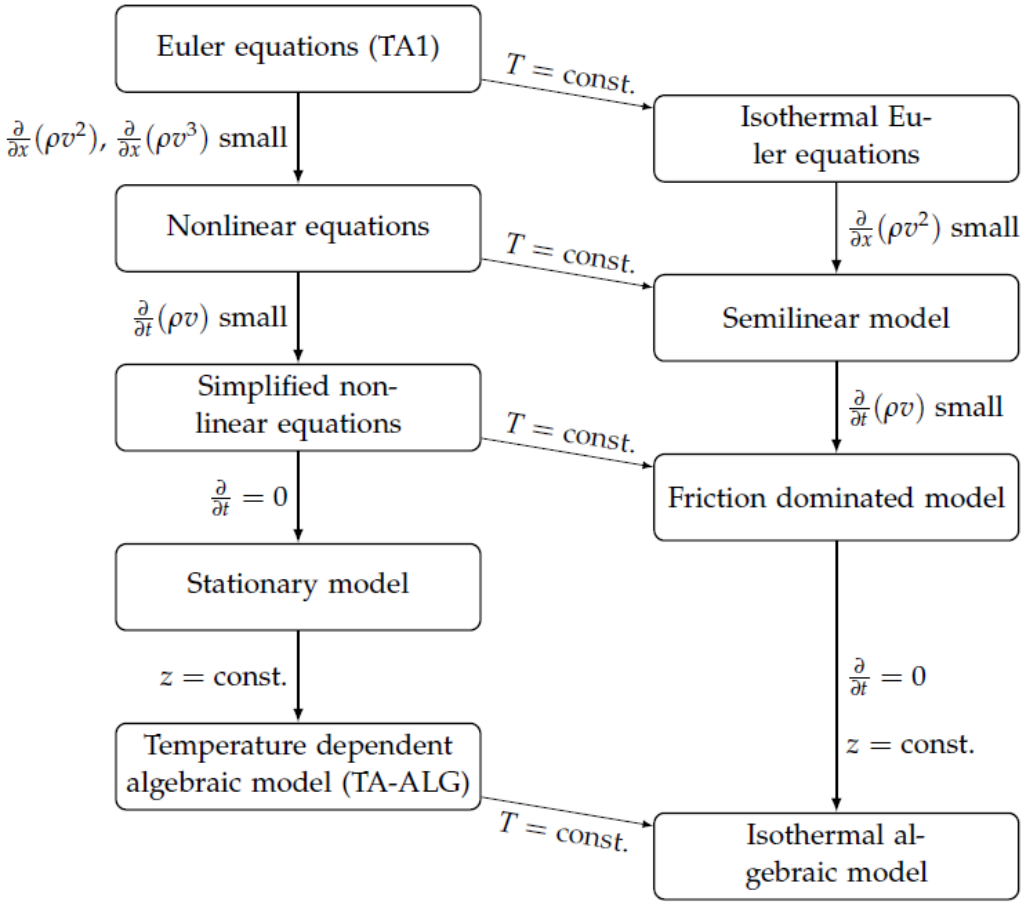
together with the state equation for real gases

$$p = R\rho T z(p, T).$$

Here,

λ = pipe friction coefficient, D = diameter, h' = slope of the pipe, e = internal energy, k_w = heat conductivity coefficient, T_w = wall temperature, R = gas constant, $z(p, T)$ = compressibility factor.

Model Hierarchy



The model hierarchy is used to find an appropriate trade-off between accuracy and computational complexity. Moreover, only the algebraic models are (currently) feasible for optimization.

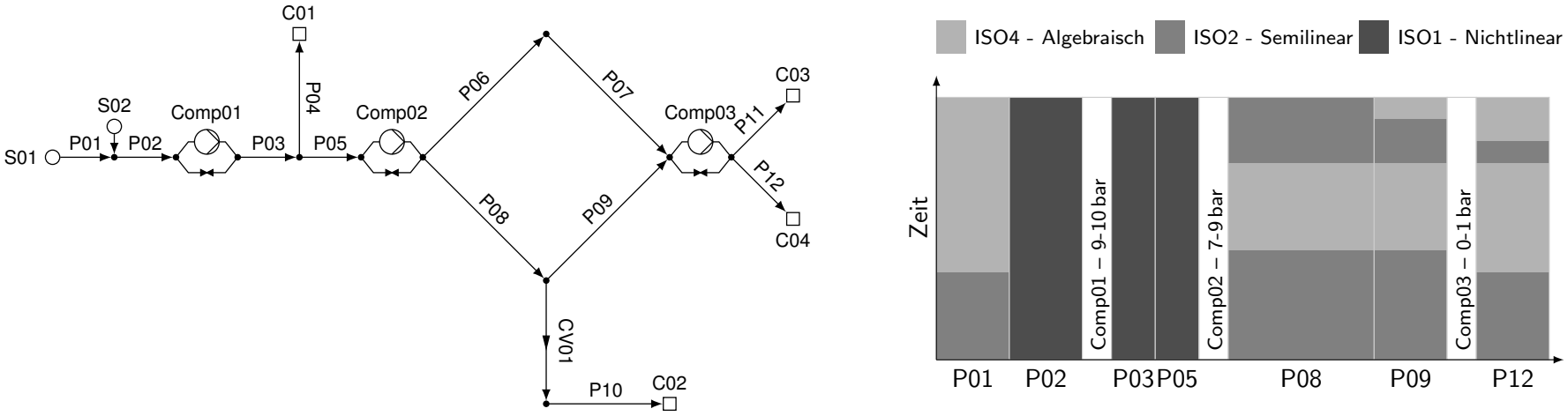


Figure: Realistic gas network with three compressors, together with the choice of models in the pipelines over time for a moderate accuracy [2].

[2] P. Domschke, O. Kolb, and J. Lang. Adjoint-based control of model and discretisation errors for gas flow in networks. *Int. J. Mathematical Modelling and Numerical Optimisation*, 2(2):175-193, 2011.

The simplifications in the hierarchy lead to the algebraic model

$$
\begin{aligned}
\rho v &= \rho_{in} v_{in}, \\
p(x) &= \sqrt{p_{in}^2 - \frac{\lambda c^2}{2D} \rho v |\rho v| (x - x_0)}, \\
T(x) &= (T_{in} - T_w) e^{-\frac{k_w}{D c_v \rho_v}(x - x_0)} + T_w,
\end{aligned}
\tag{TA-ALG}
$$

which is commercially utilized for optimization and now further examined.

Backward Error

Backward error: Computational rounding errors and measurement errors are interpreted as errors/perturbations in the input parameters.

For computing the pressure

$$p(q) = \sqrt{p_{in}^2 - \frac{\lambda c^2}{2D}\rho v|\rho v|(x - x_0)},$$

the following algorithm can be applied:

$z_1 := p_{in} \cdot p_{in};$	$z_5 := \frac{z_3}{z_4};$	$z_9 := z_8 \cdot \lvert z_6 \rvert;$
$z_2 := c \cdot c;$	$z_6 := \rho \cdot v;$	$z_{10} := z_9 \cdot z_7;$
$z_3 := \lambda \cdot z_2;$	$z_7 := x - x_0;$	$z_{11} := z_1 - z_{10};$
$z_4 := 2 \cdot D;$	$z_8 := z_5 \cdot z_6;$	$p(q) := z_{12} = \sqrt{z_{11}}.$

Every step causes a relative error of machine precision ε.

Thus,

$$\tilde{p}(q) = \sqrt{p_{in}^2(1+\varepsilon_1)(1+\varepsilon_{11}) - \frac{\lambda c^2(1+\varepsilon_2)(1+\varepsilon_3)}{2D(1-\varepsilon_4)}(1+\varepsilon_5)\rho v(1+\varepsilon_6)(1+\varepsilon_8)|\rho v|}$$
$$\cdot \sqrt{(1+\varepsilon_6)(1+\varepsilon_9)(x-x_0)(1+\varepsilon_7)(1+\varepsilon_{10})(1+\varepsilon_{11})}(1+\varepsilon_{12})$$
$$= \sqrt{(p_{in}(1+\varepsilon_{13}))^2 - \frac{\lambda(1+\varepsilon_{14})c^2}{2D}\rho v|\rho v|(x-x_0)}.$$

Including measurement errors for the input parameters gives

$$\tilde{p}(q) = \sqrt{(p_{in}(1+\varepsilon_{p_{in}})(1+\varepsilon_{\mathbf{13}}))^{\mathbf{2}} - \frac{\lambda(1+\varepsilon_\lambda)(1+\varepsilon_{\mathbf{14}})c^{\mathbf{2}}(1+\varepsilon_c)^{\mathbf{2}}}{2D(1-\varepsilon_D)}\rho v|\rho v|(1+\varepsilon_\rho)^{\mathbf{2}}(1+\varepsilon_v)^{\mathbf{2}}(x(1+\varepsilon_x) - x_{\mathbf{0}}(1+\varepsilon_{x_{\mathbf{0}}}))}$$
$$= \sqrt{(p_{in}(1+\varepsilon_{\mathbf{15}}))^{\mathbf{2}} - \frac{\lambda(1+\varepsilon_{\mathbf{16}})c^{\mathbf{2}}}{2D}\rho v|\rho v|(x(1+\varepsilon_x) - x_{\mathbf{0}}(1+\varepsilon_{x_{\mathbf{0}}}))}.$$

So the backward error for $\tilde{p}(q)$ is given by

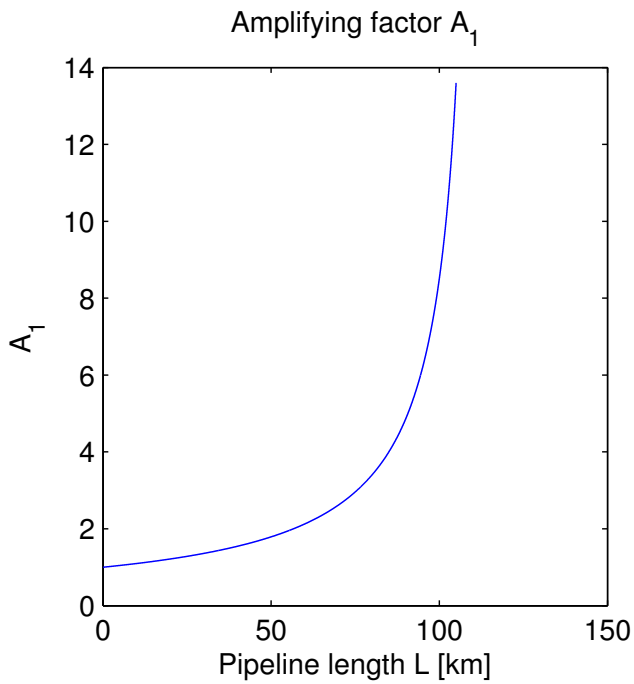
$$\tilde{p}(p_{in}, \lambda, x, x_0) = p(p_{in}(1+\varepsilon_{15}), \lambda(1+\varepsilon_{16}), x(1+\varepsilon_x), x_0(1+\varepsilon_{x_{\mathbf{0}}})).$$

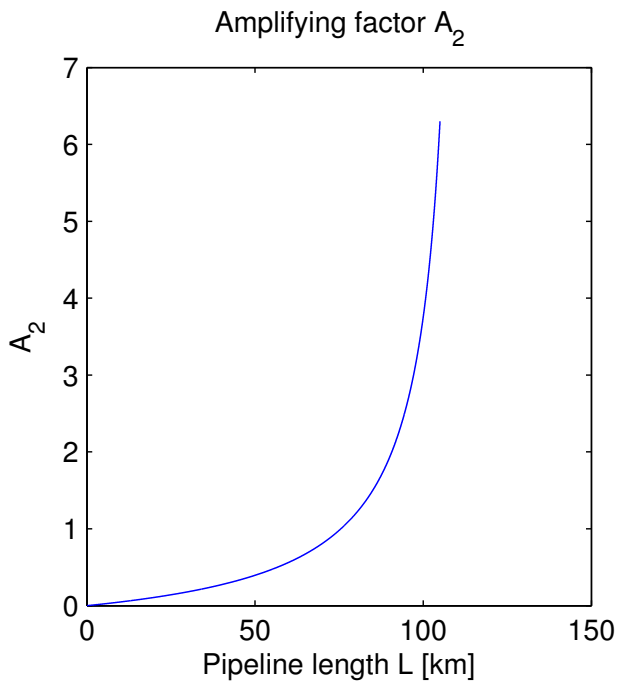
Relative error in the pressure due to perturbations in the data:

$$\frac{p(q)-p(q+\Delta q)}{p(q)} = \frac{\partial p(q)}{\partial p_{in}}\frac{p_{in}}{p(q)}\frac{\Delta p_{in}}{p_{in}} + \frac{\partial p(q)}{\partial \lambda}\frac{\lambda}{p(q)}\frac{\Delta\lambda}{\lambda} + \frac{\partial p(q)}{\partial x}\frac{x}{p(q)}\frac{\Delta x}{x} + \frac{\partial p(q)}{\partial x_0}\frac{x_0}{p(q)}\frac{\Delta x_0}{x_0} + O\left((\Delta q)^2\right)$$

$$= \underbrace{\left(\frac{p_{in}}{p(q)}\right)^2}_{A_1}\varepsilon_{15} - \underbrace{\frac{\lambda c^2\rho^2 v^2(x-x_0)}{4Dp(q)^2}}_{A_2}\varepsilon_{16} - \underbrace{\frac{\lambda c^2\rho^2 v^2 x}{4Dp(q)^2}}_{A_3}\varepsilon_x + \underbrace{\frac{\lambda c^2\rho^2 v^2 x_0}{4Dp(q)^2}}_{A_4}\varepsilon_{x_0} + \text{h.o.t.}$$

Amplifying factors for the relative error ($L = x - x_0$):





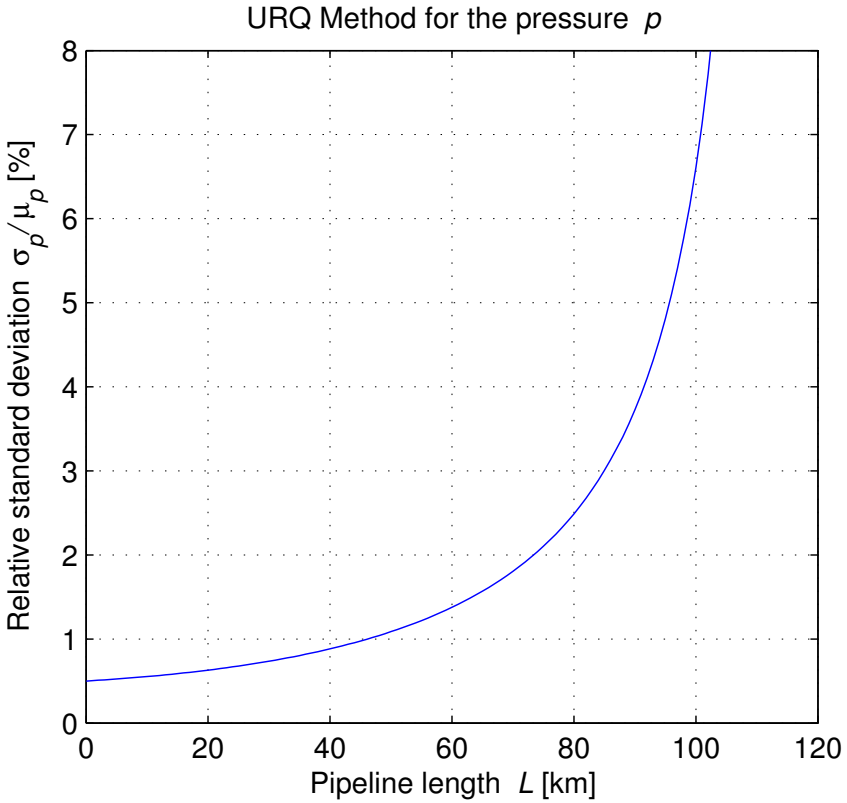
Univariate Reduced Quadrature (URQ) Method:

$$\mu_{f_{\mathbf{URQ}}}(\boldsymbol{x}_0) = W_0 f(\boldsymbol{x}_0) + \sum_{i=1}^{n} W_i \left[\frac{f(\boldsymbol{x}_i^+)}{h_i^+} - \frac{f(\boldsymbol{x}_i^-)}{h_i^-} \right]$$

and

$$\sigma^2_{f_{\mathbf{URQ}}}(\boldsymbol{x}_0) = \sum_{i=1}^{n} \left\{ W_i^+ \left[\frac{f(\boldsymbol{x}_i^+) - f(\boldsymbol{x}_0)}{h_i^+} \right]^2 + W_i^- \left[\frac{f(\boldsymbol{x}_i^-) - f(\boldsymbol{x}_0)}{h_i^-} \right]^2 \right.$$
$$\left. + \; W_i^{\pm} \frac{[f(\boldsymbol{x}_i^+) - f(\boldsymbol{x}_0)][f(\boldsymbol{x}_i^-) - f(\boldsymbol{x}_0)]}{h_i^+ h_i^-} \right\} .$$

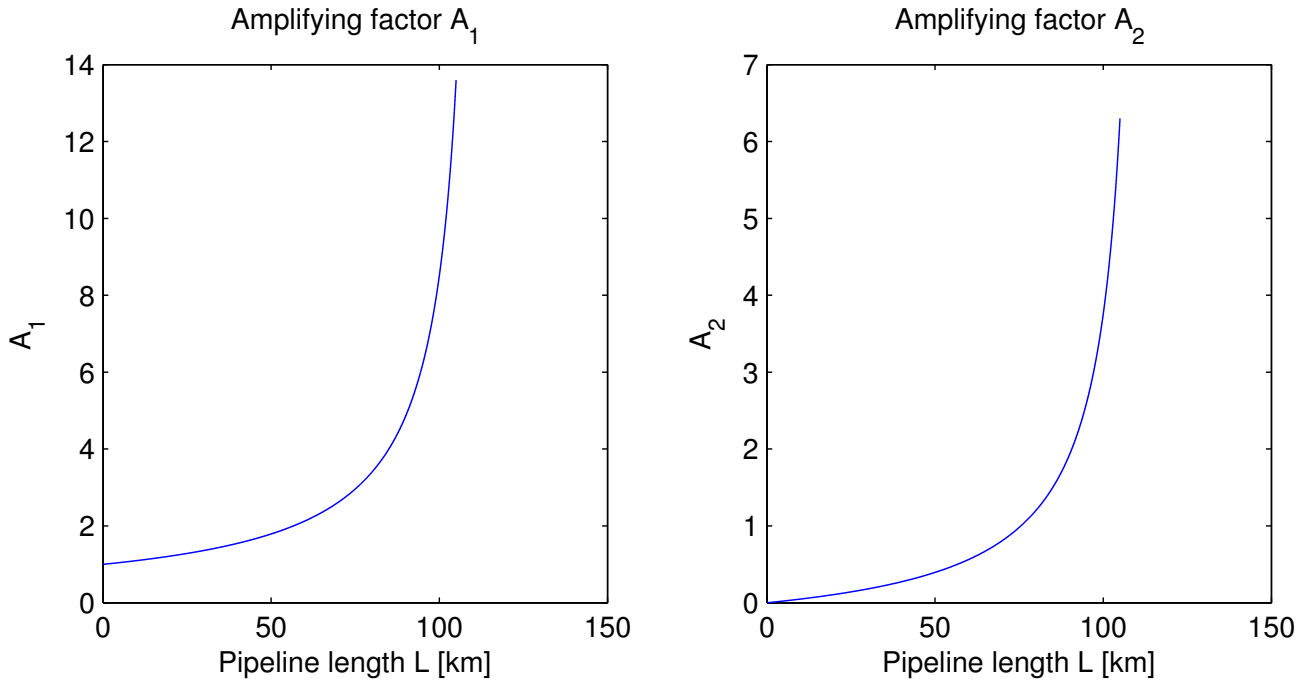
The theoretical result for the pressure $p(\boldsymbol{x}) = \sqrt{p_{in}^2 - \frac{\lambda c^2 L}{2D}\rho v|\rho v|}$ is illustrated using the URQ Method. The relative standard deviation $\frac{\sigma_x}{\mu_x}$ of the input parameters $\boldsymbol{x}$ is 0.5%.



$\Rightarrow$ A length of 70 km is the absolute maximum (40 km is preferable).

Results

- The error for the pressure in the algebraic model grows quickly if the pipeline length is increased.
- This has been shown both theoretically and in a simulation.



Conclusions

- The work of Domschke/Kolb/Lang (2011) shows that a model hierarchy can be used to reach an appropriate trade-off between accuracy and computational complexity.
- The algebraic model can be used safely for pipelines up to 70 km length.

Future Work

- Perform the error analysis on the more advanced (time-dependent) models in the hierarchy.
- Write a user-friendly MATLAB toolbox that quickly determines which model can be used where in the network, based on an error analysis.
- Use model adaptivity to determine which model to use for a desired accuracy.


ANY
QUESTIONS
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