Low rank perturbation of canonical forms

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Low rank modifications of a physical system that depends on many parameters arise when only a few parameters are modified, regardless of their magnitude (in the sense of norm). When the physical system is modeled by a system of linear differential (or differential-algebraic) equations of degree d:

$$A_d x^{(d)} + \cdots + A_1 x' + A_0 x = f, \qquad A_0, A_1, \ldots, A_n \in \mathbb{C}^{m\times n}, \tag{1}$$

then this kind of modifications result in low rank perturbations of the associated matrix polynomial $A_0 + \lambda A_1 + \cdots + \lambda^d A_d$.
It is of particular interest the case of linear differential-algebraic equations of degree 1:

$$Bx' + Ax = f, \qquad A, B \in \mathbb{C}^{m\times n},$$

where the associated polynomial is a *pencil*, $A + \lambda B$.
The behavior of the solution of the equation (1) can be described using the canonical form of the matrix polynomial (the *Smith form* for general polynomials, and the *Kronecker canonical form* (KCF) for pencils, which, for regular pencils, is also known as the *Weierstrass canonical form* (WCF)). Hence, the study of how this canonical form changes after low rank perturbations is interesting, not only as a theoretical problem, but also in a practical setting.
In this talk, we will review know results that describe the change of the following canonical forms under low rank perturbations:

- The Jordan canonical form of a matrix [4, 5].
- The WCF of a regular pencil [3].
- The KCF of a singular pencil without full rank [1].
- The Smith form of a regular matrix polynomial [2].

We will also relate some of these results with recent work by Volker and collaborators that deal with structured matrices (like selfadjoint, symplectic, orthogonal, or unitary).

This talk is mainly based on joint work with F. M. Dopico and J. Moro.

References

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