Compressing the coefficient matrices of a singular matrix polynomial

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Following the idea in [1], we present a compressing procedure for singular matrix polynomials of the form

$$P(\lambda) = \sum_{j=0}^{k} \lambda^j A_j = \lambda^k A_k + \lambda^{k-1} A_{k-1} + \ldots + \lambda A_1 + A_0.$$

where $A_0, \ldots, A_k \in \mathbb{C}^{m\times n}$. The procedure applies a sequence of $k-1$ unitary transformations simultaneously on the coefficient matrices to form a matrix polynomial

$$\widetilde{P}(\lambda) = U^* P(\lambda) V =: \lambda^k \widetilde{A}_k + \ldots + \lambda \widetilde{A}_1 + \widetilde{A}_0,$$

where U, V are unitary and all the coefficient matrices are in block forms $\widetilde{A}_j = [A_{pq}^{(j)}]_{k\times k}$. A so-called trimmed linearization $\lambda\widetilde{N} + \widetilde{M}$ can be formed with the blocks from the transformed coefficient matrices, where

$$\widetilde{N} = \begin{bmatrix} \widetilde{A}_1 & A^{(2)}_{1:k,1:k-1} & A^{(3)}_{1:k,1:k-2} & \cdots & A^{(k-1)}_{1:k,1:2} & A^{(k)}_{1:k,1:1} \\ A^{(2)}_{1:k-1,1:k} & A^{(3)}_{1:k-1,1:k-1} & A^{(4)}_{1:k-1,1:k-2} & \cdots & A^{(k)}_{1:k-1,1:2} & \\ A^{(3)}_{1:k-2,1:k} & A^{(4)}_{1:k-2,1:k-1} & \iddots & \iddots & & \\ \vdots & \vdots & \iddots & & & \\ A^{(k-1)}_{1:2,1:k} & A^{(k)}_{1:2,1:k-1} & & & & \\ A^{(k)}_{1:1,1:k} & & & & & \end{bmatrix}$$

$$\widetilde{M} = \begin{bmatrix} \widetilde{A}_0 & & & & & \\ & -A^{(2)}_{1:k-1,1:k-1} & -A^{(3)}_{1:k-1,1:k-2} & \cdots & -A^{(k-1)}_{1:k-1,1:2} & -A^{(k)}_{1:k-1,1:1} \\ & -A^{(3)}_{1:k-2,1:k-1} & -A^{(4)}_{1:k-2,1:k-2} & \cdots & -A^{(k)}_{1:k-2,1:2} & \\ & \vdots & \vdots & \iddots & & \\ & -A^{(k-1)}_{1:2,1:k-1} & -A^{(k)}_{1:2,1:k-2} & & & \\ & -A^{(k)}_{1:1,1:k-1} & & & & \end{bmatrix},$$

and $A^{(j)}_{1:p,1:q}$ is the submatrix of $\widetilde{A}_j$ on the leading p block rows and q block columns. This trimmed linearization preserves all the eigenstructure information of the original matrix polynomial, except the leading $k-1$ Jordan chains of the eigenvalue infinity are deflated. In contrast, conventional linearizations may increase the length of singular chains [2, 3].
The compressing procedure can be simply applied to structured matrix polynomials for constructing structured trimmed linearizations.

References

[1] R. Byers, V. Mehrmann, and H. Xu. Trimmed linearizations for structured matrix polynomials. *Linear Algebra Appl.*, 429:2373 - 2400, 2008.

[2] F. De Terán, F. M. Dopico, and D. S. Mackey. Fiedler companion linearizations and the recovery of minimal indices. *Electr. J. Lin. Alg.*, 17:518 - 531, 2009.

[3] F. De Terán, F. M. Dopico, and D. S. Mackey. Linearizations of singular matrix polynomials and the recovery of minimal indices. *Electr. J. Lin. Alg.*, 18:371 - 402, 2009.