Parametric PDEs: Sparse or low-rank approximations?

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Abstract

This talk centers on accuracy certified adaptive approximation of parameter-to-solution maps for parametric families of elliptic PDEs with coefficients depending on a large or even infinite number of parameters. A generic adaptive algorithm serves as a unifying thread through the various aspects of this problem. In fact, in its general form it is guaranteed to converge in the sense that it realizes any given target accuracy after finitely many steps. Moreover, its core subroutines for adaptively applying the operator and tensor recompression can be specified so as to produce approximations of essentially three types, namely 1) best n-term approximations for a given background wavelet-Legendre basis, 2) low-rank approximations based on separating spatial and parametric variables, 3) stable hierarchical tensor formats based on fully separating all parametric variables. For all three specifications the algorithm is shown to perform with (near-)optimal complexity with respect to suitable benchmark approximability properties of the solutions. The identification and justification of these benchmark properties ties into the second central theme, namely the understanding of which approximation types are best suited for which problem characteristics including the role of different types of parametric coefficient representations. Combining wavelet and tensor concepts, corresponding findings enter, in particular, the design and analysis of new matrix compression techniques which are fundamental for the optimal performance of both variants 1) and 2).