Tensor numerical methods for multi-dimensionaal PDEs

Boris N. Khoromskij

Max-Planck-Institute for Mathematics in the Sciences,
Inselstr. 22-26, D-04103 Leipzig, Germany
bokh@mis.mpg.de; ***http://personal-homepages.mis.mpg.de/bokh***

The breaking through approach to low-parametric representation of multivariate functions and operators is based on the principle of separation of variables which can be realized by using approximation in rank-structured tensor formats [5,2]. This allows the linear complexity scaling in dimension, hence breaking the "curse of dimensionality". The method of quantized tensor train (QTT) approximation is proven to provide the logarithmic data-compression on a wide class of discretized functions and operators [1]. The range-separated tensor formats [8] allow efficient representation of many-particle interaction potentials.

We discuss how the tensor methods based on the canonical, Tucker, TT and QTT approximation apply to calculation of electrostatic potential of many-particle systems, to the (post) Hartree-Fock eigenvalue problem for large 3D lattice-structured molecules (electrostatic potential on a lattice in electronic structure calculations) [2,4,6], and in the reduced basis approach to the Bethe-Salpeter spectral problem (excitation energies) [7]. We also address tensor methods for the elliptic equations with highly-oscillating coefficients (homogenization theory), and for parametric/stochastic elliptic PDEs. The other direction is related to tensor approach for parabolic equations [3] including the space-time $d+1$ formulation.

The efficiency of the tensor methods for PDEs is illustrated by numerical examples.

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