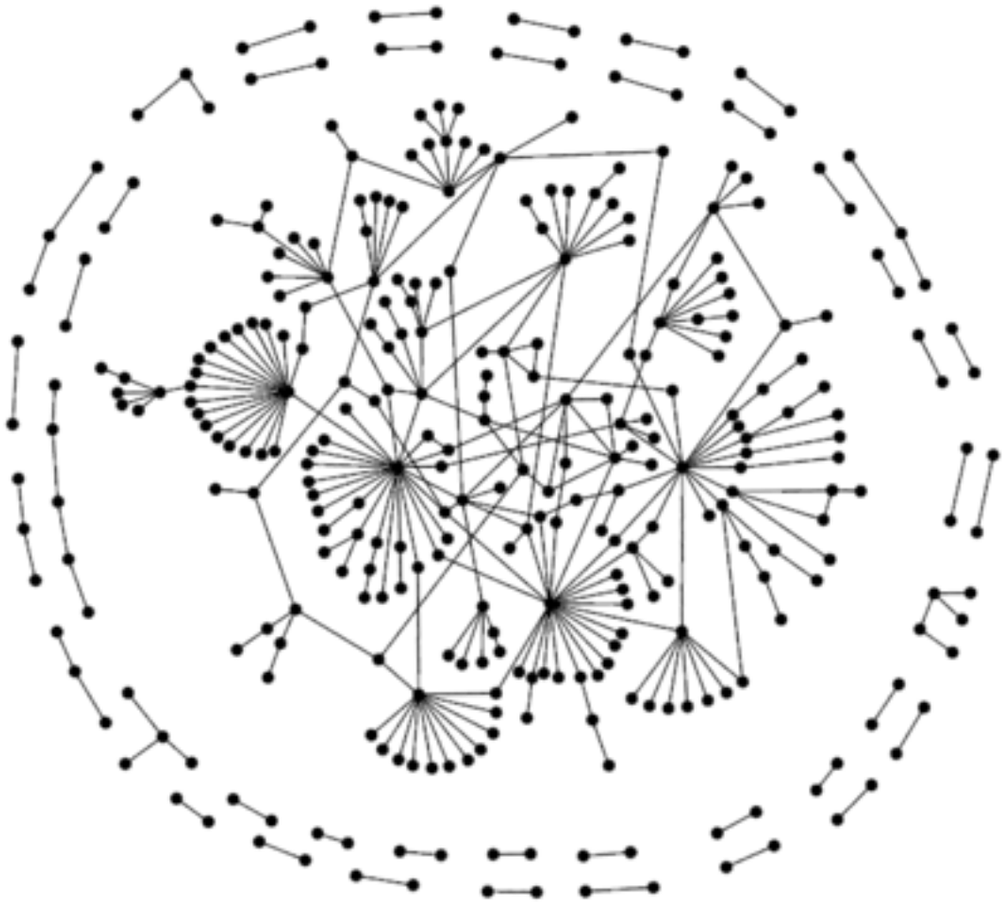
Composite Self-concordant Minimization

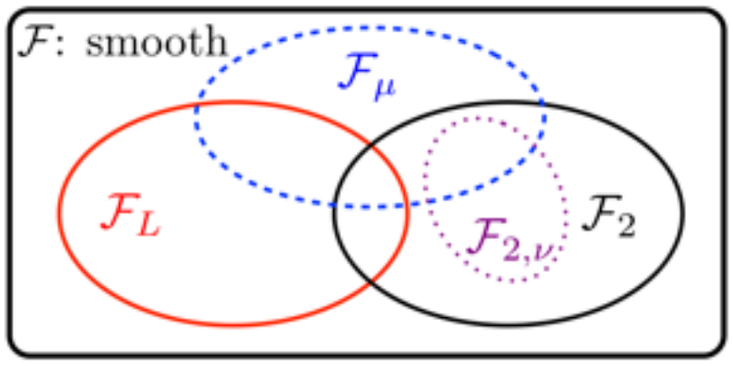


Volkan Cevher

*Laboratory for Information and Inference Systems-**LIONS***

Ecole Polytechnique Federale de Lausanne (EPFL)

volkan.cevher@epfl.ch



joint work with

Quoc Tran Dinh

Anastasios Kyrillidis

Yen-Huan Li

ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

lions@epfl

MARIE CURIE ACTIONS

FNSNF FONDS NATIONAL SUISSE SCHWEIZERISCHER NATIONALFONDS FONDO NAZIONALE SVIZZERO SWISS NATIONAL SCIENCE FOUNDATION

erc

(P) Composite minimization

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

f

convex and **smooth**

g

convex and possibly **nonsmooth**

Motivation

Problem (P) *covers many practical problems*:

- Unconstrained basic LASSO / logistic regression
- Graphical model selection / latent variable graphical model selection
- Poisson imaging reconstruction / LASSO problem with unknown variance
- Low-rank recovery / clustering
- Atomic norm regularization / off-the-grid array processing

g: ℓ_1-norm, nuclear norm or indicator functions

(P) Composite minimization

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

f

convex and **smooth**

g

convex and possibly **nonsmooth**

Motivation

Problem (P) *covers many practical* **LARGE SCALE** *problems*:

- Unconstrained basic LASSO / logistic regression
- Graphical model selection / latent variable graphical model selection
- Poisson imaging reconstruction / LASSO problem with unknown variance
- Low-rank recovery / clustering
- Atomic norm regularization / off-the-grid array processing

need scalable algorithms

g: ℓ_1-norm, nuclear norm or indicator functions

(P) Composite minimization: modus operandi

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

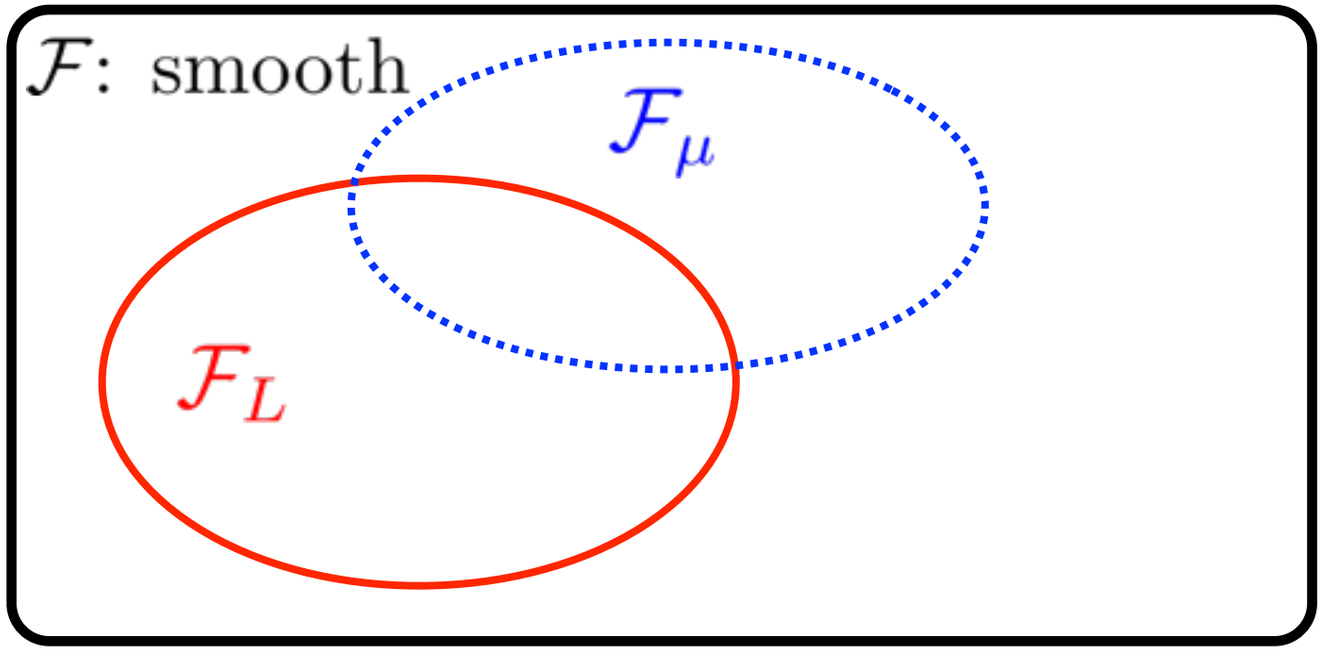
f

convex and **smooth**

g

convex and possibly **nonsmooth**

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

(P) Composite minimization: modus operandi

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

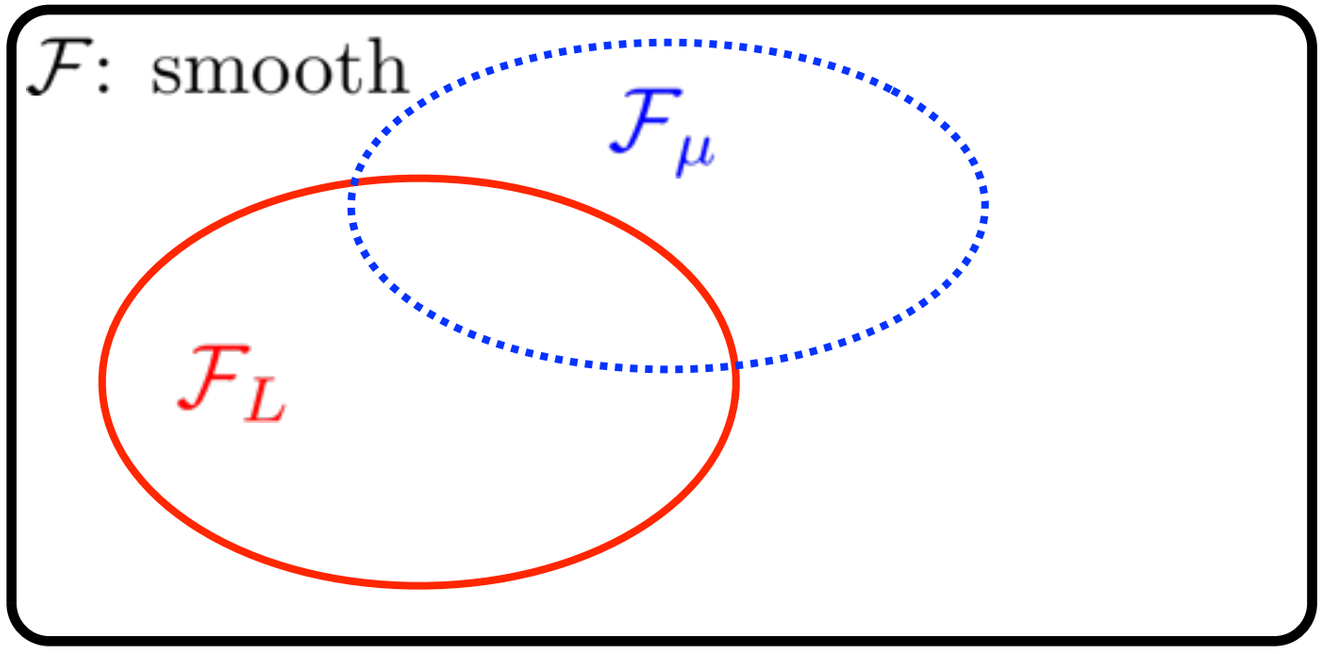
f

convex and **smooth**

g

convex and possibly **nonsmooth**

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

$$\|\nabla f(x) - \nabla f(y)\| \le L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

(P) Composite minimization: modus operandi

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

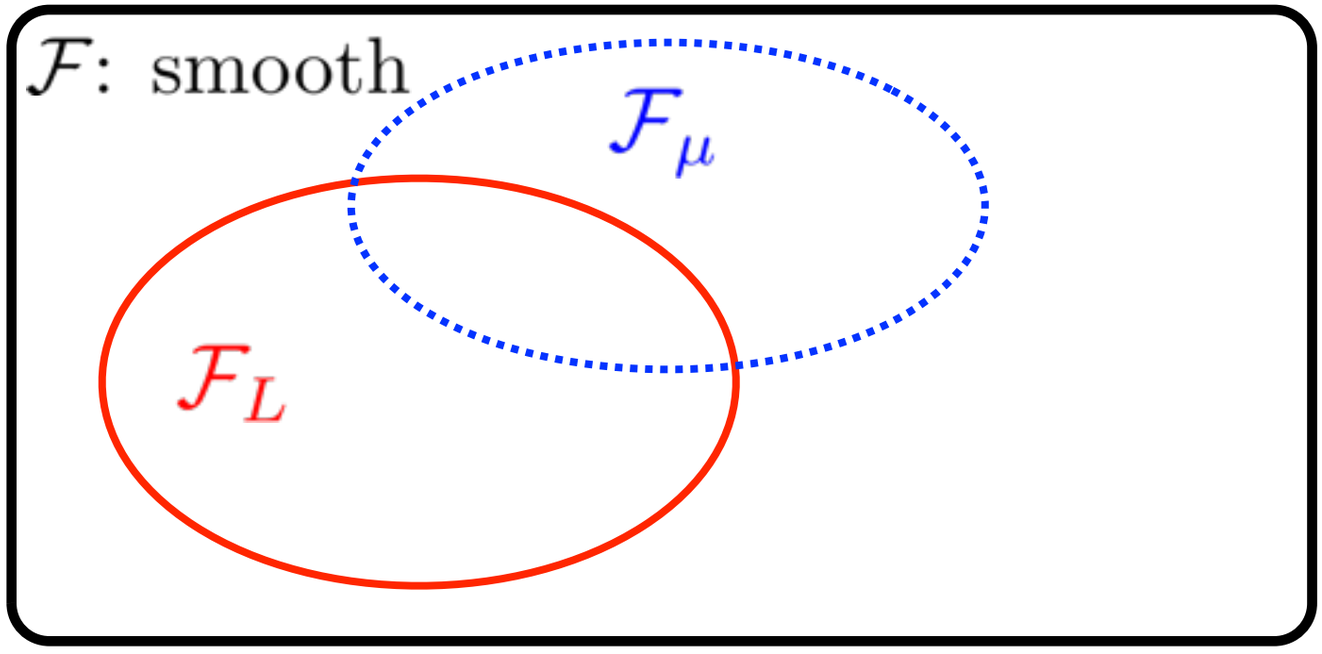
f

convex and **smooth**

g

convex and possibly **nonsmooth**

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

Following **prox** computation is ***tractable:***

$$\text{prox}_{\gamma g}(\mathbf{s}) := \arg\min_{\mathbf{x}} \left\{ g(\mathbf{x}) + \frac{1}{2\gamma}\|\mathbf{x} - \mathbf{s}\|_2^2 \right\}$$

(P) Composite minimization: modus operandi

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

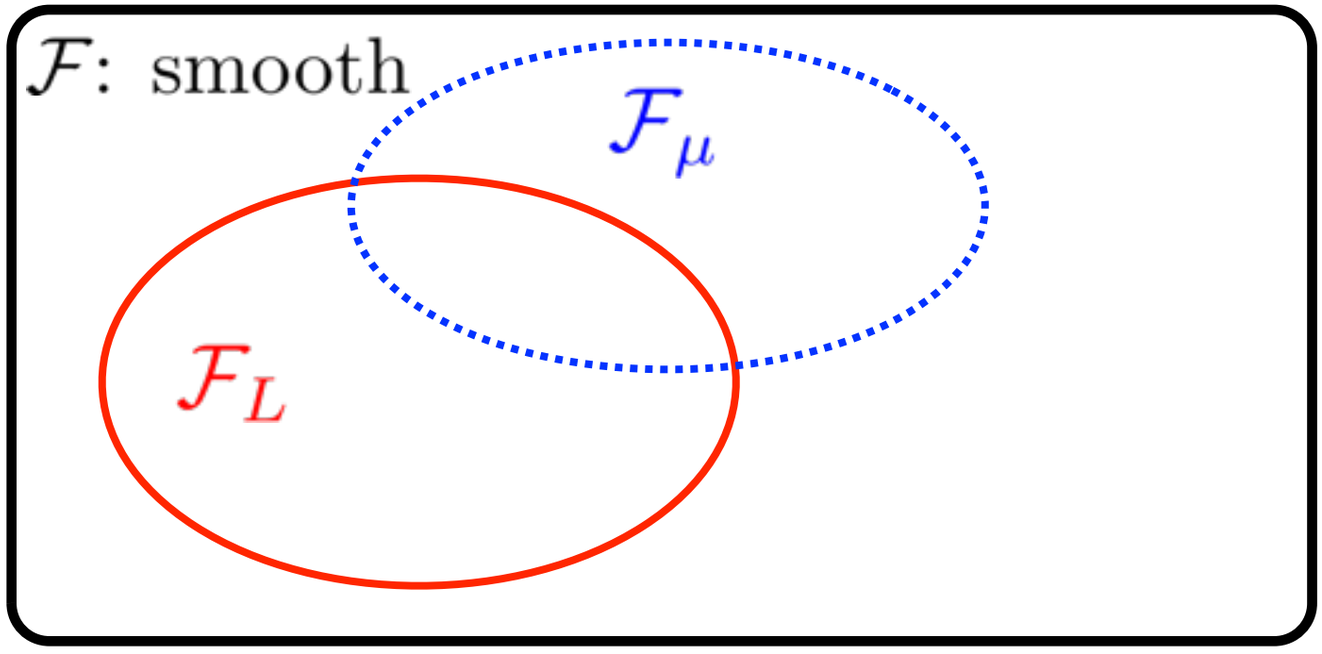
f

convex and **smooth**

g

convex and possibly **nonsmooth**

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

$$\|\nabla f(x) - \nabla f(y)\| \le L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

Following **prox** computation is ***tractable:***

$$\text{prox}_{\gamma g}(\mathbf{s}) := \arg\min_{\mathbf{x}} \left\{ g(\mathbf{x}) + \frac{1}{2\gamma}\|\mathbf{x} - \mathbf{s}\|_2^2 \right\}$$

Example:

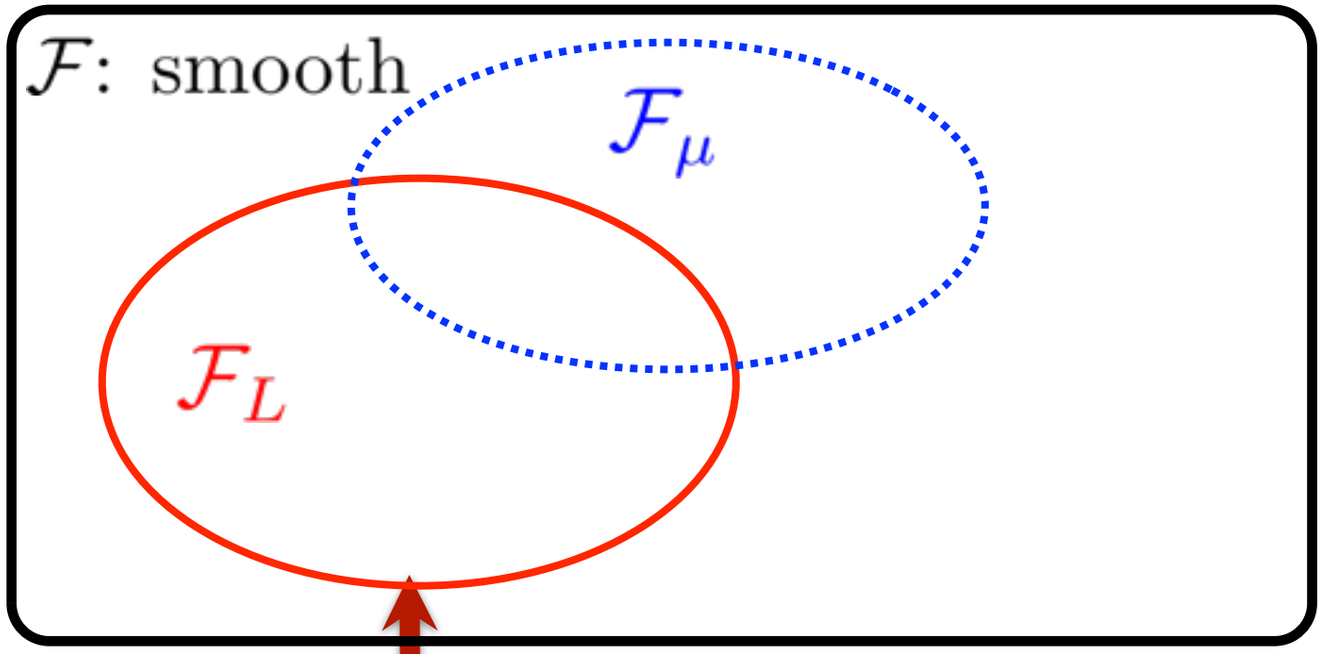
if $g(\mathbf{x}) = \|\mathbf{x}\|_1$, then
$\text{prox}_{\gamma g}(\mathbf{s}) = \text{SoftThresh}(\mathbf{s}, \gamma)$

(P) Composite minimization: modus operandi

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- **f**: convex and **smooth**
- **g**: convex and possibly **nonsmooth** with **"tractable"** prox

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

$$\|\nabla f(x) - \nabla f(y)\| \le L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

well-understood → **Fast gradient schemes (Nesterov's methods)**

Newton/quasi Newton schemes

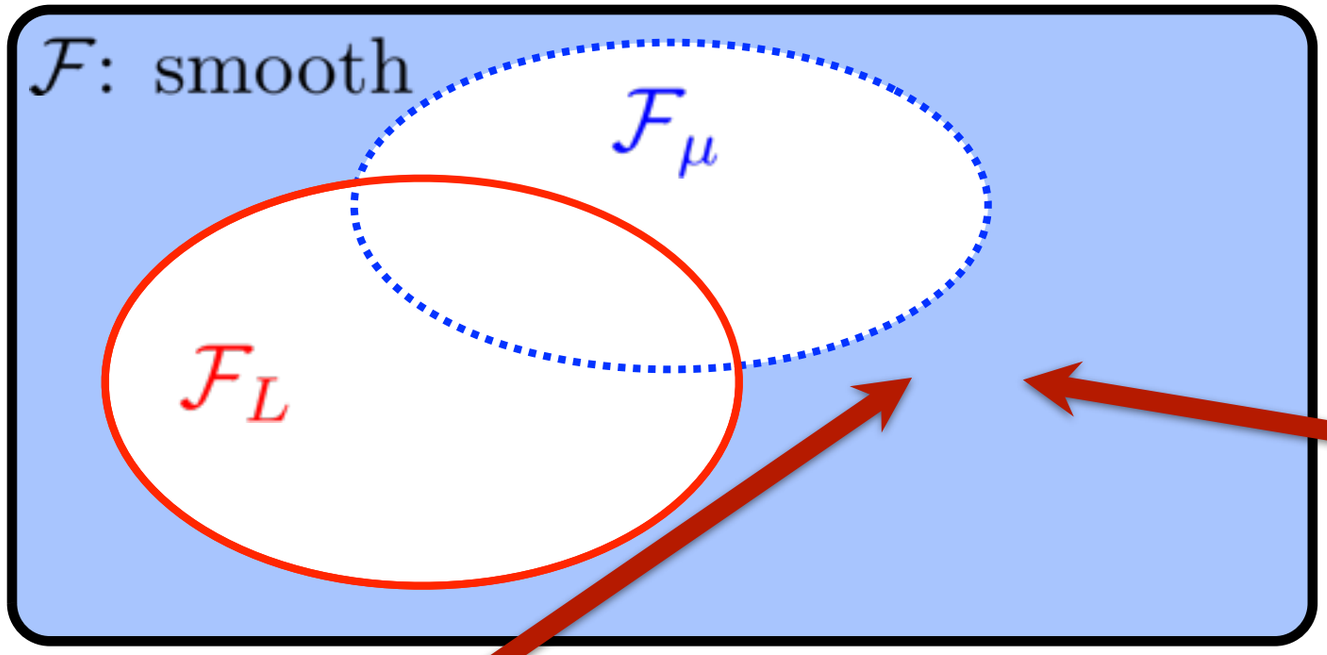
(P) Composite minimization: an uncharted region

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

f
convex and **smooth**

g
convex and possibly **nonsmooth** with **"tractable"** prox

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex

~~$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$~~

~~$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$~~

Scalability is NOT great

~~**Fast gradient schemes (Nesterov's methods)**~~

~~**Newton/quasi Newton schemes**~~

(P) Composite self-concordant minimization

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

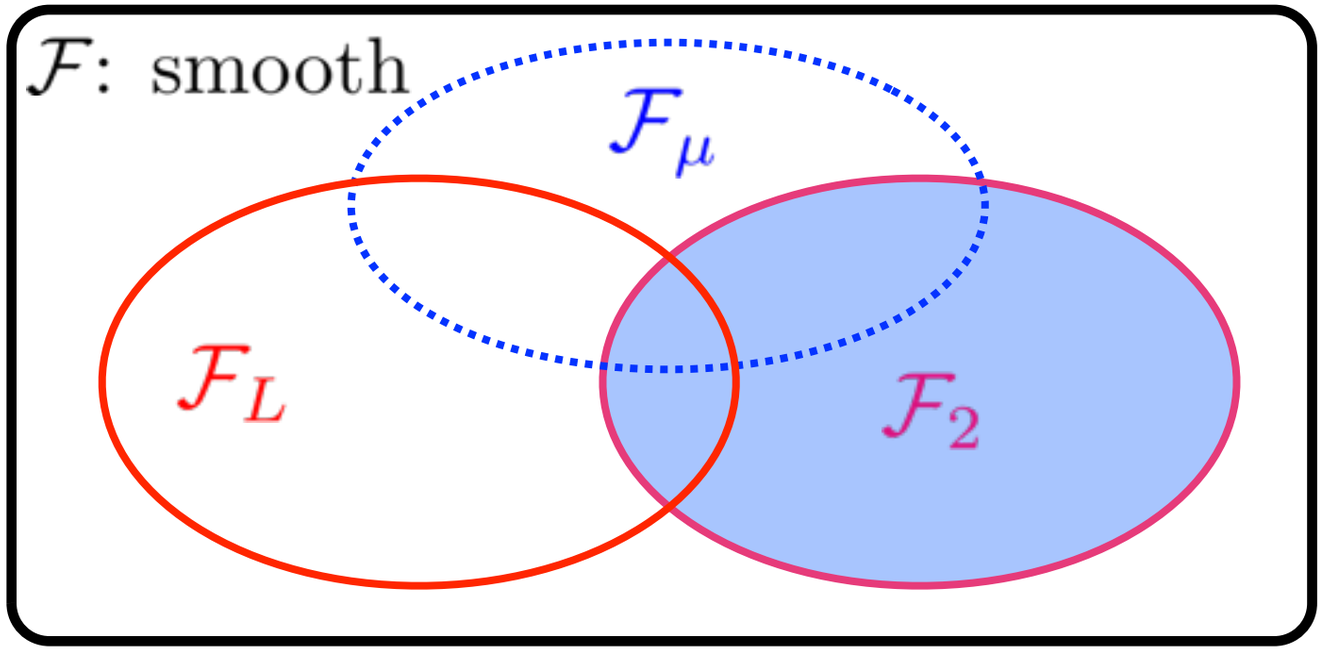
f

convex and **self-concordant**

g

convex and possibly **nonsmooth** with **"tractable"** prox

Classes of smooth functions (f)



- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_\mu$ - μ-strongly convex
- $\mathcal{F}_2$ - self-concordant

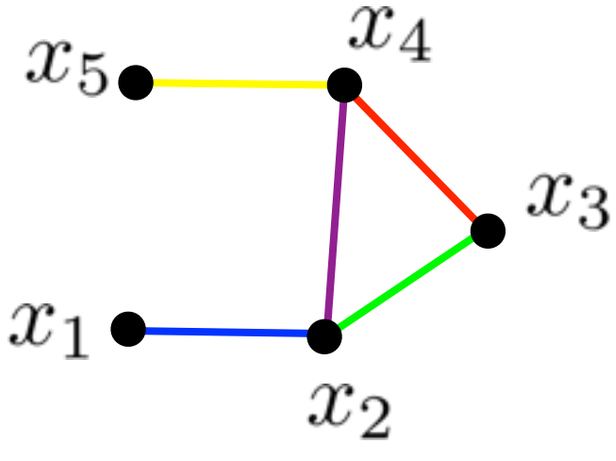
Key structure for the interior point method

---

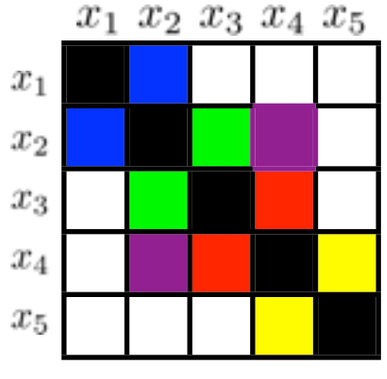
f is self-concordant if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$

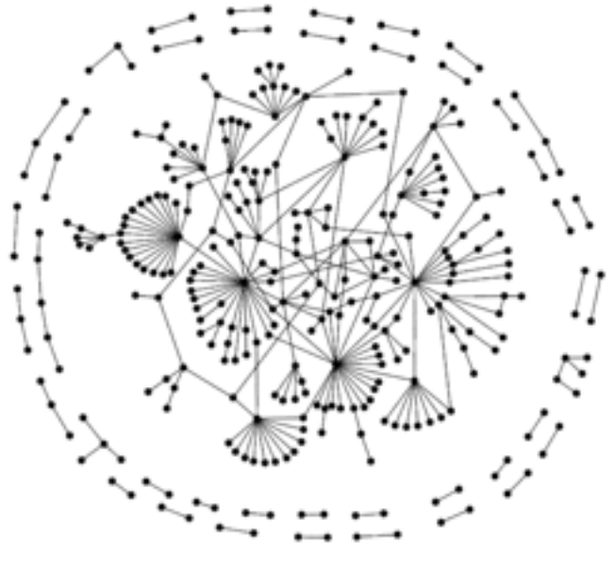
Example: Log-determinant for LMIs

- ***Application***: Graphical model selection



$$\Theta =$$





Given a data set $\mathcal{D} := \{\mathbf{x}_1, \ldots, \mathbf{x}_N\}$, where $\mathbf{x}_i$ is a Gaussian random variable. Let $\mathbf{\Sigma}$ be the **covariance matrix** corresponding to the **graphical model** of the Gausian Markov random field. The aim is to learn a **sparse matrix** Θ that approximates the inverse $\mathbf{\Sigma}^{-1}$.

Optimization problem

$$\min_{\mathbf{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\mathbf{\Theta}) + \operatorname{trace}(\mathbf{\Sigma}\mathbf{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\mathbf{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

Log-barrier for linear/quadratic inequalities

- Poisson imaging reconstruction via TV regularization
- -a

$$x^* \in \operatorname*{argmin}_x \Big\{ \underbrace{\sum_{i=1}^{m} a_i^T x - \sum_{i=1}^{m} y_i \log(a_i^T x + b_i)}_{f(x)} + g(x) \Big\}$$

- Basic pursuit denoising problem (BPDP): Barrier formulation

$$\mathbf{x}_t^* = \operatorname*{argmin}_{\mathbf{x}} \Big\{ \underbrace{-t \log\left(\sigma^2 - \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2\right)}_{=: f(\mathbf{x})} + g(\mathbf{x}) \Big\}$$

- LASSO problem with unknown variance

$$\mathbf{x}^* \equiv (\phi^*, \gamma^*) = \operatorname*{argmin}_{\phi,\gamma} \Big\{ \underbrace{-\log(\gamma) + \frac{1}{2n}\|\gamma\mathbf{y} - \mathbf{X}\phi\|_2^2}_{=: f(\mathbf{x})} + \underbrace{\lambda\|\phi\|_1}_{=: g(\mathbf{x})} \Big\}$$

(P) Composite self-concordant minimization

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

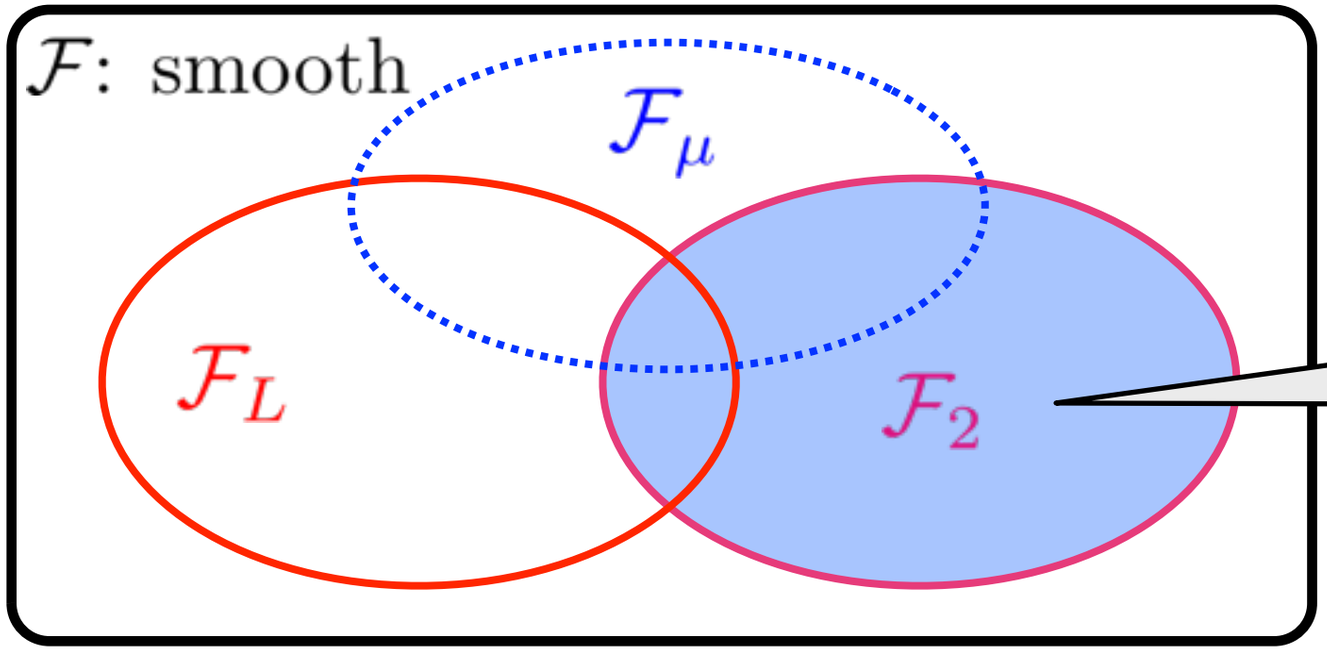
f

convex and **self-concordant**

g

convex and possibly **nonsmooth** with **"tractable"** prox

Classes of smooth functions (f)



Our contributions:

i) a ***variable metric*** (path following) forward-backward framework

ii) convergence theory ***without the Lipschitz gradient assumption***

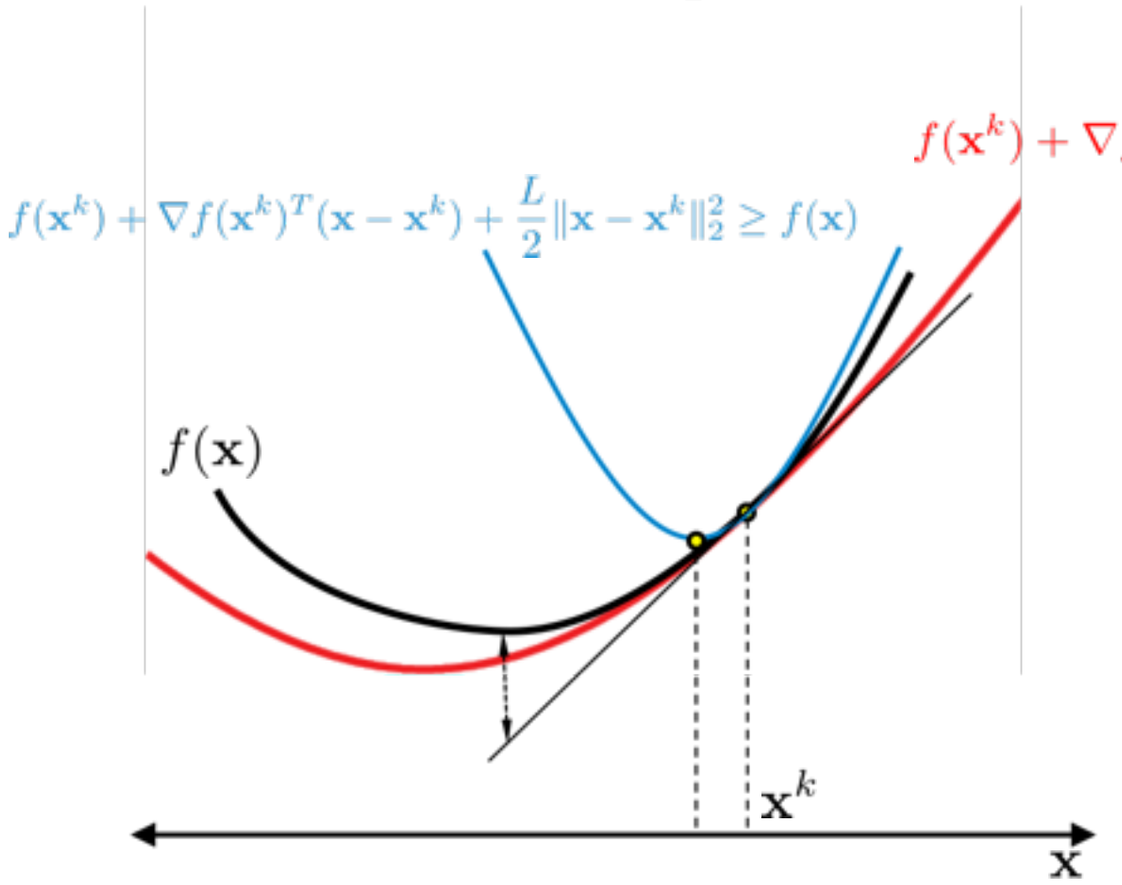
iii) novel variants and extensions for several applications & SCOPT

---

f is self-concordant if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$

Basic algorithmic framework

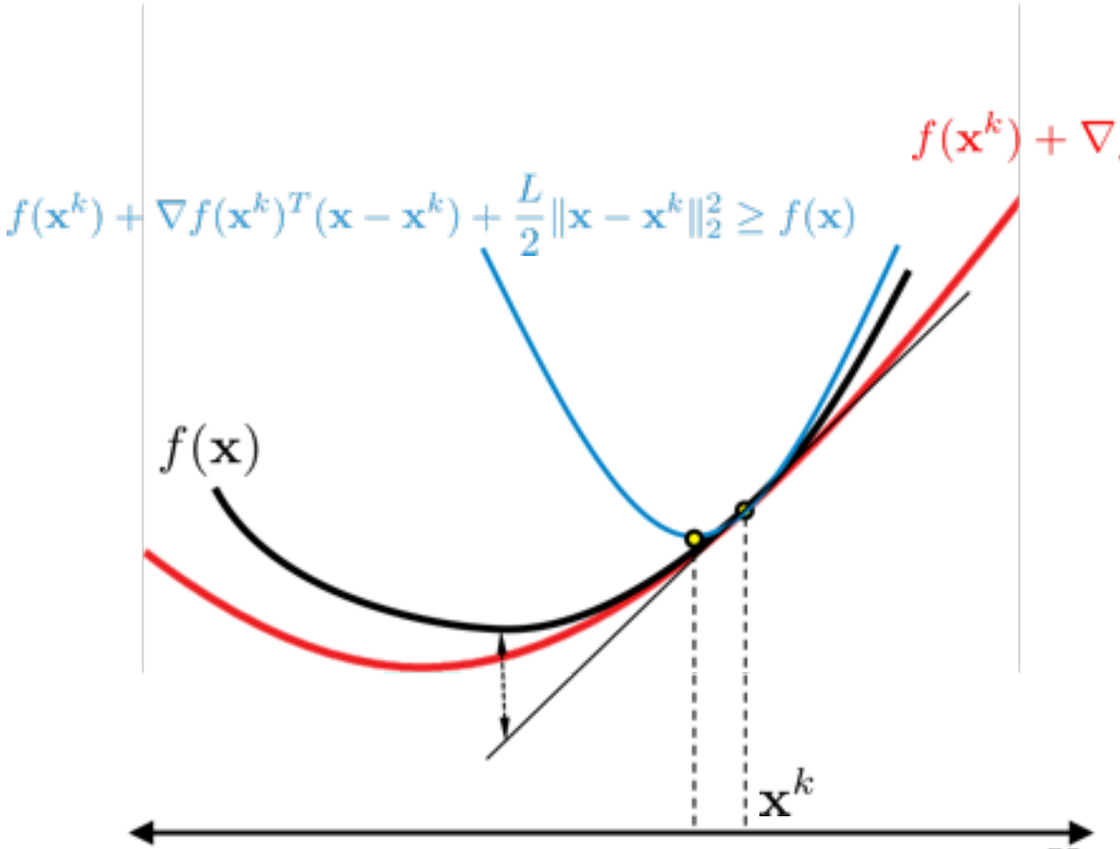
A basic composite minimization framework



- **Main properties of $\mathcal{F}_{\mu,L}$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \mathrm{dom}(f)$

A basic composite minimization framework

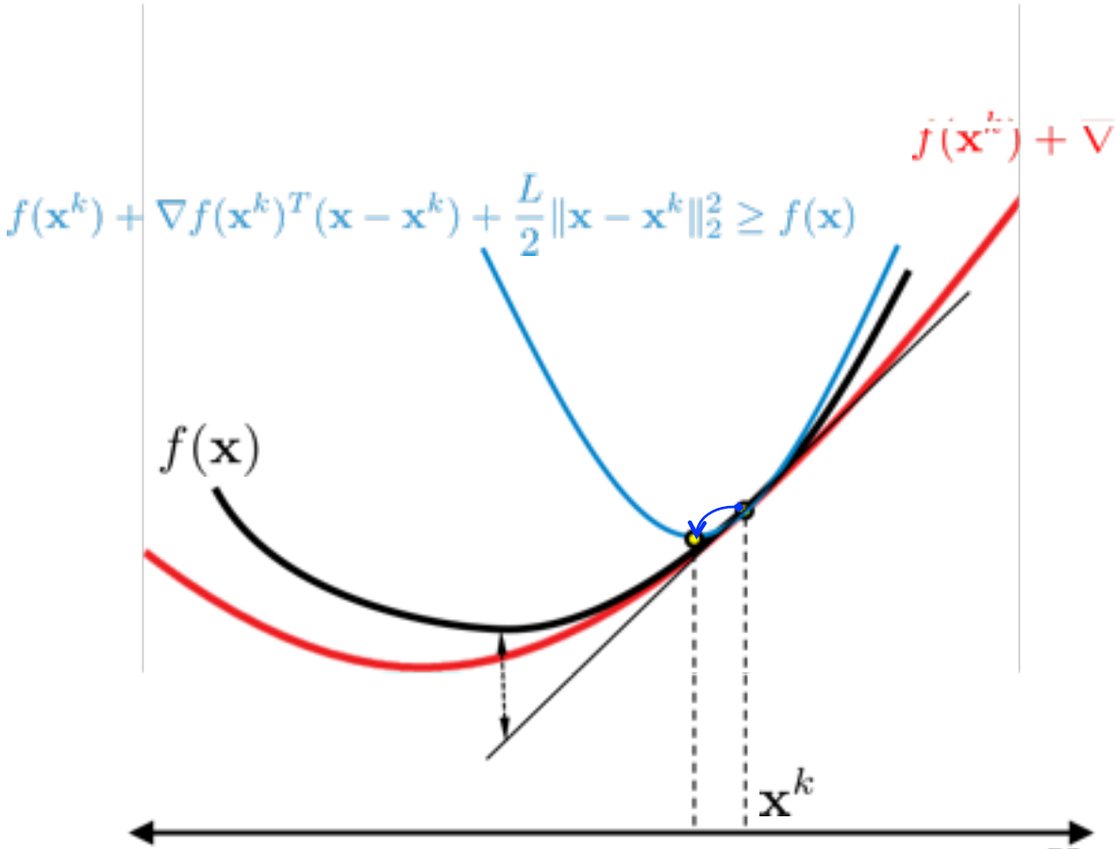


$$\min_{\mathbf{x}} \{f(\mathbf{x}) + g(\mathbf{x})\}$$

- **Main properties of $\mathcal{F}_{\mu,L}$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \mathrm{dom}(f)$

A basic composite minimization framework



$$\mathbf{x}^{k+1} = \mathrm{prox}_g\left(\mathbf{x}^k - \frac{1}{L}\nabla f(\mathbf{x}^k)\right)$$

$$\min_{\mathbf{x}} \{f(\mathbf{x}) + g(\mathbf{x})\}$$

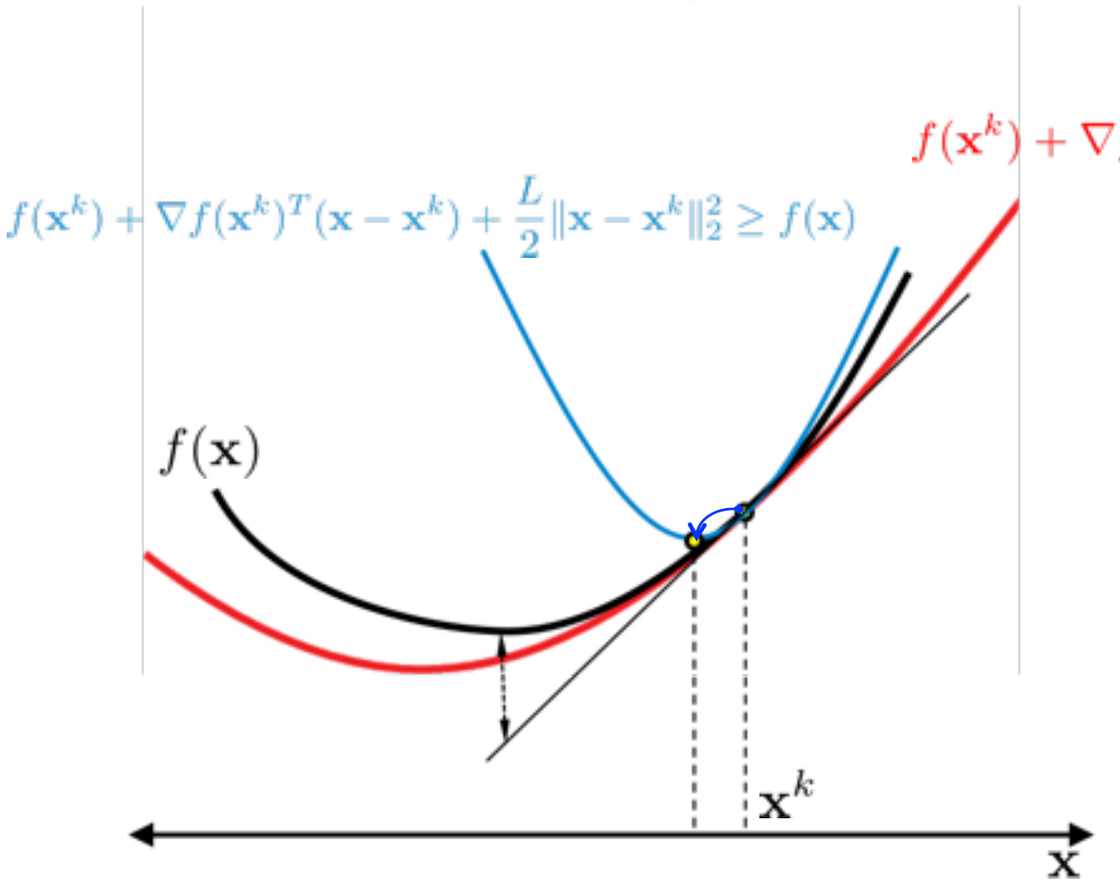
$$\mathbf{x}^{k+1} := \arg\min_{\mathbf{x}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2 + g(\mathbf{x}) \right\}$$

- **Main properties of $\mathcal{F}_{\mu,L}$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \mathrm{dom}(f)$

$\mathcal{F}_L$

A basic composite minimization framework



ISTA

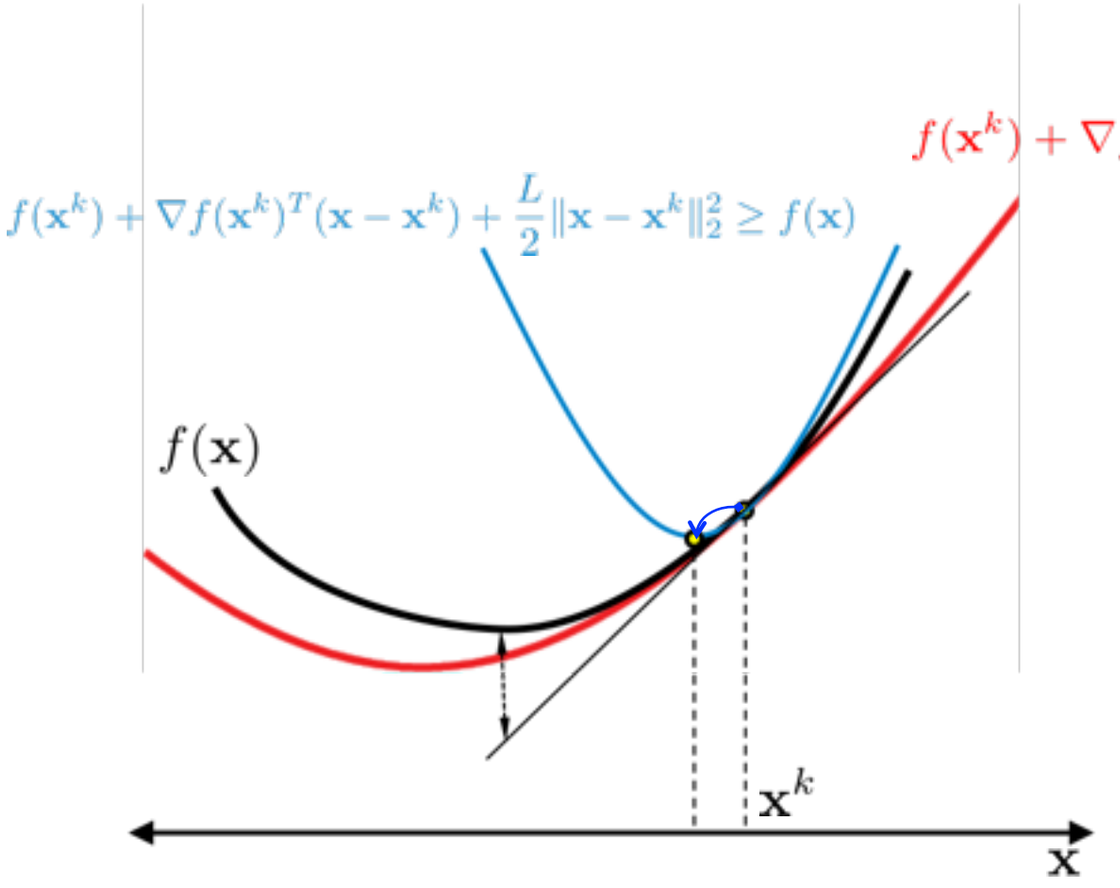
$$\mathbf{x}^{k+1} = \text{prox}_g\left(\mathbf{x}^k - \frac{1}{L}\nabla f(\mathbf{x}^k)\right)$$

- **Main properties of** $\mathcal{F}_{\mu,L}$

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \text{dom}(f)$

$\mathcal{F}_L$

A basic composite minimization framework



ISTA

$$\mathbf{x}^{k+1} = \text{prox}_g\left(\mathbf{x}^k - \frac{1}{L}\nabla f(\mathbf{x}^k)\right)$$

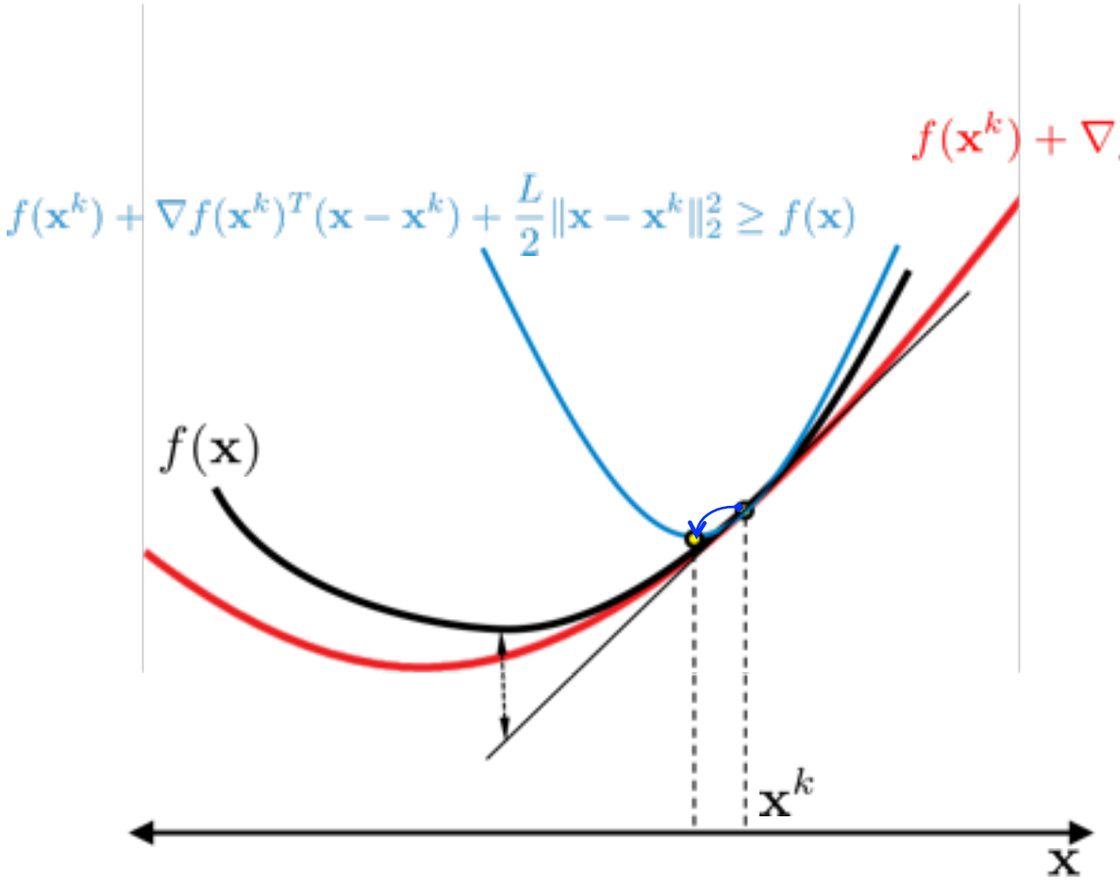
$$f(\mathbf{x}^k) - f(\mathbf{x}^*) \leq \frac{L\|\mathbf{x}^0 - \mathbf{x}^*\|^2}{2k} \Rightarrow \text{iterations} = \mathcal{O}(\epsilon^{-1})$$

- **Main properties of $\mathcal{F}_{\mu,L}$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \text{dom}(f)$

$\mathcal{F}_L$

A basic composite minimization framework



ISTA

$$\mathbf{x}^{k+1} = \text{prox}_g\left(\mathbf{x}^k - \frac{1}{L}\nabla f(\mathbf{x}^k)\right)$$

$$f(\mathbf{x}^k) - f(\mathbf{x}^*) \leq \frac{L\|\mathbf{x}^0 - \mathbf{x}^*\|^2}{2k} \Rightarrow \text{iterations} = \mathcal{O}(\epsilon^{-1})$$

acceleration is possible

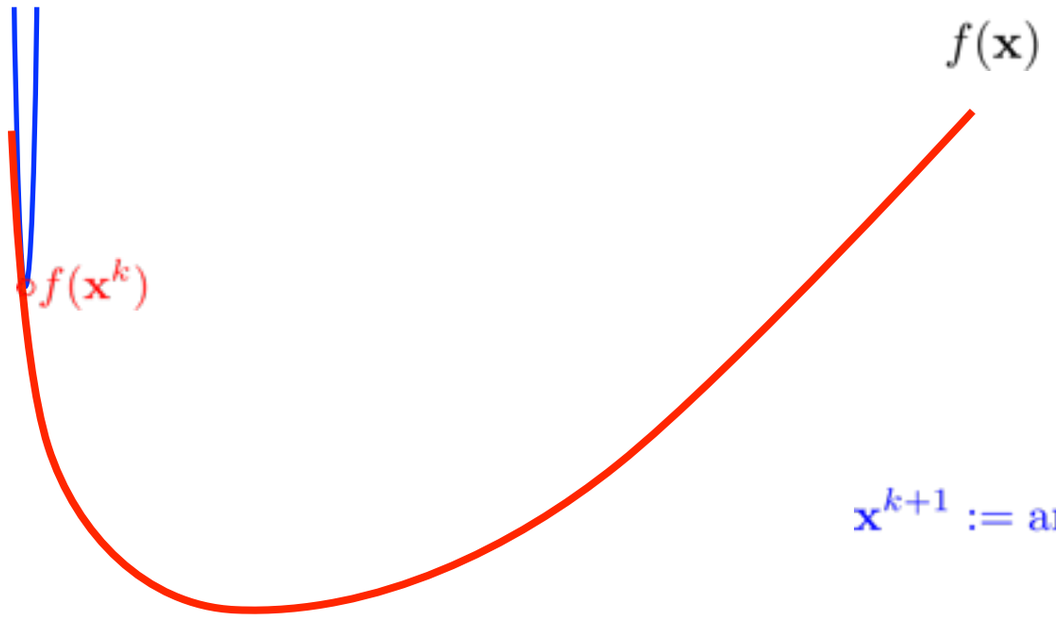
FISTA

- **Main properties of $\mathcal{F}_{\mu,L}$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{\mu}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y} - \mathbf{x}) + \frac{L}{2}\|\mathbf{y} - \mathbf{x}\|_2^2$	$\mathbf{x}, \mathbf{y} \in \text{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \text{dom}(f)$

$\mathcal{F}_L$

To adapt or not to adapt?

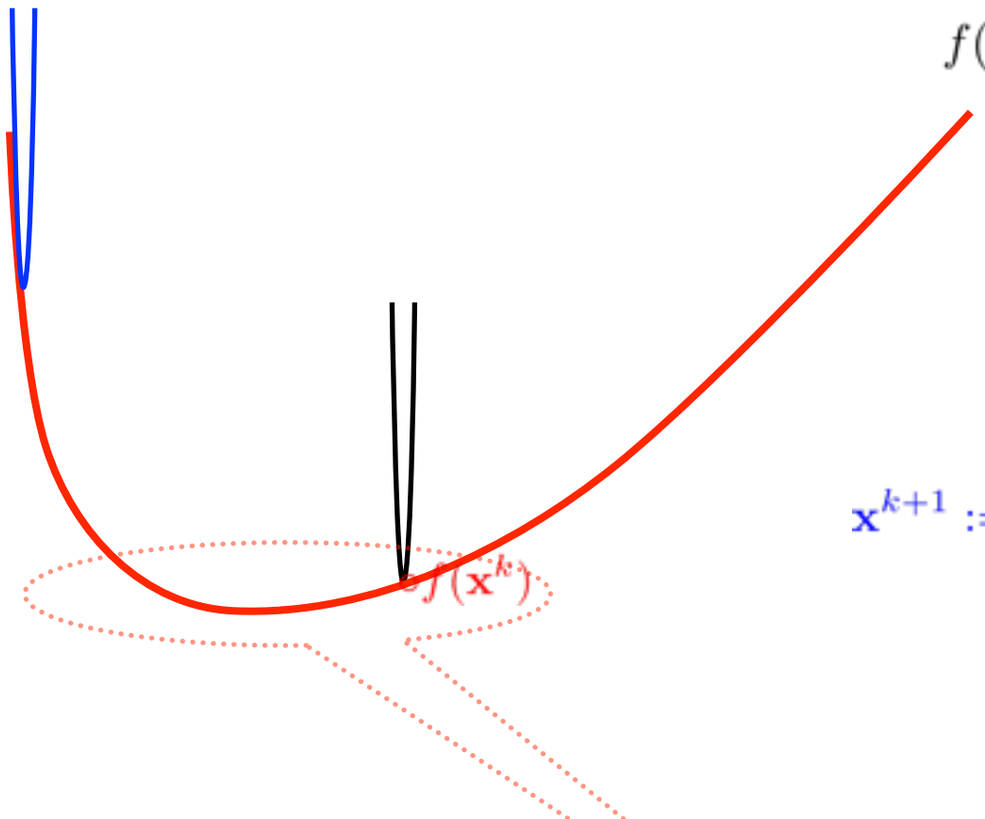


$$\mathbf{x}^{k+1} := \arg\min_{\mathbf{x}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2 + g(\mathbf{x}) \right\}$$

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

L is a global worst-case constant

To adapt or not to adapt?

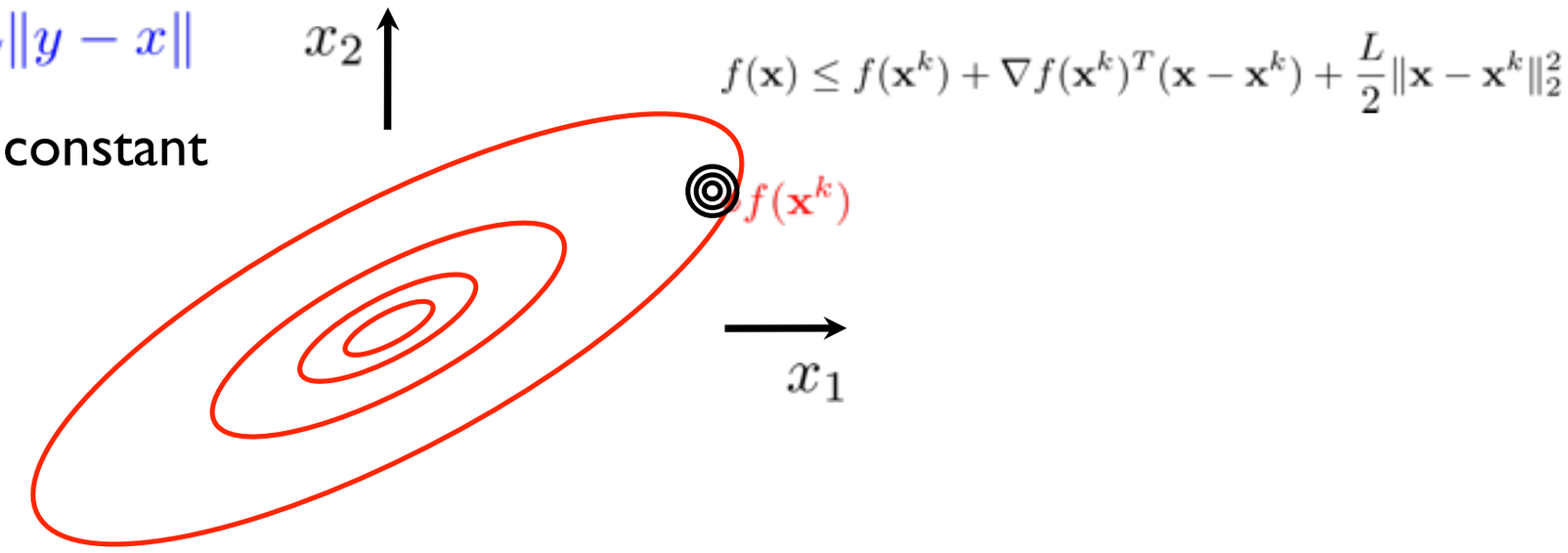


$$\mathbf{x}^{k+1} := \arg\min_{\mathbf{x}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2 + g(\mathbf{x}) \right\}$$

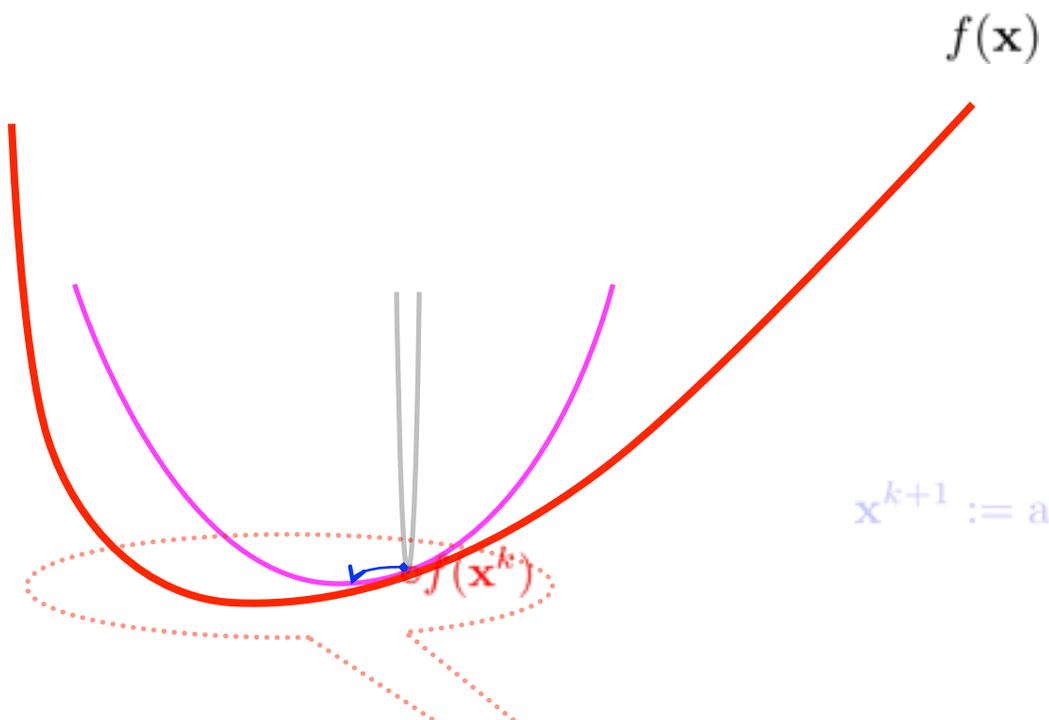
$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

L is a global worst-case constant

$$f(\mathbf{x}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2$$



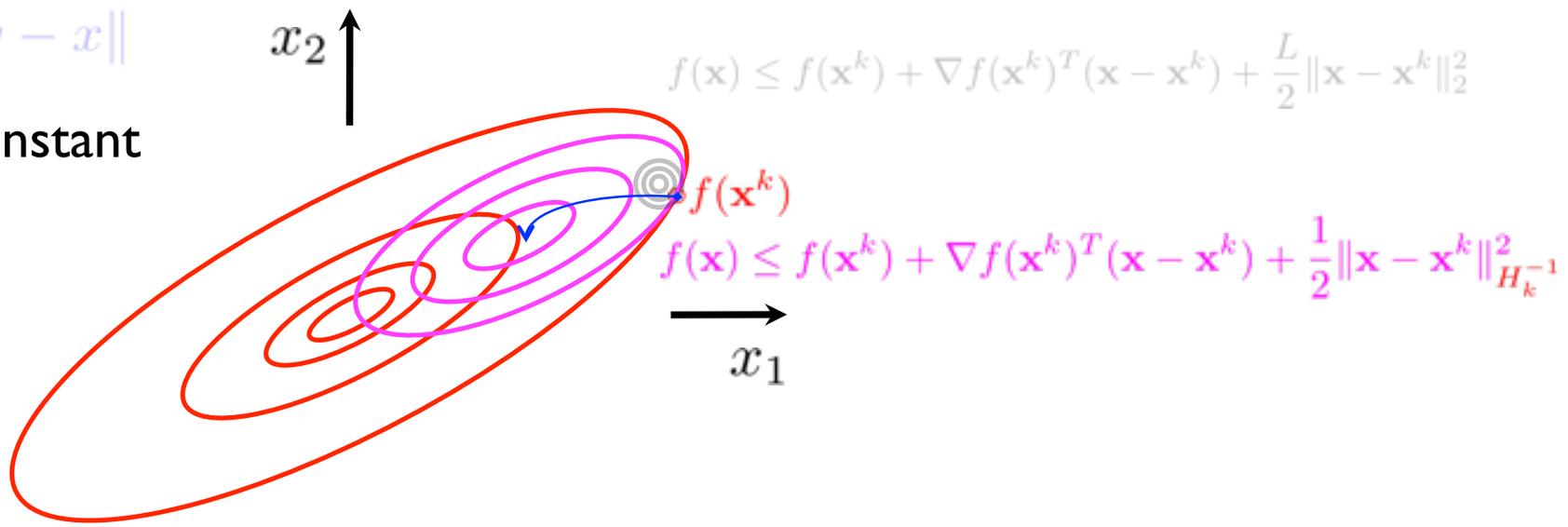
To adapt or not to adapt?



$$\mathbf{x}^{k+1} := \arg\min_{\mathbf{x}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2 + g(\mathbf{x}) \right\}$$

$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

L is a global worst-case constant



$$f(\mathbf{x}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{L}{2}\|\mathbf{x} - \mathbf{x}^k\|_2^2$$

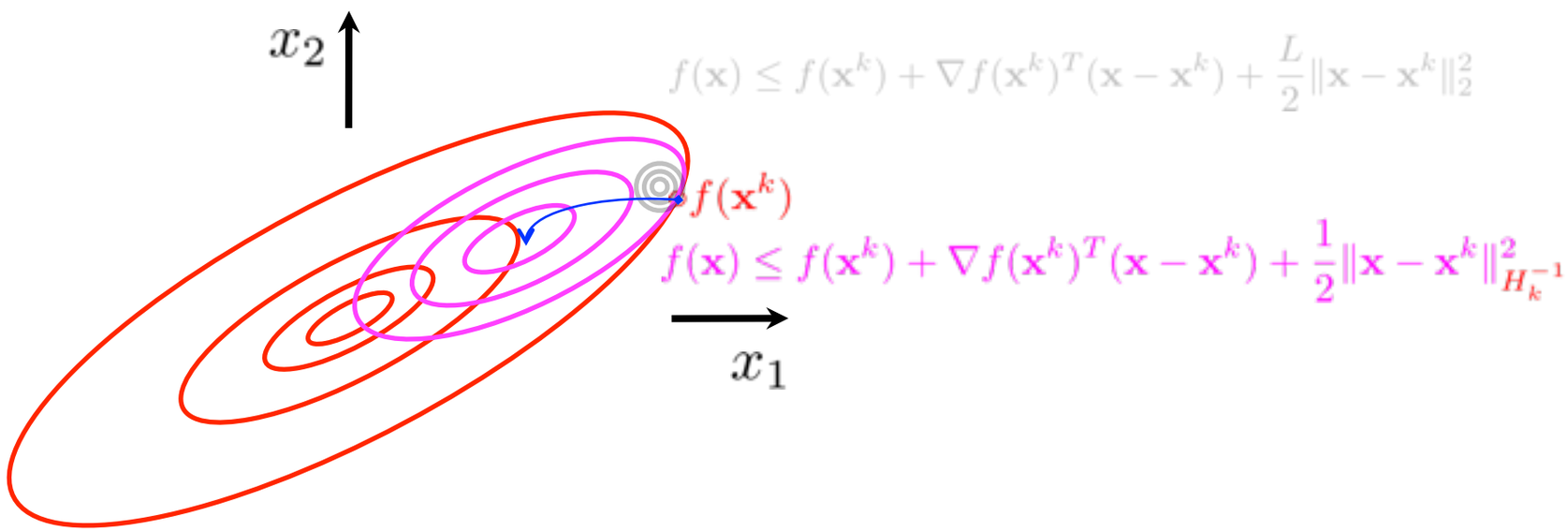
$$f(\mathbf{x}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x} - \mathbf{x}^k) + \frac{1}{2}\|\mathbf{x} - \mathbf{x}^k\|_{H_k^{-1}}^2$$

To adapt or not to adapt?

$$\text{prox}_{\gamma g}(\mathbf{s}) := \arg\min_{\mathbf{x}} \left\{ g(\mathbf{x}) + \frac{1}{2\gamma}\|\mathbf{x}-\mathbf{s}\|_2^2 \right\}$$

Variable metric proximal point operator

$$\text{prox}_{H^{-1}g}(s) := \arg\min_{x} \left\{ g(x) + \frac{1}{2}\|x-s\|_{H^{-1}}^2 \right\}$$

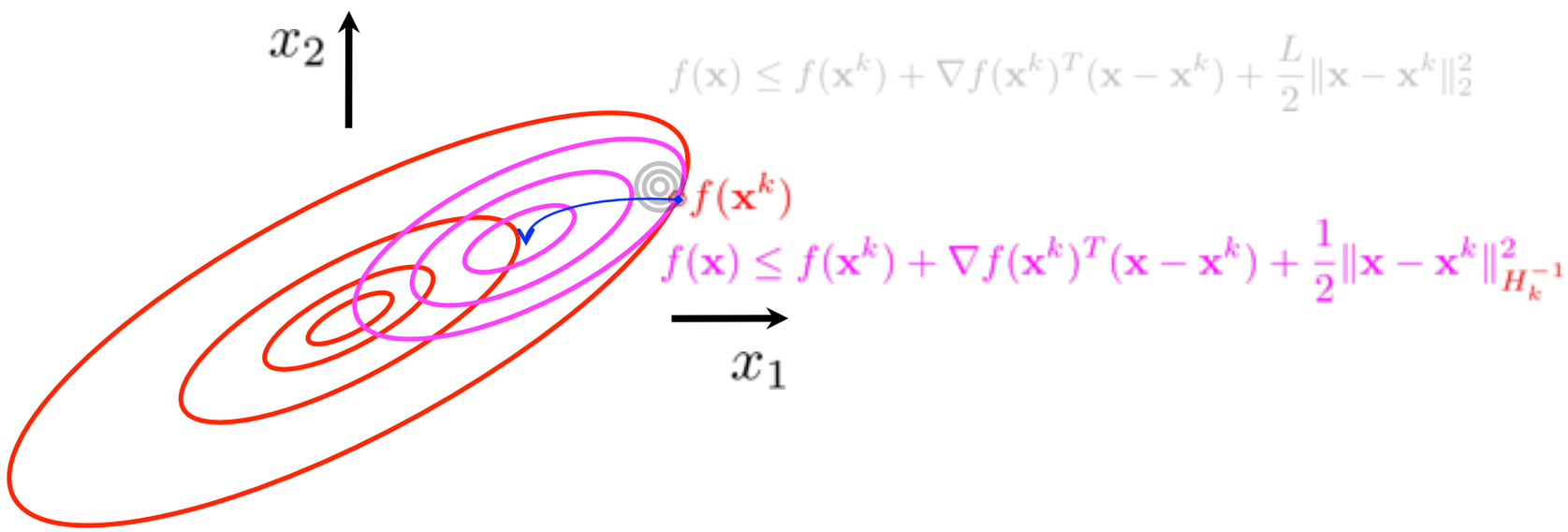


To adapt or not to adapt?

$$\text{prox}_{\gamma g}(\mathbf{s}) := \arg\min_{\mathbf{x}} \left\{ g(\mathbf{x}) + \frac{1}{2\gamma}\|\mathbf{x}-\mathbf{s}\|_2^2 \right\}$$

Variable metric proximal point operator

$$\text{prox}_{H^{-1}g}(s) := \arg\min_{x} \left\{ g(x) + \frac{1}{2}\|x-s\|_{H^{-1}}^2 \right\}$$



if $g(\mathbf{x}) = \|\mathbf{x}\|_1$, then
$\text{prox}_{\gamma g}(\mathbf{s}) = \text{SoftThresh}(\mathbf{s}, \gamma)$
$\text{prox}_{H^{-1}g}(s) = \text{LASSO}$

A basic variable metric minimization framework

- Proximal point scheme with variable metric [Bonnans, 1993]

Proximal point scheme with variable metric

Given x^0, generate a sequence $\{x^k\}_{k\geq 0}$ such that

$$x^{k+1} = \text{prox}_{H_k^{-1}}\left(x^k - {H_k}^{-1}\nabla f(x^k)\right)$$

where H_k is symmetric positive definite

Variable metric proximal point operator

$$\text{prox}_{H^{-1}g}(s) := \arg\min_x \left\{ g(x) + \frac{1}{2}\|x - s\|^2_{H^{-1}} \right\}$$

- Additional accuracy vs. computation trade-offs

Order	Example	Components	k
1-st	Accelerated gradient	$\nabla f, \text{prox}_{1/L\mathbf{I}_n}$	$\mathcal{O}(\epsilon^{-1/2})$
1^+-th	BFGS	$H_k, \nabla f, \text{prox}_{H_k^{-1}}$	$\mathcal{O}(\log \epsilon^{-1})$ or faster
2-nd	Proximal Newton, IPM	$\nabla^2 f, \nabla f, \text{prox}_{\nabla^2 f^{-1}}$	$\mathcal{O}(\log\log \epsilon^{-1})$

Self-concordance vs. Lipschitz gradient + SC

- **Main properties of** $\mathcal{F}_{\mu,L}$

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \frac{\mu}{2}\|\mathbf{y}-\mathbf{x}\|_2^2$	$\mathbf{x},\mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \frac{L}{2}\|\mathbf{y}-\mathbf{x}\|_2^2$	$\mathbf{x},\mathbf{y} \in \mathrm{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \mathrm{dom}(f)$

Self-concordance vs. Lipschitz gradient + SC

- **Main properties of** $\mathcal{F}_{\mu,L}$

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \frac{\mu}{2}\|\mathbf{y}-\mathbf{x}\|_2^2$	$\mathbf{x},\mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \frac{L}{2}\|\mathbf{y}-\mathbf{x}\|_2^2$	$\mathbf{x},\mathbf{y} \in \mathrm{dom}(f)$
Hessian surrogates	$\mu\mathbb{I} \preceq \nabla^2 f(\mathbf{x}) \preceq L\mathbb{I}$	$\mathbf{x} \in \mathrm{dom}(f)$

Global

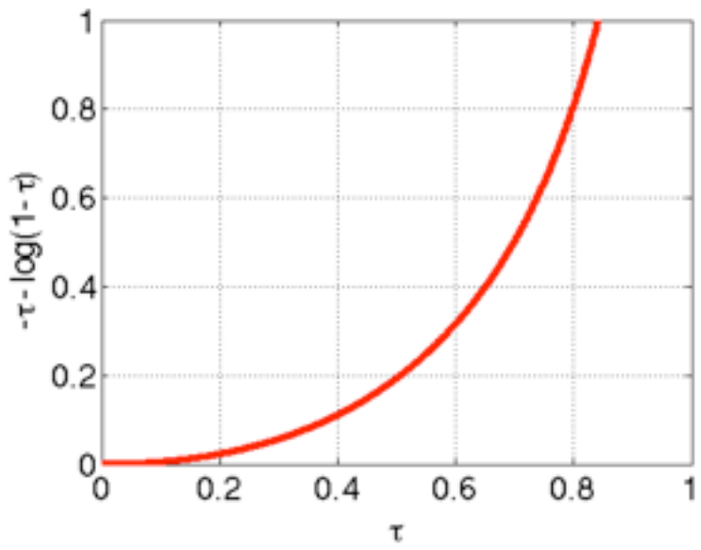
Self-concordance vs. Lipschitz gradient + SC

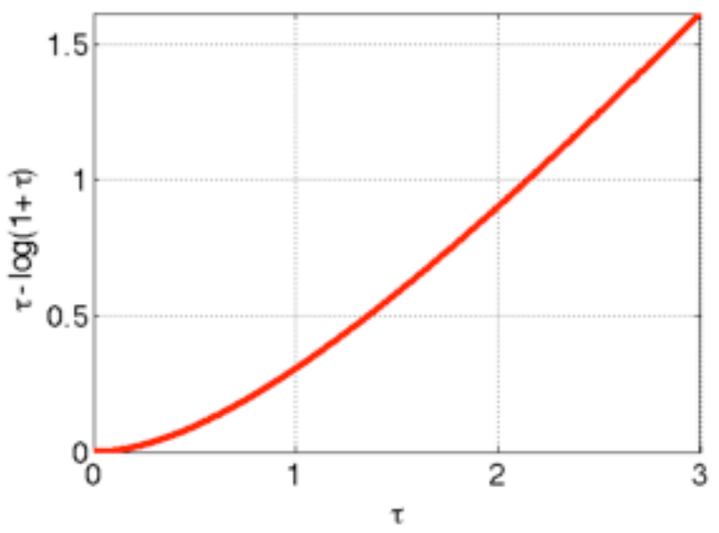
- **Main properties of $\mathcal{F}_2$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega_*\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$
Hessian surrogates	$(1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^2 \nabla^2 f(\mathbf{x}) \preceq \nabla^2 f(\mathbf{y}) \preceq (1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^{-2} \nabla^2 f(\mathbf{x})$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$

Local norm: $\|\mathbf{u}\|_{\mathbf{x}} := \left[\mathbf{u}^T \nabla^2 f(\mathbf{x}) \mathbf{u}\right]^{1/2}$

Utility functions: $\omega_*(\tau) = -\tau - \ln(1-\tau),\ \tau \in [0,1)$ $\omega(\tau) = \tau - \ln(1+\tau),\ \tau \geq 0$





f is self-concordant if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$

Self-concordance vs. Lipschitz gradient + SC

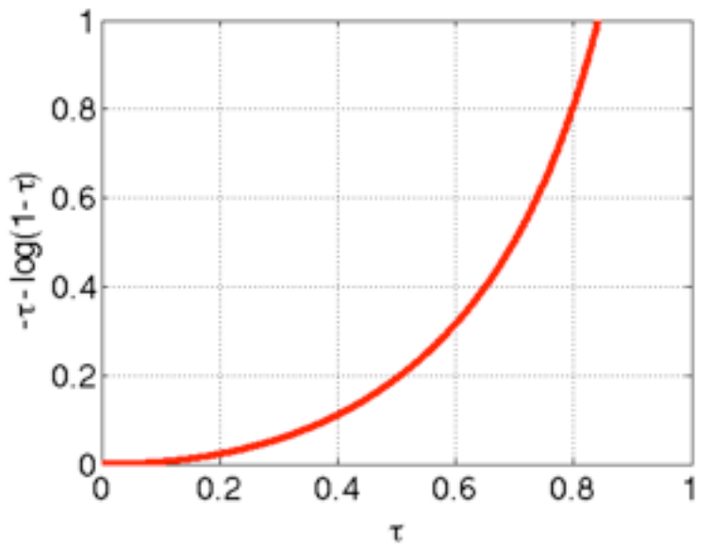
- **Main properties of** $\mathcal{F}_2$

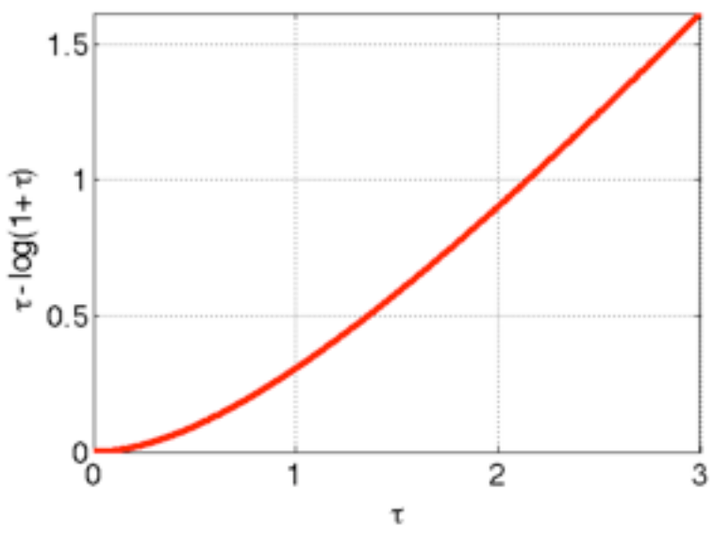
Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega_*\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$
Hessian surrogates	$(1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^2 \nabla^2 f(\mathbf{x}) \preceq \nabla^2 f(\mathbf{y}) \preceq (1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^{-2} \nabla^2 f(\mathbf{x})$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$

Local

Local norm: $\|\mathbf{u}\|_{\mathbf{x}} := \left[\mathbf{u}^T \nabla^2 f(\mathbf{x}) \mathbf{u}\right]^{1/2}$

Utility functions: $\omega_*(\tau) = -\tau - \ln(1-\tau),\ \tau \in [0,1)$ $\omega(\tau) = \tau - \ln(1+\tau),\ \tau \geq 0$





f is self-concordant if $\varphi(t) := f(\mathbf{x}+t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$

Self-concordance: A mathematical tool

- **Main properties of $\mathcal{F}_2$**

Lower surrogate	$f(\mathbf{y}) \geq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\mathbf{x}, \mathbf{y} \in \mathrm{dom}(f)$
Upper surrogate	$f(\mathbf{y}) \leq f(\mathbf{x}) + \nabla f(\mathbf{x})^T(\mathbf{y}-\mathbf{x}) + \omega_*\left(\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}}\right)$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$
Hessian surrogates	$(1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^2\nabla^2 f(\mathbf{x}) \preceq \nabla^2 f(\mathbf{y}) \preceq (1-\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}})^{-2}\nabla^2 f(\mathbf{x})$	$\|\mathbf{y}-\mathbf{x}\|_{\mathbf{x}} < 1$

- New variable metric framework with rigorous convergence guarantees

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

Includes several algorithms: Newton, quasi-Newton, and gradient methods...

---

f is self-concordant if $\varphi(t) := f(\mathbf{x}+t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$

Our composite self-concordant minimization framework

- Proximal Newton scheme $\mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$

Given $\mathbf{x}^0$, generate a sequence $\{\mathbf{x}^k\}_{k\geq 0}$ such that

$$\boxed{\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \mathbf{d}^k_{\mathbf{H}_k}}$$

where $\alpha_k \in (0, 1]$ is step-size, $\mathbf{d}^k_{\mathbf{H}_k}$ is a search direction

Our composite self-concordant minimization framework

- Proximal Newton scheme $\mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$

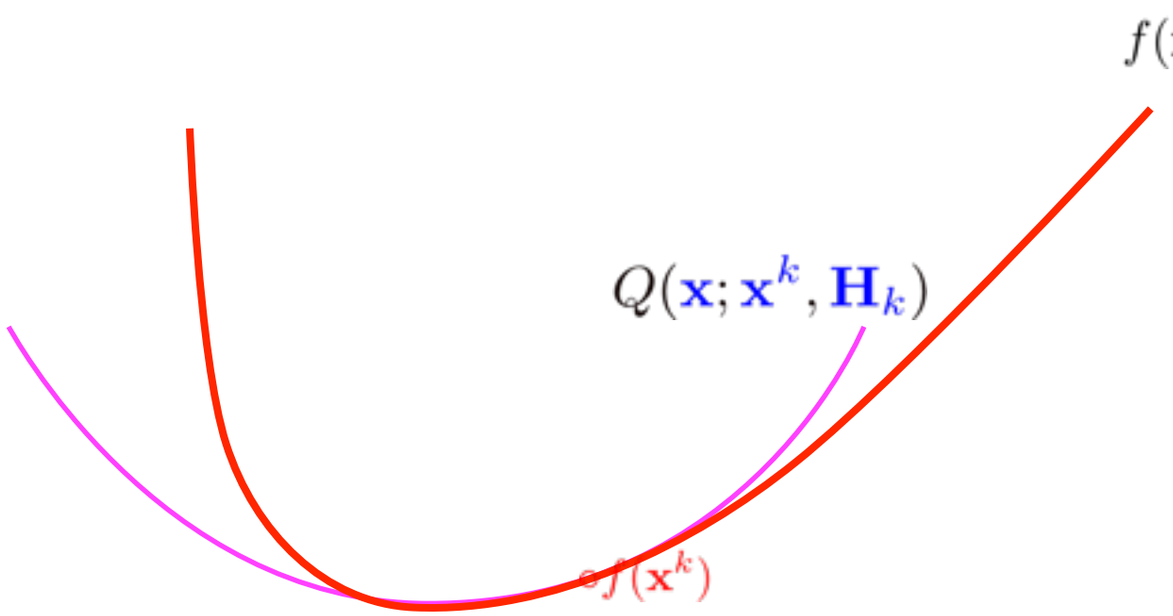
Given $\mathbf{x}^0$, generate a sequence $\{\mathbf{x}^k\}_{k\geq 0}$ such that

$$\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \mathbf{d}^k_{\mathbf{H}_k}$$

where $\alpha_k \in (0, 1]$ is step-size, $\mathbf{d}^k_{\mathbf{H}_k}$ is a search direction

- How to compute the Proximal Newton direction?

$$\mathbf{d}^k_{\mathbf{H}_k} := \arg\min_{\mathbf{d}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{H}_k \mathbf{d} + g(\mathbf{x}^k + \mathbf{d}) \right\}, \quad \mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$$



$$\mathbf{s}^k_g := \arg\min_{\mathbf{x}\in \mathrm{dom}(F)} \left\{ Q(\mathbf{x}; \mathbf{x}^k, \mathbf{H}_k) + g(\mathbf{x}) \right\}$$

$$\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \left(\mathbf{s}^k_g - \mathbf{x}^k \right)$$

Our composite self-concordant minimization framework

- Proximal Newton scheme $\mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$

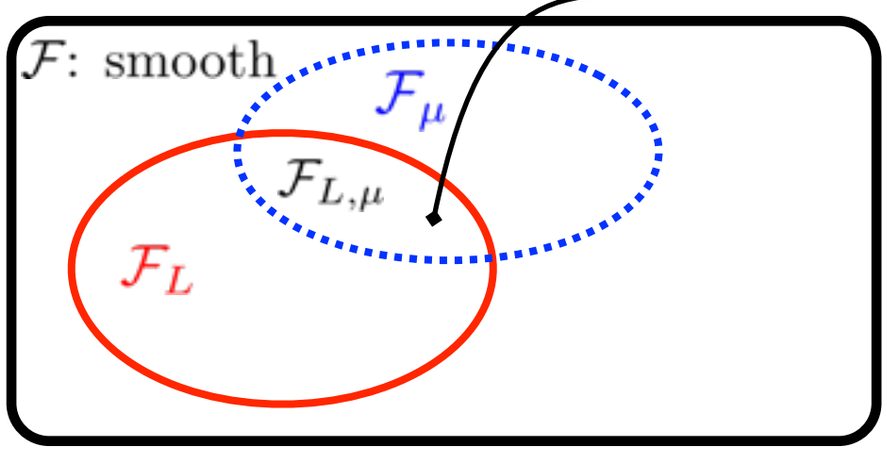
Given $\mathbf{x}^0$, generate a sequence $\{\mathbf{x}^k\}_{k\geq 0}$ such that

$$\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \mathbf{d}^k_{\mathbf{H}_k}$$

where $\alpha_k \in (0, 1]$ is step-size, $\mathbf{d}^k_{\mathbf{H}_k}$ is a search direction

- How to compute the Proximal Newton direction?

$$\mathbf{d}^k_{\mathbf{H}_k} := \arg\min_{\mathbf{d}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{H}_k \mathbf{d} + g(\mathbf{x}^k + \mathbf{d}) \right\}, \quad \mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$$



$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

Fast gradient schemes (Nesterov's methods)

Newton/quasi Newton schemes

Our composite self-concordant minimization framework

- Proximal Newton scheme

Key contribution:
step size selection procedure

$\mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$

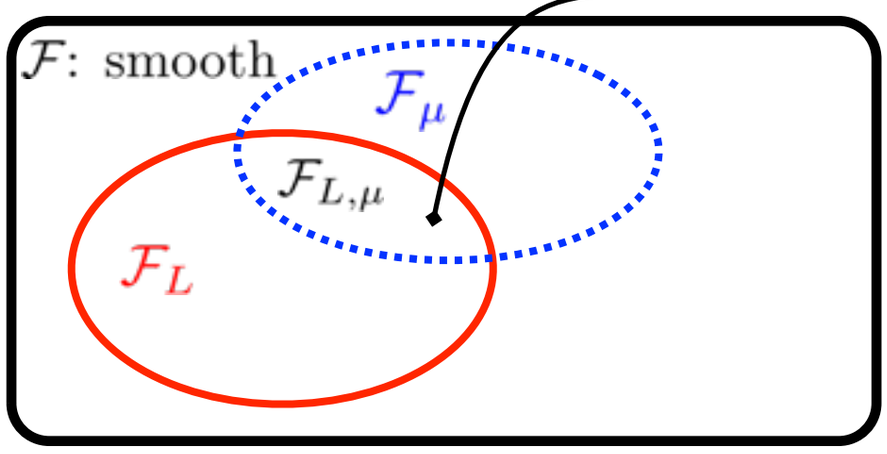
Given $\mathbf{x}^0$, generate a sequence $\{\mathbf{x}^k\}_{k\geq 0}$ such that

$$\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \mathbf{d}^k_{\mathbf{H}_k}$$

where $\alpha_k \in (0, 1]$ is step-size, $\mathbf{d}^k_{\mathbf{H}_k}$ is a search direction

- How to compute the Proximal Newton direction?

$$\mathbf{d}^k_{\mathbf{H}_k} := \arg\min_{\mathbf{d}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{H}_k \mathbf{d} + g(\mathbf{x}^k + \mathbf{d}) \right\}, \quad \mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$$



$$\|\nabla f(x) - \nabla f(y)\| \leq L\|y - x\|$$

$$\mu\mathbb{I} \preceq \nabla^2 f(x) \preceq L\mathbb{I}$$

Fast gradient schemes (Nesterov's methods)

Newton/quasi Newton schemes

How do we compute the step-size?

- Upper surrogate of f

$$f(\mathbf{x}^{k+1}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x}^{k+1} - \mathbf{x}^k) + \omega^*(\|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k}), \quad \|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k} < 1$$

- Convexity of g and optimality condition of the subproblem

$$g(\mathbf{x}^{k+1}) - g(\mathbf{x}^k) \leq -\alpha_k \nabla f(\mathbf{x}^k)^T \mathbf{d}^k_{\mathbf{H_k}} - \alpha_k \|\mathbf{d}^k_{\mathbf{H_k}}\|^2_{\mathbf{H}_k}.$$

How do we compute the step-size?

- Upper surrogate of f

$$f(\mathbf{x}^{k+1}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x}^{k+1} - \mathbf{x}^k) + \omega^*(\|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k}), \quad \|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k} < 1$$

- Convexity of g and optimality condition of the subproblem

$$g(\mathbf{x}^{k+1}) - g(\mathbf{x}^k) \leq -\alpha_k \nabla f(\mathbf{x}^k)^T \mathbf{d}^k_{\mathbf{H_k}} - \alpha_k \|\mathbf{d}^k_{\mathbf{H_k}}\|^2_{\mathbf{H}_k}.$$

$$\phi(\mathbf{x}^{k+1}) \leq \phi(\mathbf{x}^k) - \alpha_k \|\mathbf{d}^k_{\mathbf{H}_k}\|^2_{\mathbf{H}_k} + \omega_* \left(\alpha_k \|\mathbf{d}^k_{\mathbf{H}_k}\|_{\mathbf{x}^k}\right)$$

How do we compute the step-size?

- Upper surrogate of f

$$f(\mathbf{x}^{k+1}) \leq f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T(\mathbf{x}^{k+1} - \mathbf{x}^k) + \omega^*(\|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k}), \quad \|\mathbf{x}^{k+1} - \mathbf{x}^k\|_{\mathbf{x}^k} < 1$$

- Convexity of g and optimality condition of the subproblem

$$g(\mathbf{x}^{k+1}) - g(\mathbf{x}^k) \leq -\alpha_k \nabla f(\mathbf{x}^k)^T \mathbf{d}^k_{\mathbf{H}_k} - \alpha_k \|\mathbf{d}^k_{\mathbf{H}_k}\|^2_{\mathbf{H}_k}.$$

$$\phi(\mathbf{x}^{k+1}) \leq \phi(\mathbf{x}^k) - \alpha_k \|\mathbf{d}^k_{\mathbf{H}_k}\|^2_{\mathbf{H}_k} + \omega_* \left(\alpha_k \|\mathbf{d}^k_{\mathbf{H}_k}\|_{\mathbf{x}^k}\right)$$

- When $\mathbf{H}_k \equiv \nabla^2 f(\mathbf{x}^k)$, $\lambda_k := \|\mathbf{d}^k_{\mathbf{H}_k}\|_{\mathbf{x}^k}$

$$\phi(\mathbf{x}^{k+1}) \leq \phi(\mathbf{x}^k) - \underbrace{[\alpha_k \lambda_k - \omega_* (\alpha_k \lambda_k)]}_{\psi(\alpha_k)}$$

maximize $\psi(\alpha_k)$ to get optimal α_k^*

$$\alpha_k^* = \frac{1}{\lambda_k + 1} \in (0, 1]$$

Analytic complexity

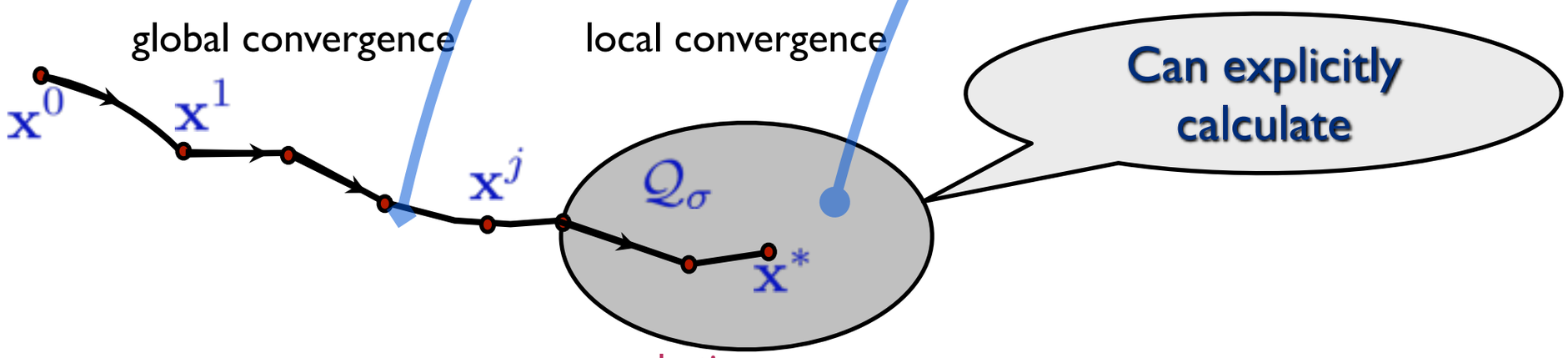
- Worst-case complexity to obtain an ε-approximate solution

$$\#\text{iterations} = \left\lfloor \frac{\phi(\mathbf{x}^0) - \phi(\mathbf{x}^*)}{0.021} \right\rfloor + O\left(\ln\ln\left(\frac{4.56}{\varepsilon}\right)\right)$$

Analytic complexity

- Worst-case complexity to obtain an ε-approximate solution

$$\#\text{iterations} = \left\lfloor \frac{\phi(\mathbf{x}^0) - \phi(\mathbf{x}^*)}{0.021} \right\rfloor + O\left(\ln\ln\left(\frac{4.56}{\varepsilon}\right)\right)$$

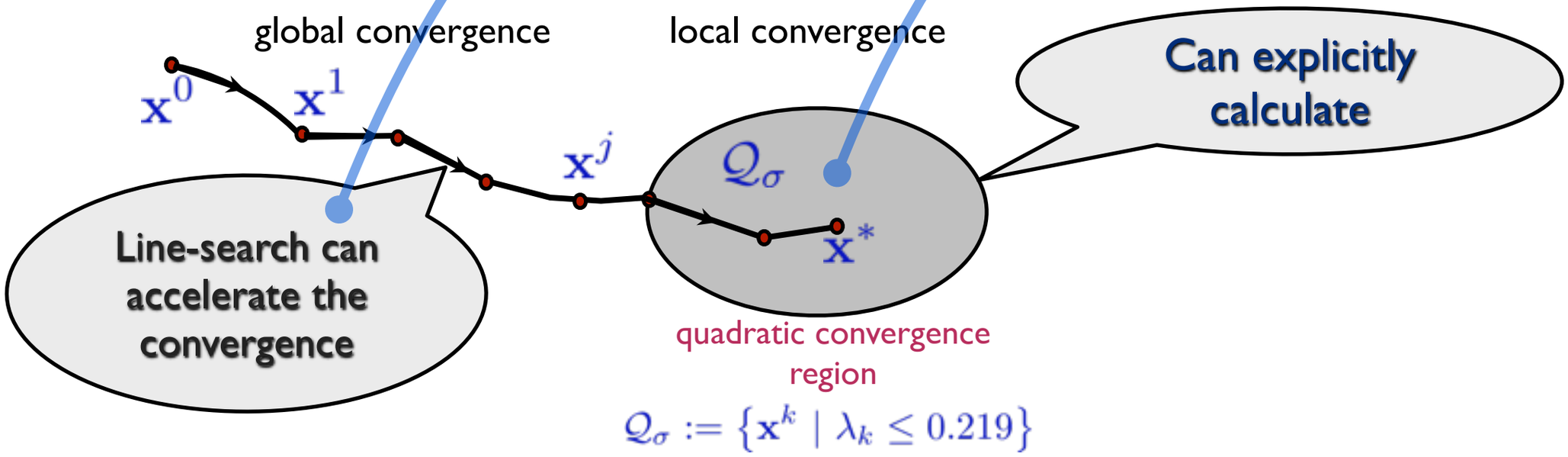


$$\mathcal{Q}_\sigma := \{\mathbf{x}^k \mid \lambda_k \leq 0.219\}$$

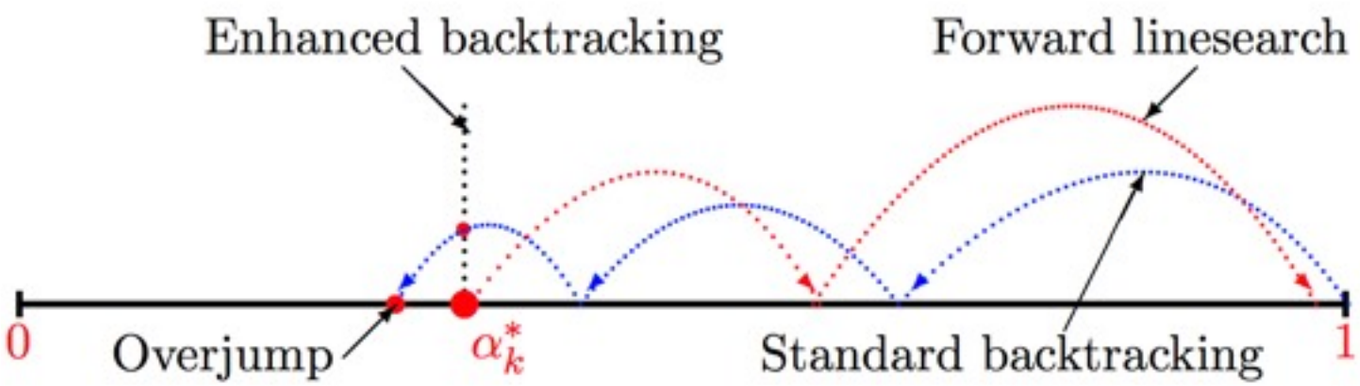
Analytic complexity

- Worst-case complexity to obtain an ε-approximate solution

$$\#\text{iterations} = \left\lfloor \frac{\phi(\mathbf{x}^0) - \phi(\mathbf{x}^*)}{0.021} \right\rfloor + O\left(\ln\ln\left(\frac{4.56}{\varepsilon}\right)\right)$$



- Line-search enhancement



Graphical model selection

- Objective:

$$\min_{\mathbf{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\mathbf{\Theta}) + \mathrm{trace}(\mathbf{\Sigma}\mathbf{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\mathrm{vec}(\mathbf{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

Graphical model selection

- Objective:

$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

- Gradient and Hessian (large-scale, special structure)
  - Gradient of f: $\nabla f(\mathbf{x}) = \mathbf{vec}(\boldsymbol{\Sigma} - \boldsymbol{\Theta}^{-1})$.
  - Hessian of f: $\nabla^2 f(\mathbf{x}) = \boldsymbol{\Theta}^{-1} \otimes \boldsymbol{\Theta}^{-1}$

Graphical model selection

- Objective:

$$\min_{\boldsymbol{\Theta}\succ 0}\left\{\underbrace{-\log\det(\boldsymbol{\Theta})+\operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})}+\underbrace{\rho\|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})}\right\}$$

- Gradient and Hessian (large-scale, special structure)
  - Gradient of f: $\nabla f(\mathbf{x}) = \mathbf{vec}(\boldsymbol{\Sigma} - \boldsymbol{\Theta}^{-1})$.
  - Hessian of f: $\nabla^2 f(\mathbf{x}) = \boldsymbol{\Theta}^{-1} \otimes \boldsymbol{\Theta}^{-1}$

- How to compute the Proximal Newton direction?

$$\mathbf{d}_{\mathbf{H}_k}^k := \arg\min_{\mathbf{d}}\left\{f(\mathbf{x}^k)+\nabla f(\mathbf{x}^k)^T\mathbf{d}+\frac{1}{2}\mathbf{d}^T\mathbf{H}_k\mathbf{d}+g(\mathbf{x}^k+\mathbf{d})\right\}, \quad \mathbf{H}_k = \nabla^2 f(\mathbf{x}^k)$$

Graphical model selection

- Objective:

$$\min_{\boldsymbol{\Theta}\succ 0}\left\{\underbrace{-\log\det(\boldsymbol{\Theta})+\operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})}+\underbrace{\rho\|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})}\right\}$$

- Gradient and Hessian (large-scale, special structure)
  - Gradient of f: $\nabla f(\mathbf{x})=\mathbf{vec}(\boldsymbol{\Sigma}-\boldsymbol{\Theta}^{-1})$.
  - Hessian of f: $\nabla^2 f(\mathbf{x})=\boldsymbol{\Theta}^{-1}\otimes\boldsymbol{\Theta}^{-1}$

- Dual approach for solving subproblem (SP)

Primal subproblem	Dual subproblem (SPGL)
$\min_{\Delta}\left\{\frac{1}{2}\operatorname{trace}((\boldsymbol{\Theta}_i^{-1}\Delta)^2)+\operatorname{trace}(\mathbf{R}_i\Delta)+\rho\|\operatorname{vec}(\Delta)\|_1\right\}$	$\min_{\|\operatorname{vec}(\mathbf{U})\|_\infty\le 1}\left\{\frac{1}{2}\operatorname{trace}((\boldsymbol{\Theta}_i\mathbf{U})^2)+\operatorname{trace}(\mathbf{Q}_i\mathbf{U})\right\}$
$\mathbf{R}_i:=\boldsymbol{\Sigma}-2\boldsymbol{\Theta}_i^{-1}$	$\mathbf{Q}_i:=\rho^{-1}[\boldsymbol{\Theta}_i\boldsymbol{\Sigma}\boldsymbol{\Theta}_i-2\boldsymbol{\Theta}_i]$

Unconstrained LASSO problem

Graphical model selection

- Objective:

$$\min_{\boldsymbol{\Theta}\succ 0}\left\{\underbrace{-\log\det(\boldsymbol{\Theta})+\operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})}+\underbrace{\rho\|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})}\right\}$$

- Gradient and Hessian (large-scale, special structure)
  - Gradient of f: $\nabla f(\mathbf{x}) = \mathbf{vec}(\boldsymbol{\Sigma} - \boldsymbol{\Theta}^{-1})$.
  - Hessian of f: $\nabla^2 f(\mathbf{x}) = \boldsymbol{\Theta}^{-1} \otimes \boldsymbol{\Theta}^{-1}$

No Cholesky decomposition and matrix inversion

- Dual approach for solving subproblem (SP)

Primal subproblem	Dual subproblem (SPGL)
$\min_{\Delta}\left\{\frac{1}{2}\operatorname{trace}((\boldsymbol{\Theta}_i^{-1}\Delta)^2)+\operatorname{trace}(\mathbf{R}_i\Delta)+\rho\|\operatorname{vec}(\Delta)\|_1\right\}$	$\min_{\|\operatorname{vec}(\mathbf{U})\|_\infty\le 1}\left\{\frac{1}{2}\operatorname{trace}((\boldsymbol{\Theta}_i\mathbf{U})^2)+\operatorname{trace}(\mathbf{Q}_i\mathbf{U})\right\}$
$\mathbf{R}_i := \boldsymbol{\Sigma} \quad 2\boldsymbol{\Theta}_i^{-1}$	$\mathbf{Q}_i := \rho^{-1}[\boldsymbol{\Theta}_i\boldsymbol{\Sigma}\boldsymbol{\Theta}_i - 2\boldsymbol{\Theta}_i]$

Unconstrained LASSO problem

Graphical model selection

- Objective:

$$\min_{\mathbf{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\mathbf{\Theta}) + \mathrm{trace}(\mathbf{\Sigma}\mathbf{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho\|\mathrm{vec}(\mathbf{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

- Gradient and Hessian (large-scale, special structure)
  - Gradient of f: $\nabla f(\mathbf{x}) = \mathbf{vec}(\mathbf{\Sigma} - \mathbf{\Theta}^{-1})$.
  - Hessian of f: $\nabla^2 f(\mathbf{x}) = \mathbf{\Theta}^{-1} \otimes \mathbf{\Theta}^{-1}$

No Cholesky decomposition and matrix inversion

- Dual approach for solving subproblem (SP)

Primal subproblem	Dual subproblem (SPGL)
Unconstrained LASSO problem	$\min_{\|\mathrm{vec}(\mathbf{U})\|_\infty \le 1} \left\{ \frac{1}{2}\mathrm{trace}((\mathbf{\Theta}_i\mathbf{U})^2) + \mathrm{trace}(\mathbf{Q}_i\mathbf{U}) \right\}$
$\mathbf{R}_i := \mathbf{\Sigma} - 2\mathbf{\Theta}_i^{-1}$	$\mathbf{Q}_i := \rho^{-1}[\mathbf{\Theta}_i\mathbf{\Sigma}\mathbf{\Theta}_i - 2\mathbf{\Theta}_i]$

- How to compute proximal Newton decrement $\lambda_i := \|\mathbf{d}^i\|_{\mathbf{x}^i}$?

$$\lambda_i := [p - 2\mathrm{trace}(\mathbf{W}_i) + \mathrm{trace}(\mathbf{W}_i^2)]^{1/2}, \quad \mathbf{W}_i = \mathbf{\Theta}_i(\mathbf{\Sigma} - \rho\mathbf{U}^*)$$

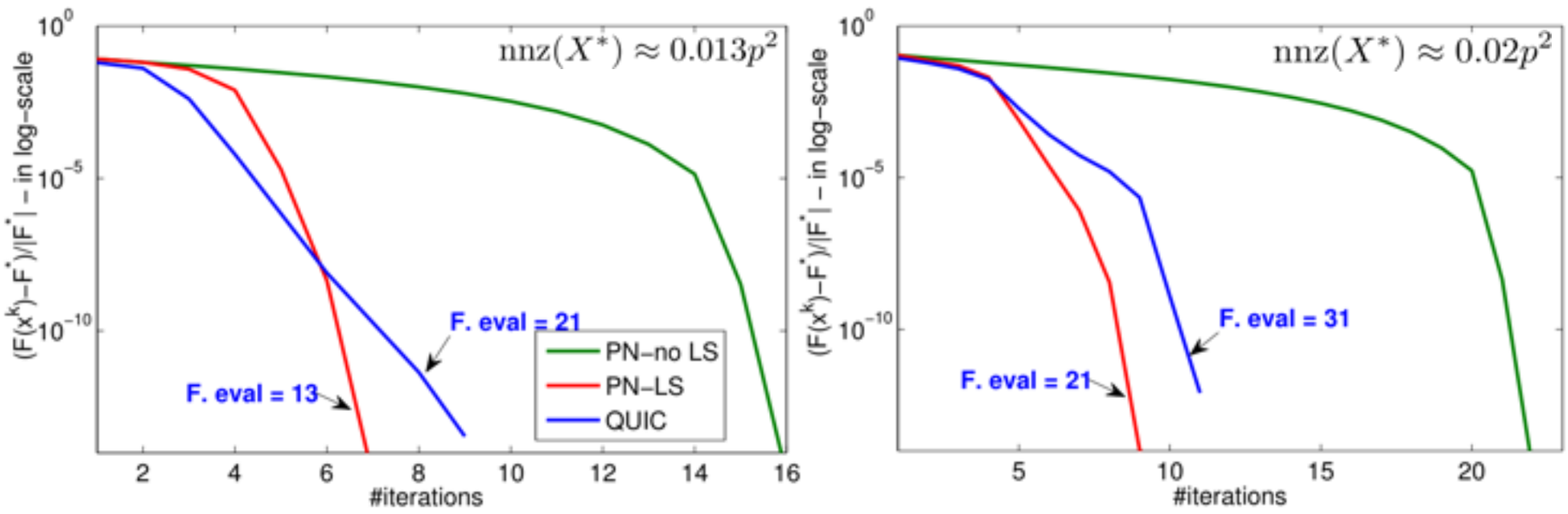
Graphical model selection: numerical examples

Our method vs QUIC [Hseih2011]

- QUIC subproblem solver: special block-coordinate descent

- Our subproblem solver: general proximal algorithms

Convergence behaviour [rho = 0.5]: Lymph [p = 587] (left), Leukemia [p = 1255] (right)



Step-size selection strategies: Arabidopsis [p = 834], Leukemia [p = 1255], Hereditary [p = 1869]

	Synthetic ($\rho = 0.01$)			Arabidopsis ($\rho = 0.5$)			Leukemia ($\rho = 0.1$)			Hereditary ($\rho = 0.1$)		
LS Scheme	#iter	#chol	#Mm	#iter	#chol	#Mm	#iter	#chol	#Mm	#iter	#chol	#Mm
NoLS	25.4	-	3400	18	-	1810	44	-	9842	72	-	20960
BtkLS	25.5	37.0	2436	11	25	718	15	50	1282	19	63	2006
E-BtkLS	25.5	36.2	2436	11	24	718	15	49	1282	15	51	1282
FwLS	18.1	26.2	1632	10	17	612	12	34	844	14	44	1126

Details:** "A proximal Newton framework for composite minimization: Graph learning without Cholesky decompositions and matrix inversions," **ICML'13 and lions.epfl.ch/publications.

Graphical model selection: numerical examples

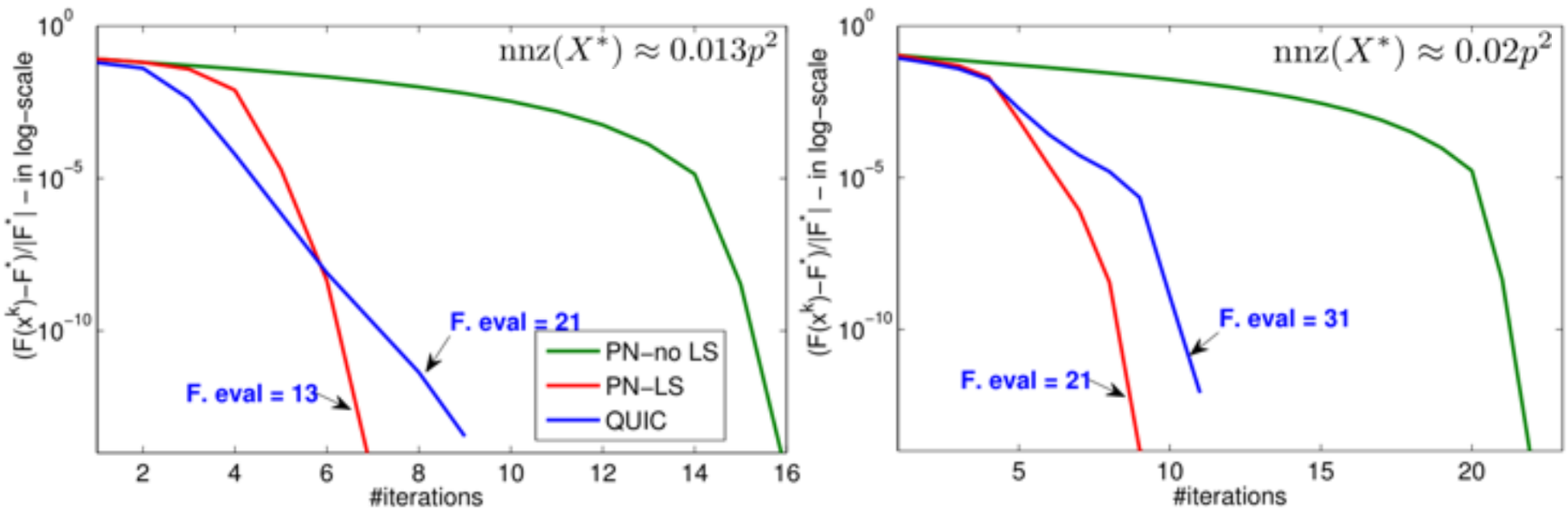
Our method vs QUIC [Hseih2011]

- QUIC subproblem solver: special block-coordinate descent

On the average x5 acceleration (up to x15) over Matlab QUIC

- Our subproblem solver: general proximal algorithms

Convergence behaviour [rho = 0.5]: Lymph [p = 587] (left), Leukemia [p = 1255] (right)



Step-size selection strategies: Arabidopsis [p = 834], Leukemia [p = 1255], Hereditary [p = 1869]

	Synthetic ($\rho = 0.01$)			Arabidopsis ($\rho = 0.5$)			Leukemia ($\rho = 0.1$)			Hereditary ($\rho = 0.1$)		
LS Scheme	#iter	#chol	#Mm	#iter	#chol	#Mm	#iter	#chol	#Mm	#iter	#chol	#Mm
NoLS	25.4	-	3400	18	-	1810	44	-	9842	72	-	20960
BtkLS	25.5	37.0	2436	11	25	718	15	50	1282	19	63	2006
E-BtkLS	25.5	36.2	2436	11	24	718	15	49	1282	15	51	1282
FwLS	18.1	26.2	1632	10	17	612	12	34	844	14	44	1126

*Details: "A proximal Newton framework for composite minimization: Graph learning without Cholesky decompositions and matrix inversions," ICML'13 and lions.epfl.ch/publications.

(P) Composite minimization: alternatives?

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$

- a

f

convex and **smooth**

g

convex and possibly **nonsmooth**

Existing numerical approaches

- Splitting methods

  - **Forward-backward**: *applicable if **f** has Lipschitz gradient*

  - **Douglas-Rachford** decomposition: *__f__ and __g__ have "tractable" proximity operators*

- Augmented Lagrangian methods (e.g., D-R again)

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x},\mathbf{y}) := f(\mathbf{x}) + g(\mathbf{y})\}$$
$$\text{s.t.} \quad \boxed{\mathbf{x}-\mathbf{y}=0}$$

Prox operator of self-concordant functions are costly!

– log det : full eigen decomposition

– log (with a linear operator): non-linear system

Our "cheaper" variable metric strategies

- Proximal **gradient** scheme*

Given $\mathbf{x}^0$, generate a sequence $\{\mathbf{x}^k\}_{k\geq 0}$ such that

$$\mathbf{x}^{k+1} := \mathbf{x}^k + \alpha_k \mathbf{d}^k_{\mathbf{H}_k}$$

where $\alpha_k \in (0,1]$ is step-size, $\mathbf{d}^k_{\mathbf{H}_k}$ is a search direction

- How to compute the search direction?

$$\mathbf{d}^k_{\mathbf{H}_k} := \arg\min_{\mathbf{d}} \left\{ f(\mathbf{x}^k) + \nabla f(\mathbf{x}^k)^T \mathbf{d} + \frac{1}{2}\mathbf{d}^T \mathbf{H}_k \mathbf{d} + g(\mathbf{x}^k + \mathbf{d}) \right\}, \quad \mathbf{H}_k = \mathbf{D}_k : \text{ diagonal}$$

- No Lipschitz assumption

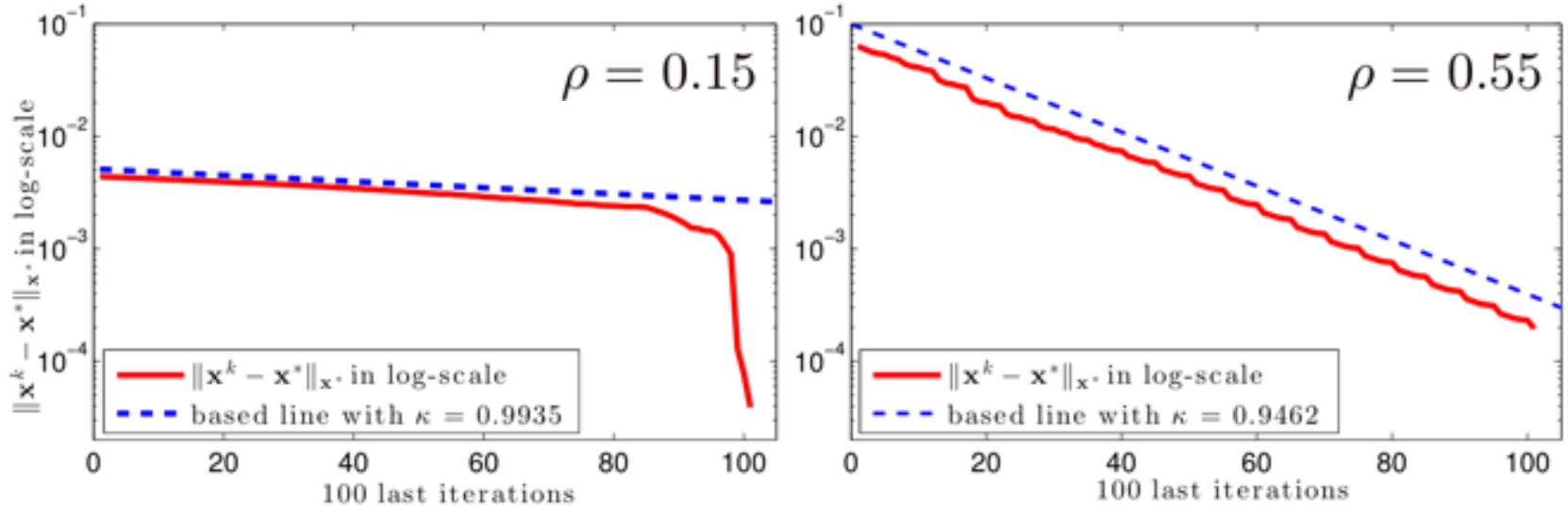
A new predictor corrector scheme (with local linear convergence*)

- Proximal **quasi**-Newton scheme (BFGS updates)*

Details:** "Composite Self-Concordant Minimization," **arxiv and lions.epfl.ch/publications.

New theory: Local linear convergence of the PG method

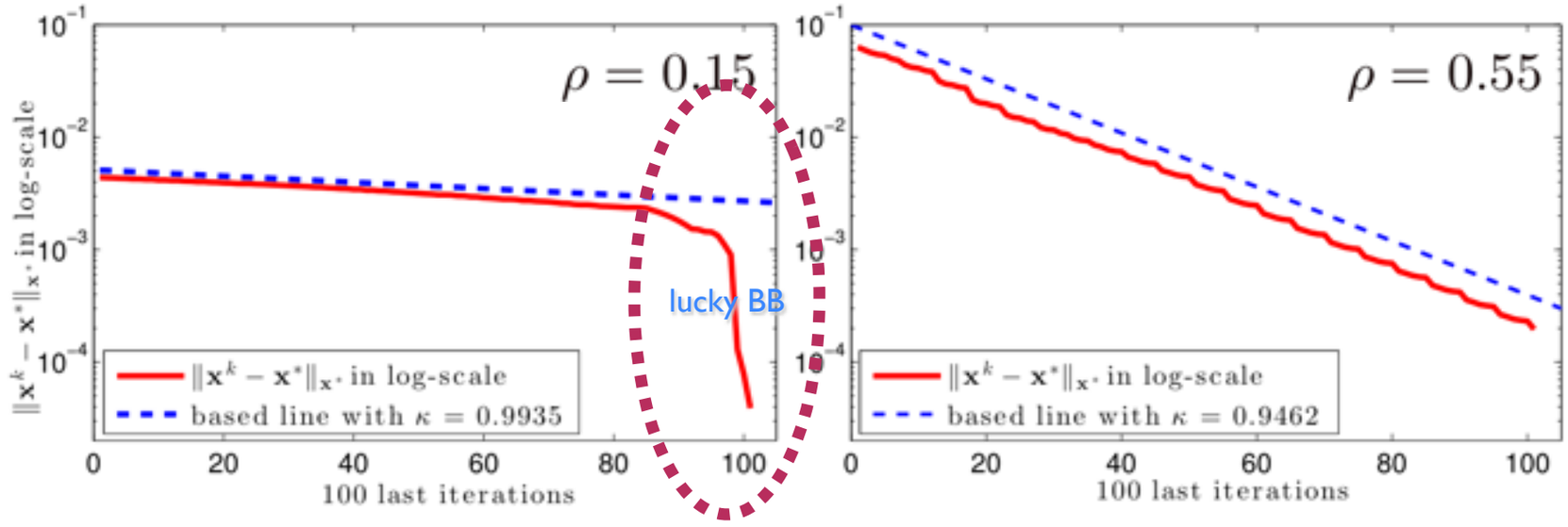
Graph learning: Lymph [p = 587]



$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

New theory: Local linear convergence of the PG method

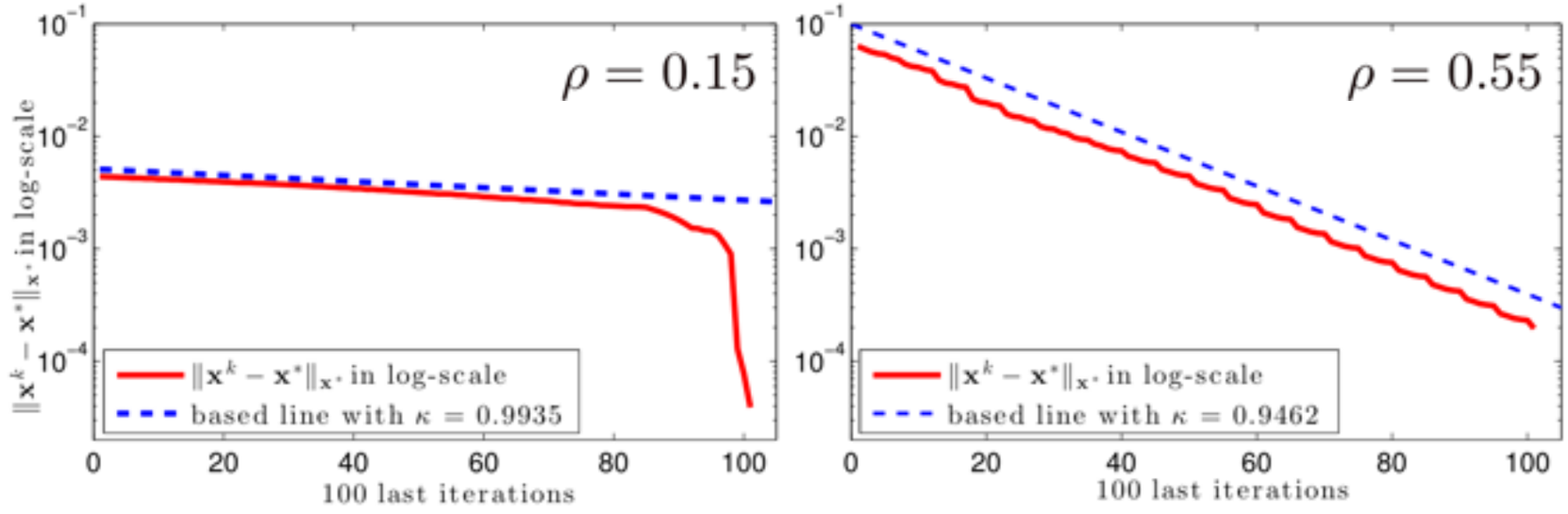
Graph learning: Lymph [p = 587]



$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

New theory: Local linear convergence of the PG method

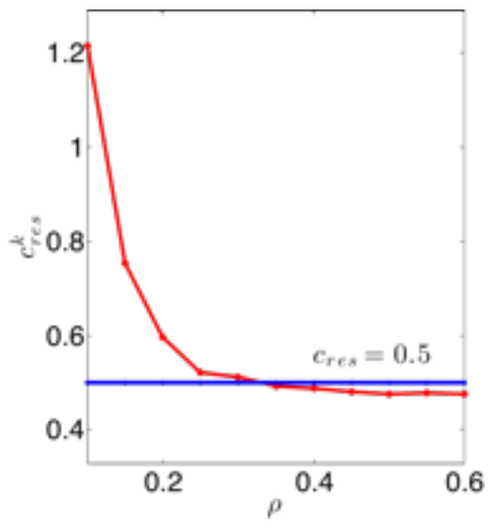
Graph learning: Lymph [p = 587]

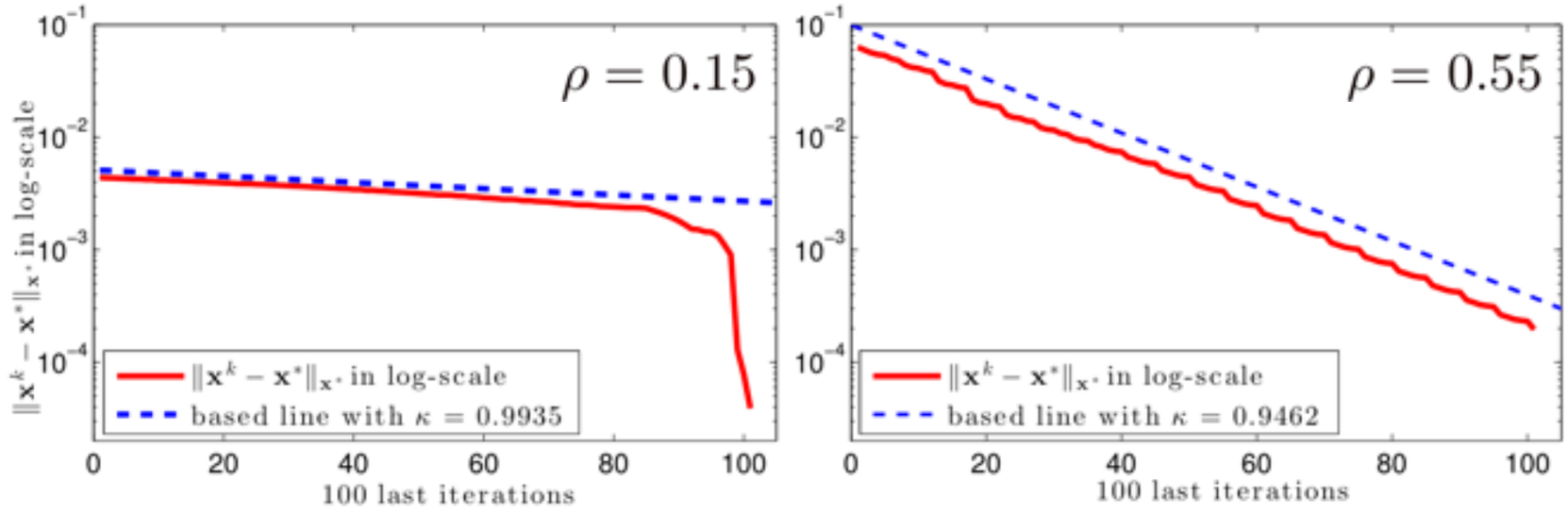


$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

New theory: Local linear convergence of the PG method

Graph learning: Lymph [p = 587]



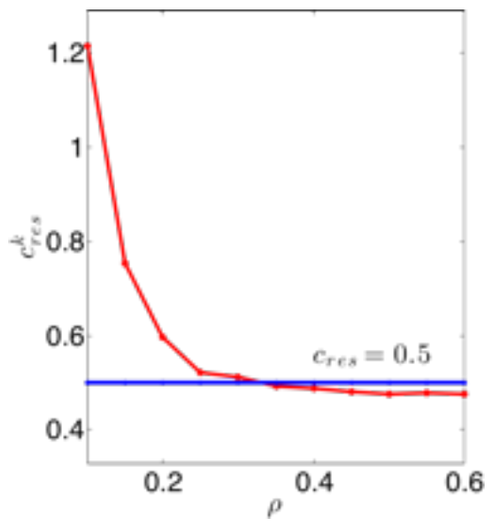


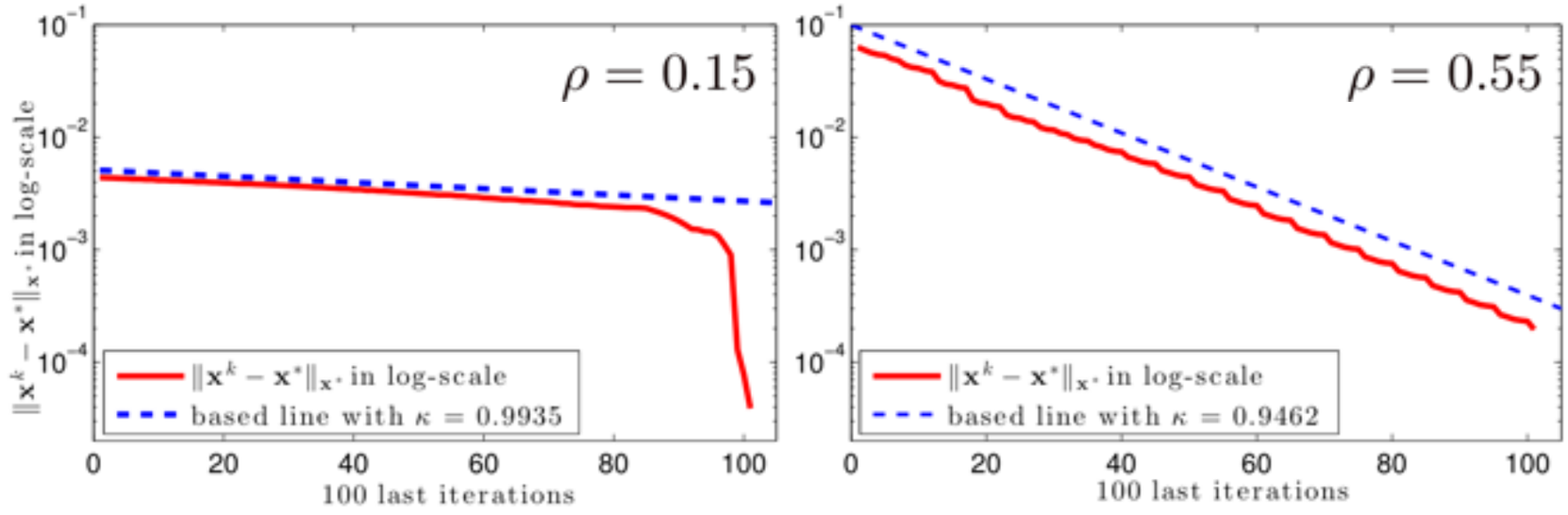
Theory is based on the notion of "restricted strong convexity"

$$\min_{\mathbf{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\mathbf{\Theta}) + \mathrm{trace}(\mathbf{\Sigma}\mathbf{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho\|\mathrm{vec}(\mathbf{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

New theory: Local linear convergence of the PG method

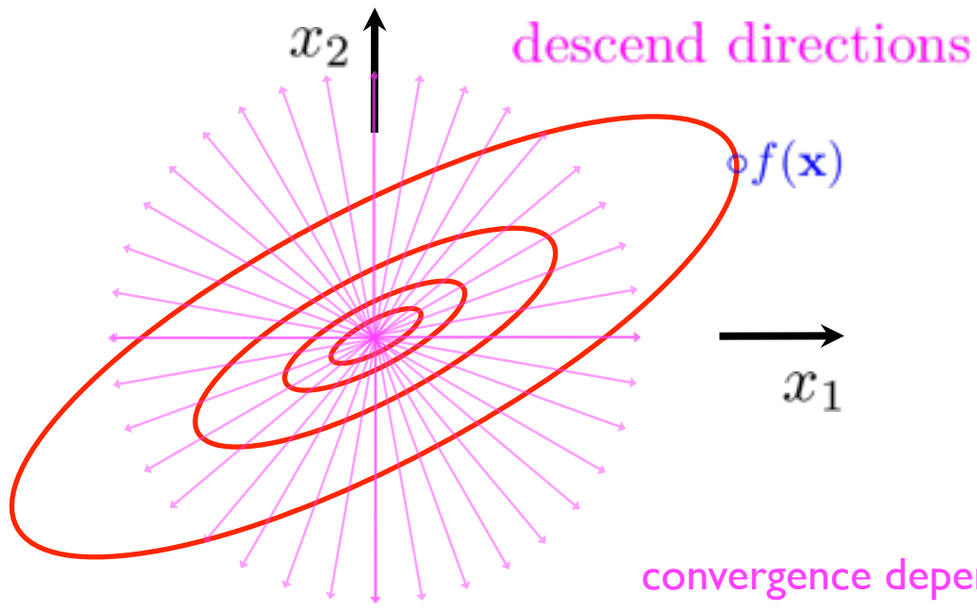
Graph learning: Lymph [p = 587]





Theory is based on the notion of "restricted strong convexity"

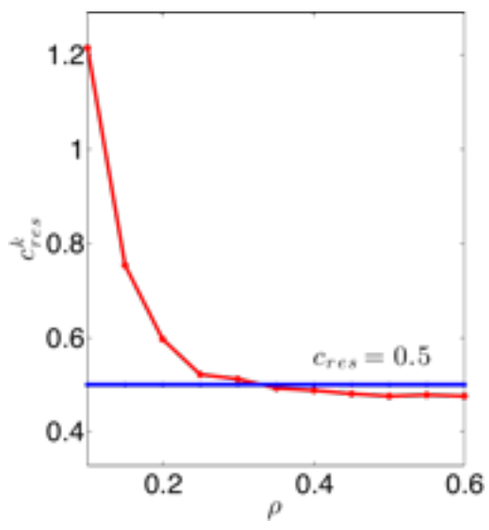
$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} \right\}$$

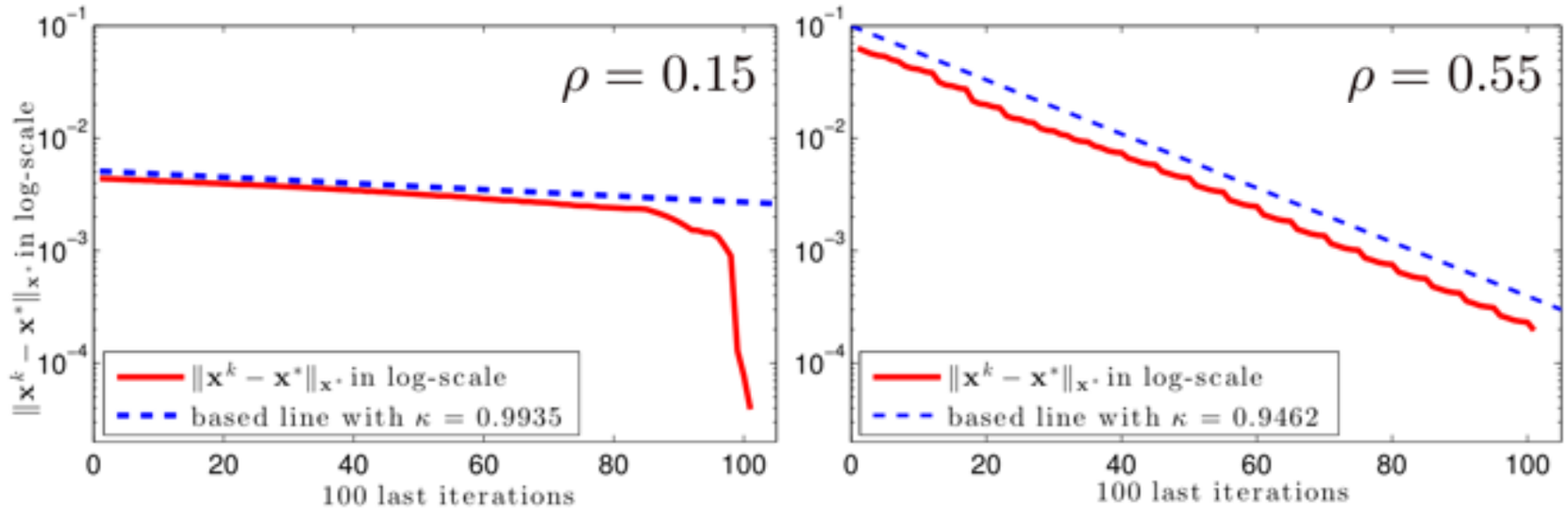


convergence depends on the full condition number

New theory: Local linear convergence of the PG method

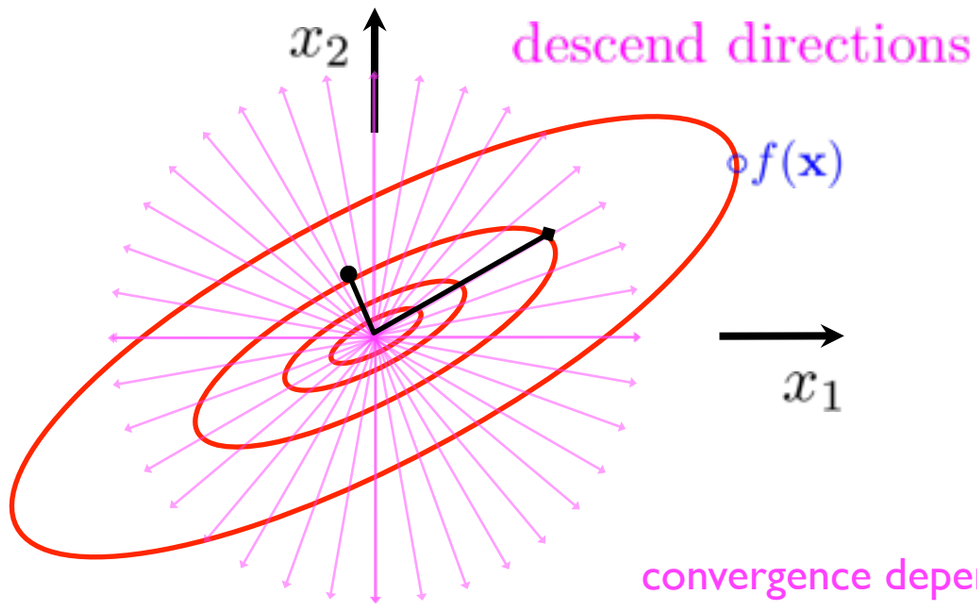
Graph learning: Lymph [p = 587]





Theory is based on the notion of "restricted strong convexity"

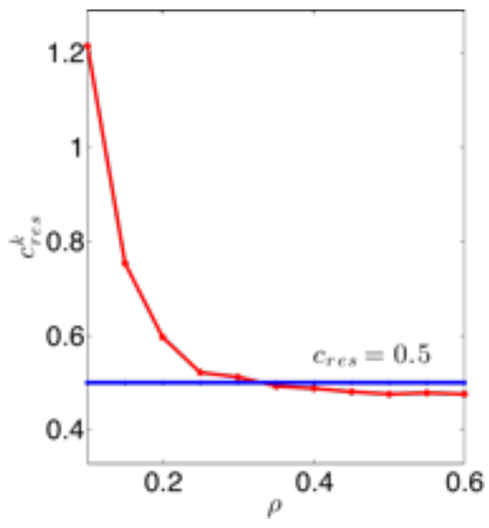
$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} \right\}$$

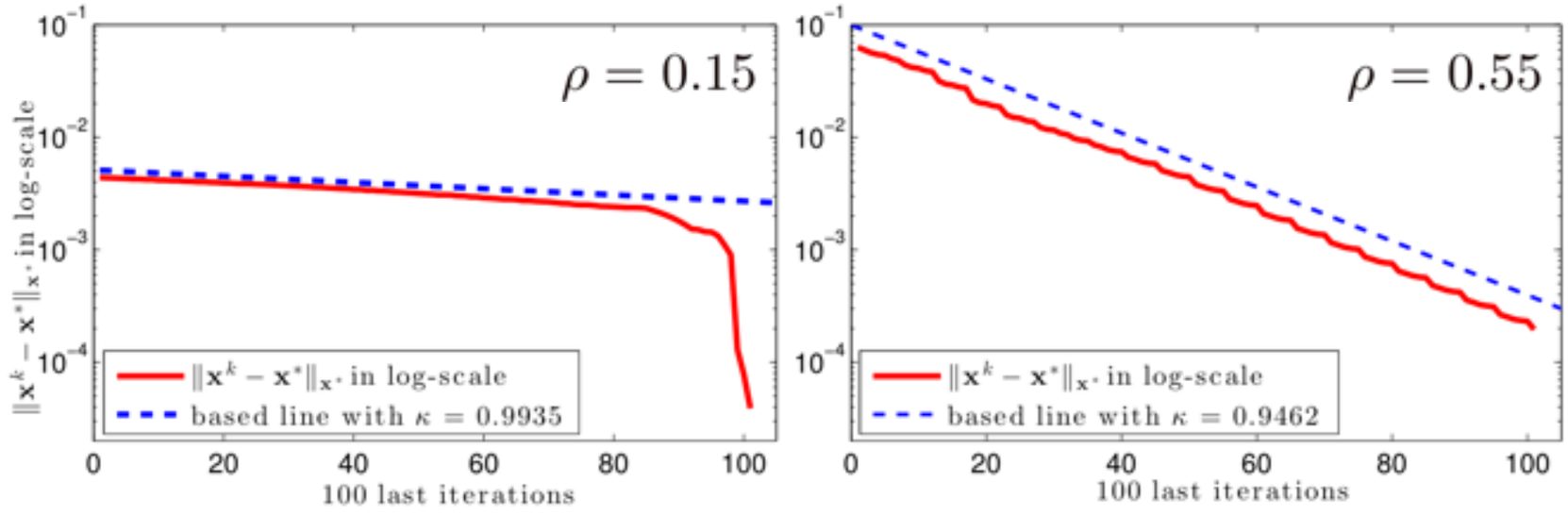


convergence depends on the full condition number

New theory: Local linear convergence of the PG method

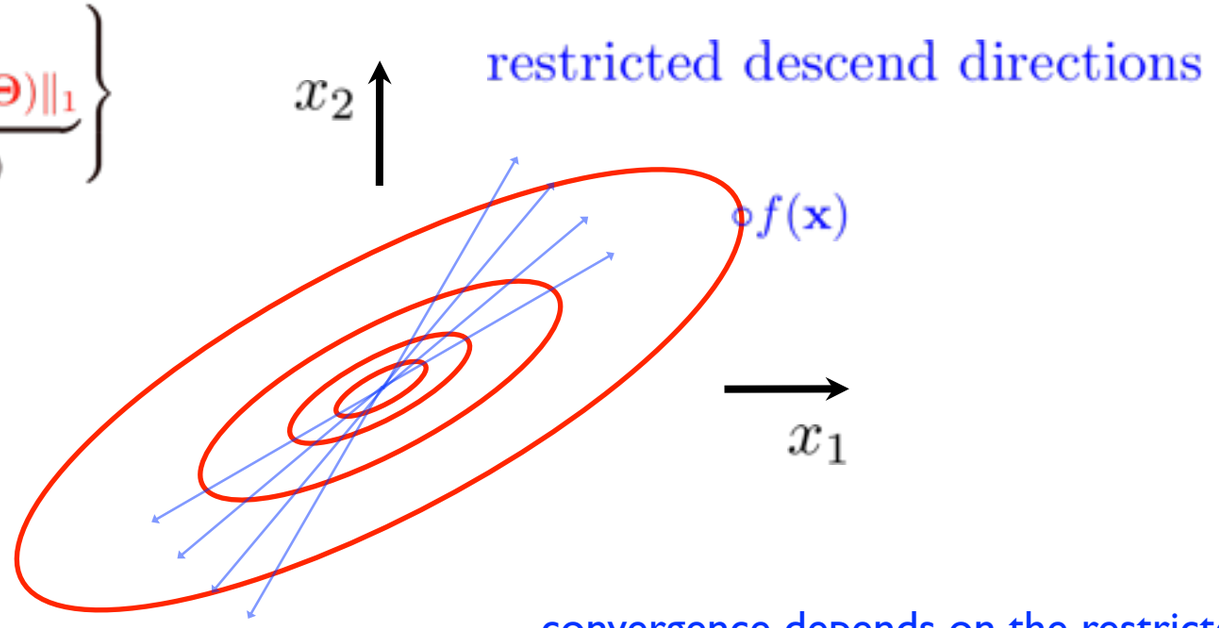
Graph learning: Lymph [p = 587]





Theory is based on the notion of "restricted strong convexity"

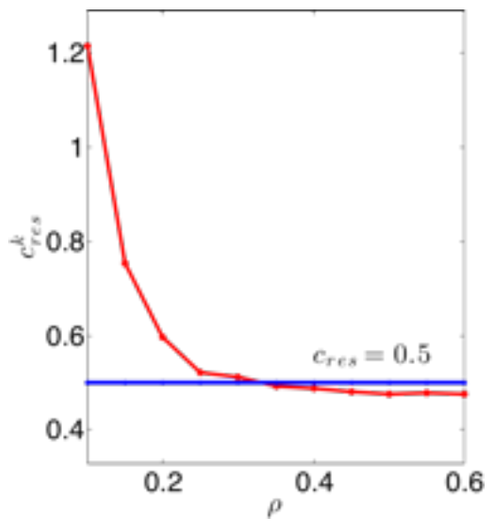
$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$

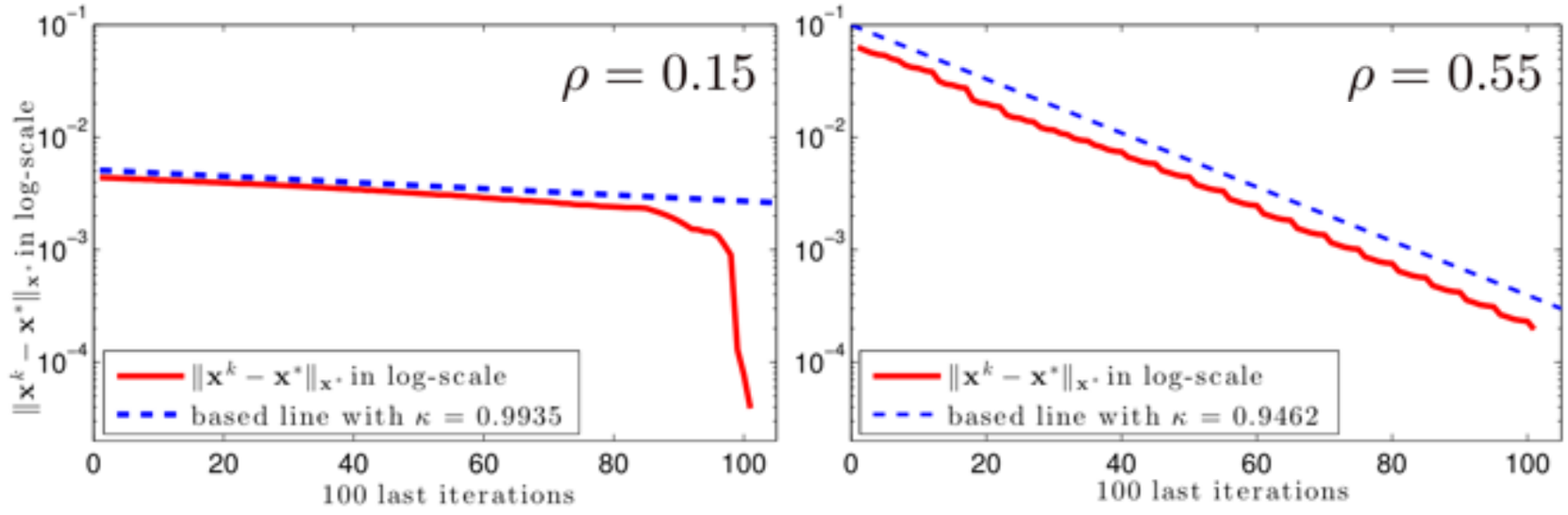


convergence depends on the restricted condition number

New theory: Local linear convergence of the PG method

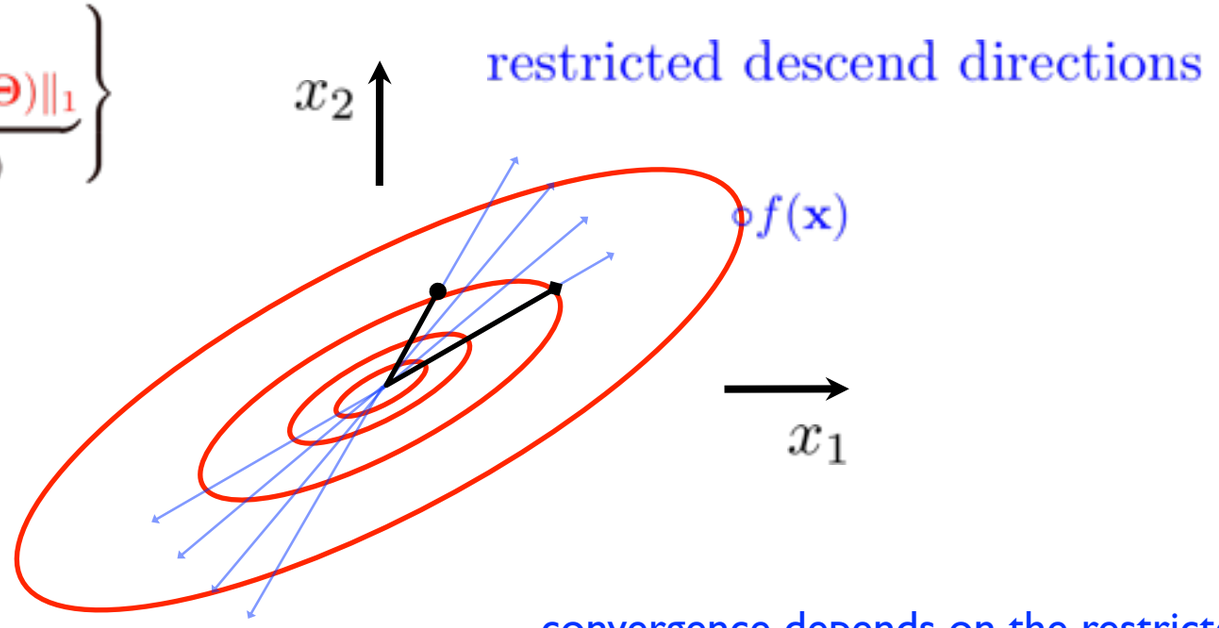
Graph learning: Lymph [p = 587]





Theory is based on the notion of "restricted strong convexity"

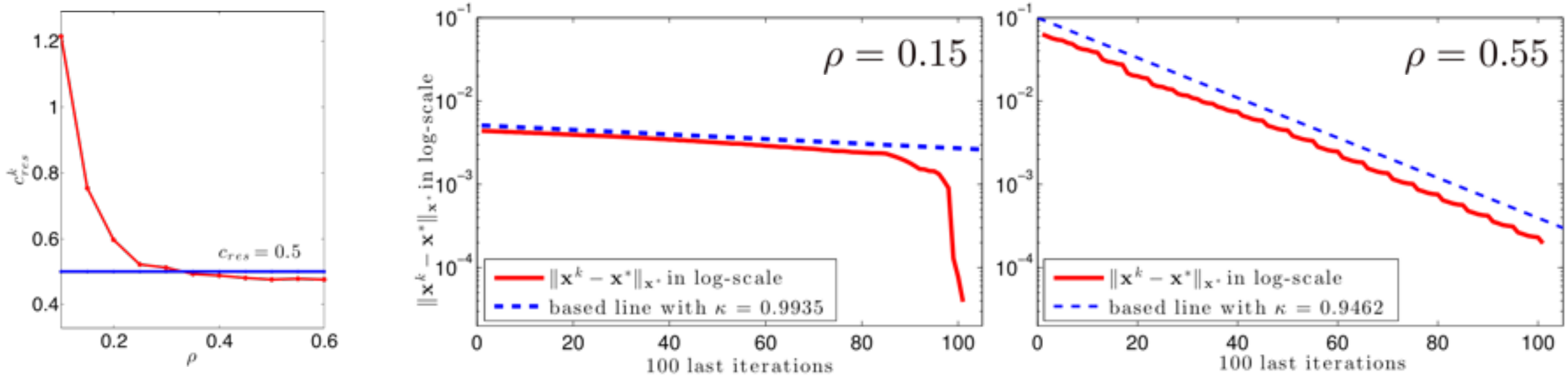
$$\min_{\boldsymbol{\Theta} \succ 0} \left\{ \underbrace{-\log\det(\boldsymbol{\Theta}) + \operatorname{trace}(\boldsymbol{\Sigma}\boldsymbol{\Theta})}_{f(\mathbf{x})} + \underbrace{\rho \|\operatorname{vec}(\boldsymbol{\Theta})\|_1}_{g(\mathbf{x})} \right\}$$



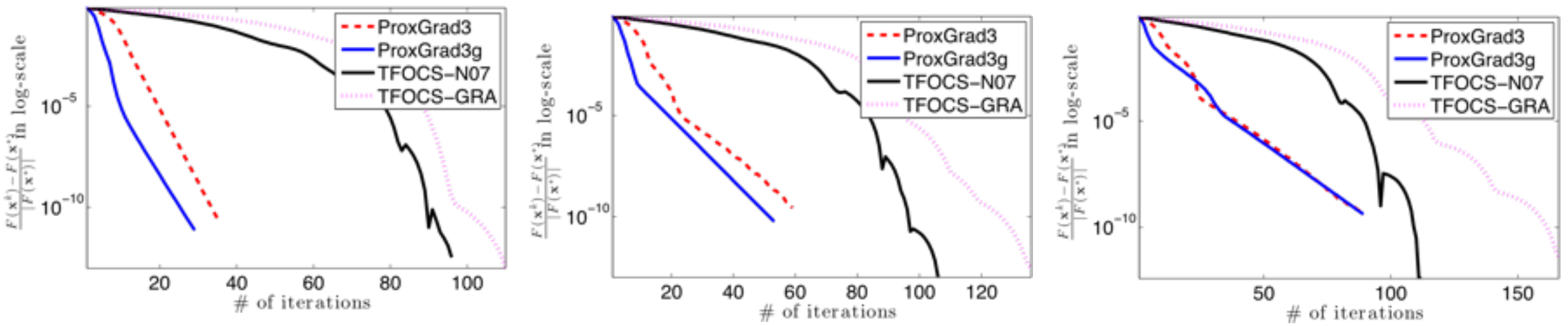
convergence depends on the restricted condition number

New theory: Local linear convergence of the PG method

Graph learning: Lymph [p = 587]



Heteroschedastic LASSO [rho decreases from left to right]

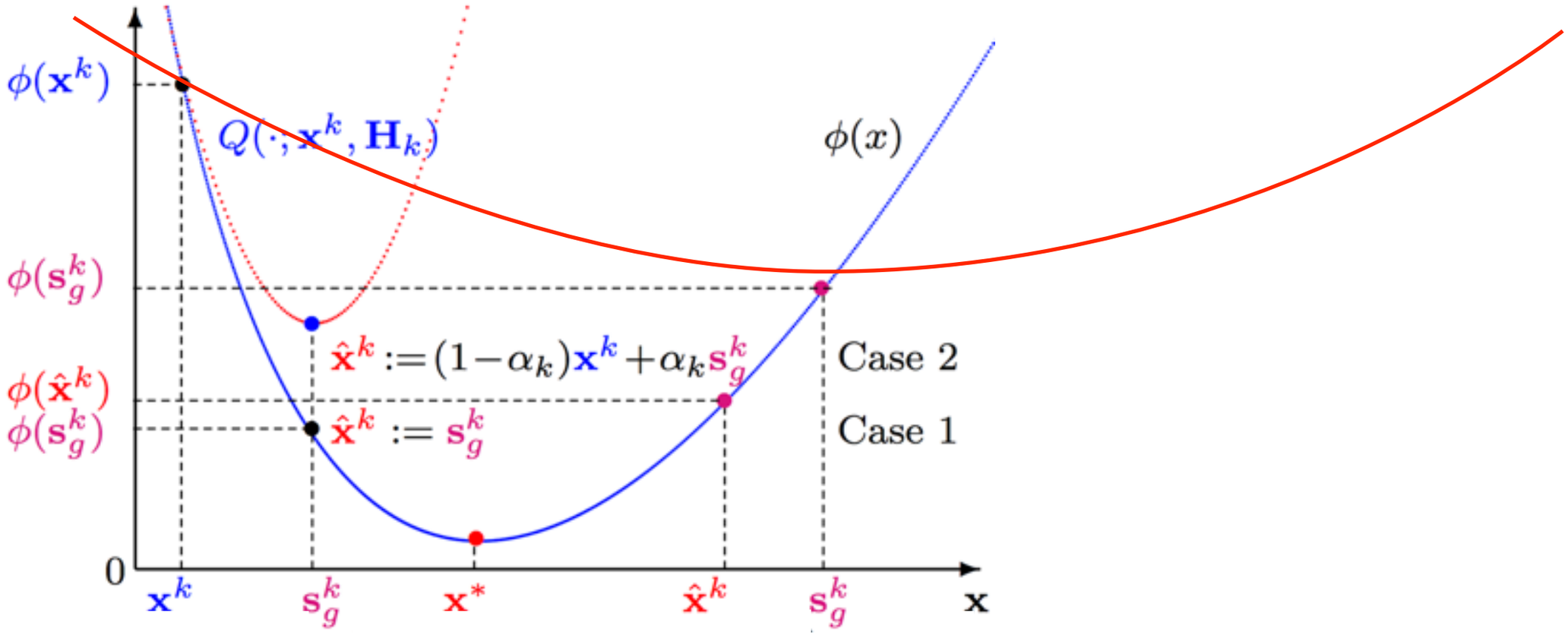


$$\mathbf{x}^* \equiv (\phi^*, \gamma^*) = \underset{\phi,\gamma}{\operatorname{argmin}} \Big\{ \underbrace{-\log(\gamma) + \frac{1}{2n}\|\gamma\mathbf{y} - \mathbf{X}\phi\|_2^2}_{=:f(\mathbf{x})} + \underbrace{\rho\|\phi\|_1}_{=:g(\mathbf{x})} \Big\}$$

Proximal gradient scheme: new engineering

- A ***greedy*** enhancement

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



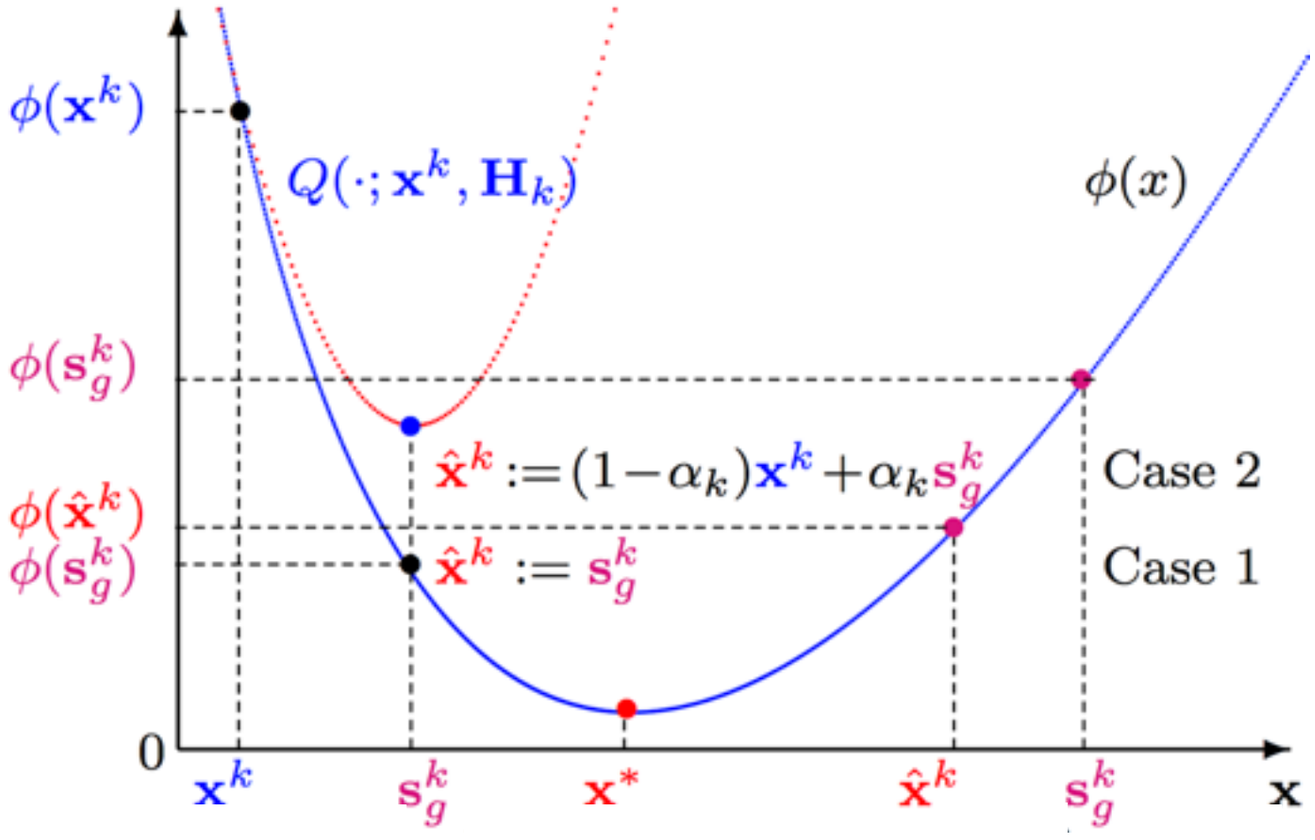
$$\hat{\mathbf{x}}^k := (1-\alpha_k)\mathbf{x}^k + \alpha_k \mathbf{s}_g^k$$

prox: $\mathbf{s}_g^k := \arg\min_{\mathbf{x}\in\mathrm{dom}(F)} \{Q(\mathbf{x};\mathbf{x}^k,\mathbf{D}_k) + g(\mathbf{x})\}$

Proximal gradient scheme: new engineering

- A ***greedy*** enhancement

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



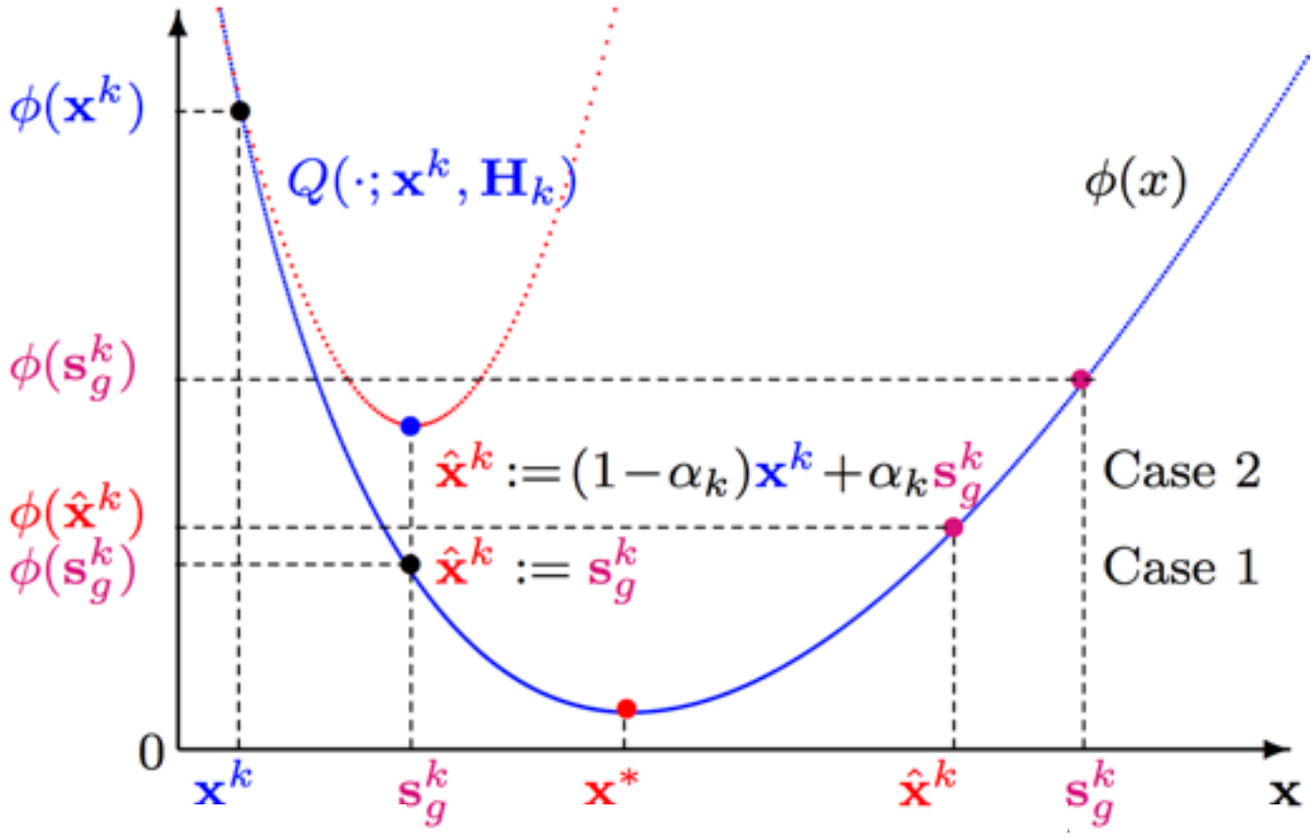
$$\hat{\mathbf{x}}^k := (1-\alpha_k)\mathbf{x}^k + \alpha_k \mathbf{s}_g^k$$

prox: $\mathbf{s}_g^k := \arg\min_{\mathbf{x}\in\mathrm{dom}(F)} \{Q(\mathbf{x};\mathbf{x}^k,\mathbf{D}_k) + g(\mathbf{x})\}$

Proximal gradient scheme: new engineering

- A ***greedy*** enhancement

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



slows down convergence!

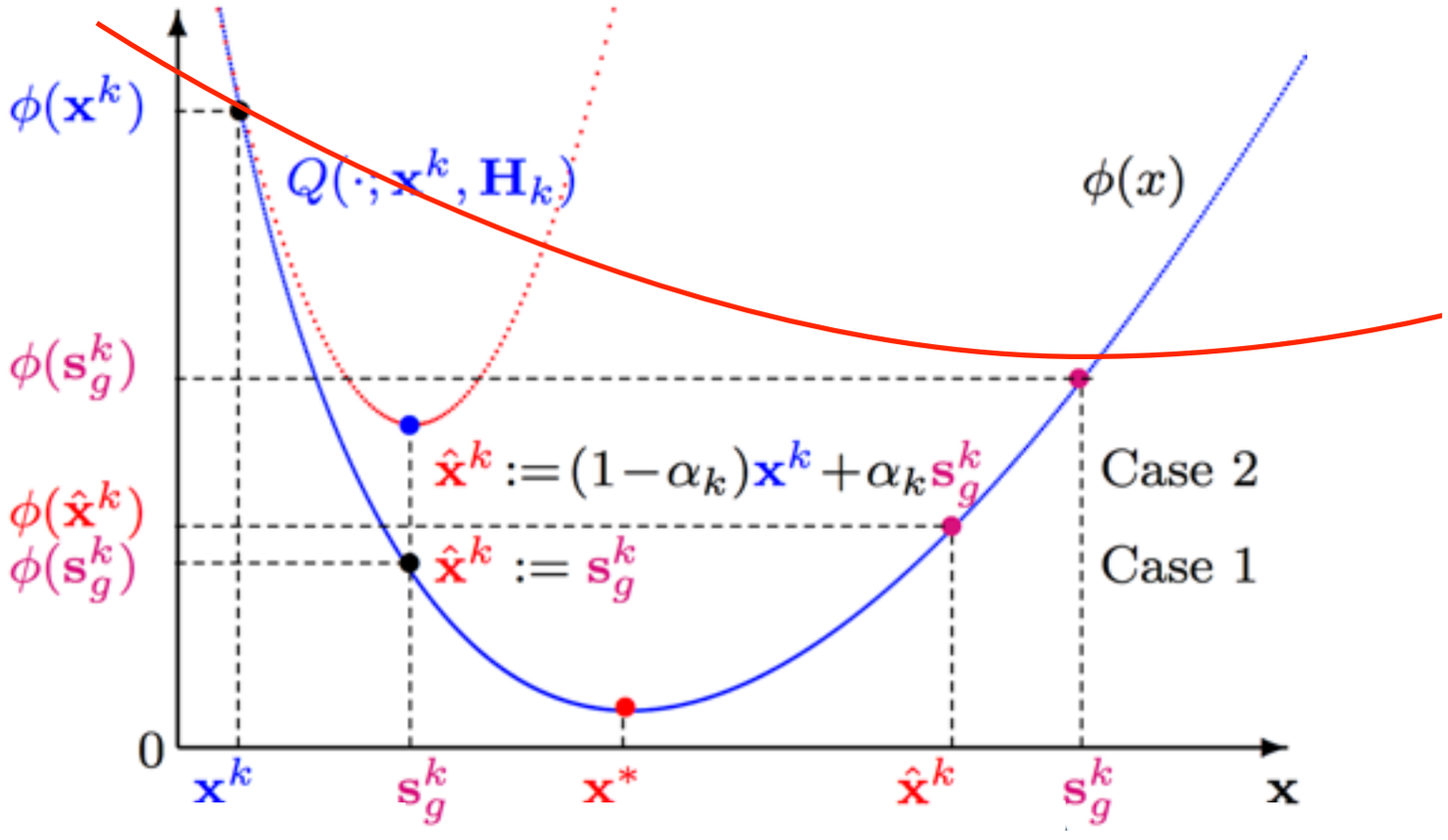
$$\hat{\mathbf{x}}^k := (1-\alpha_k)\mathbf{x}^k + \alpha_k \mathbf{s}_g^k$$

prox: $\mathbf{s}_g^k := \arg\min_{\mathbf{x}\in\mathrm{dom}(F)} \{Q(\mathbf{x};\mathbf{x}^k,\mathbf{D}_k) + g(\mathbf{x})\}$

Proximal gradient scheme: new engineering

- A ***greedy*** enhancement

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



simple decision:

based on the function values

$\phi(\mathbf{s}_g^k)$, $\phi(\hat{\mathbf{x}}^k)$ and $\phi(\mathbf{x}^k)$

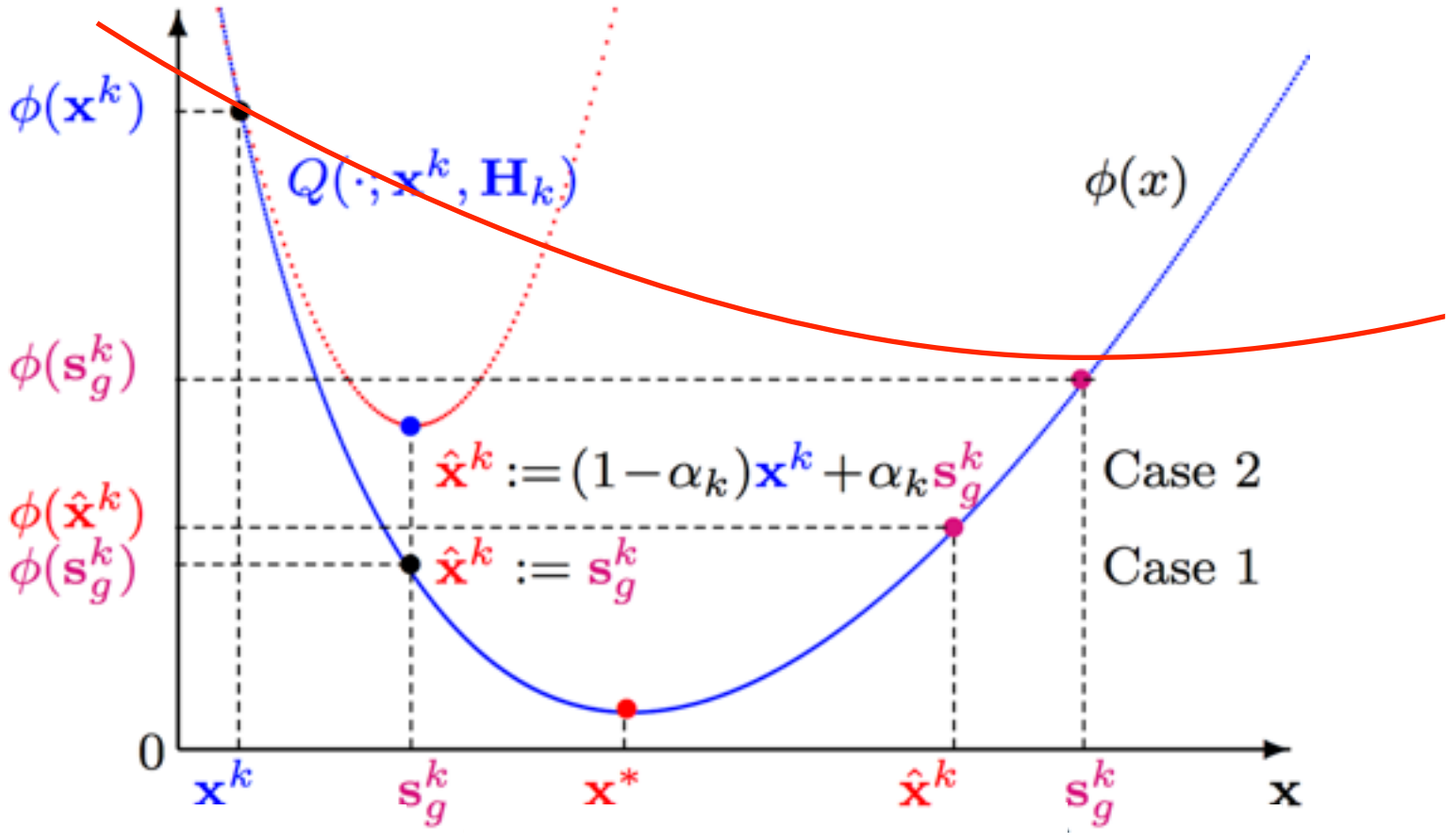
$$\hat{\mathbf{x}}^k := (1-\alpha_k)\mathbf{x}^k + \alpha_k \mathbf{s}_g^k$$

prox: $\mathbf{s}_g^k := \arg\min_{\mathbf{x}\in\mathrm{dom}(F)} \{Q(\mathbf{x};\mathbf{x}^k,\mathbf{D}_k) + g(\mathbf{x})\}$

Proximal gradient scheme: new engineering

- A ***greedy*** enhancement

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



simple decision:

based on the function values

$\phi(\mathbf{s}_g^k)$, $\phi(\hat{\mathbf{x}}^k)$ and $\phi(\mathbf{x}^k)$

cost:

practically none if implemented carefully

$$\hat{\mathbf{x}}^k := (1-\alpha_k)\mathbf{x}^k + \alpha_k \mathbf{s}_g^k$$

prox: $\mathbf{s}_g^k := \arg\min_{\mathbf{x}\in\mathrm{dom}(F)} \{Q(\mathbf{x};\mathbf{x}^k,\mathbf{D}_k) + g(\mathbf{x})\}$

Poisson imaging reconstruction via TV

Our method vs SPIRAL-TAP [Harmany2012]

Original image | Poisson noise image | Reconstructed image (ProxGrad) | Reconstructed image (ProxGradNewton) | Reconstructed image (SPIRAL–TAP

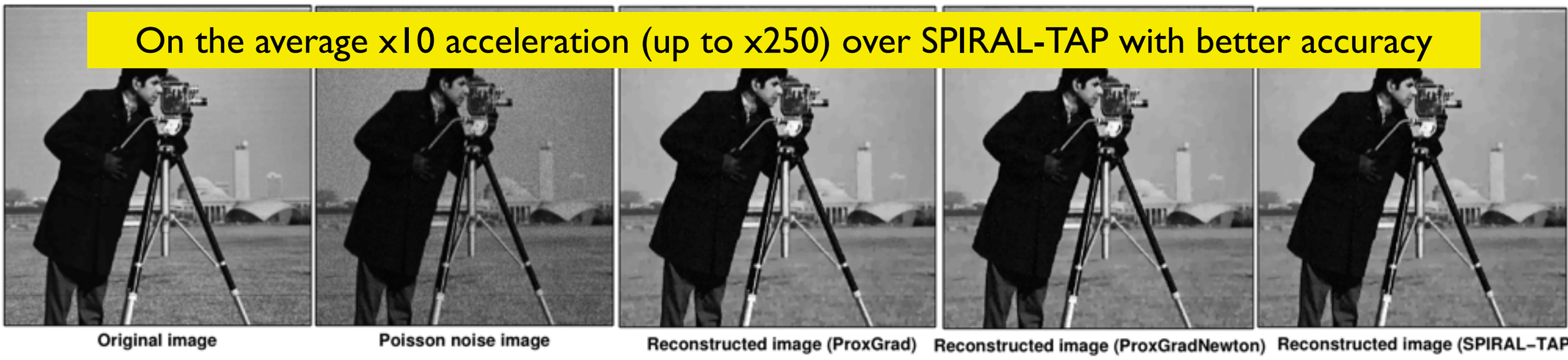
- Poisson imaging reconstruction via TV regularization
- -a

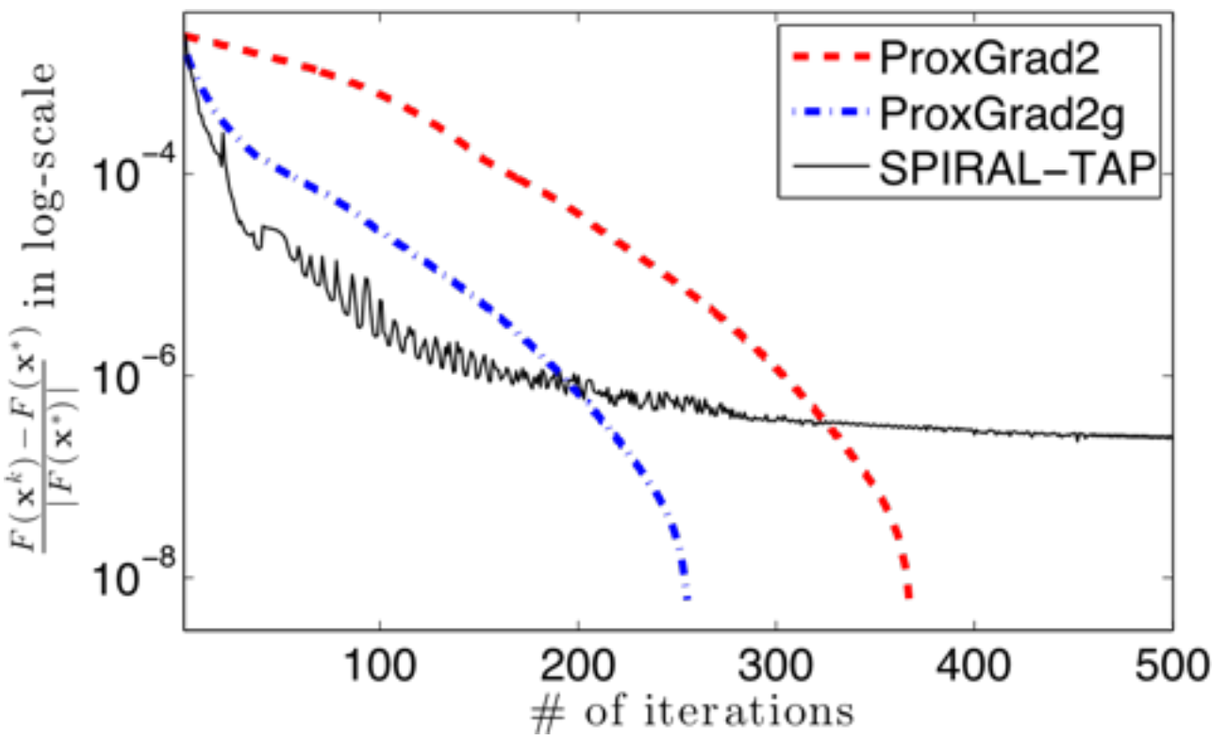
$$x^* \in \operatorname*{argmin}_x \Big\{ \underbrace{\sum_{i=1}^{m} a_i^T x - \sum_{i=1}^{m} y_i \log(a_i^T x + b_i)}_{f(x)} + g(x) \Big\}$$

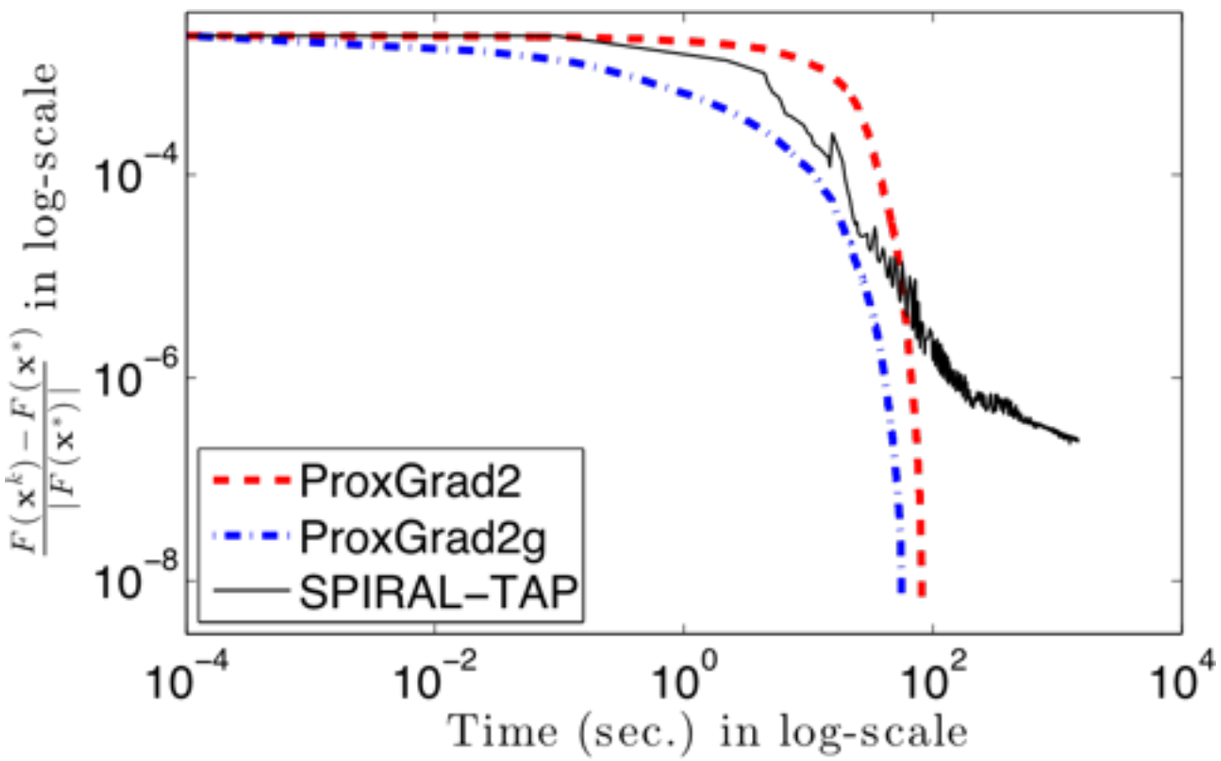
Poisson imaging reconstruction via TV

Our method vs SPIRAL-TAP [Harmany2012]

On the average x10 acceleration (up to x250) over SPIRAL-TAP with better accuracy





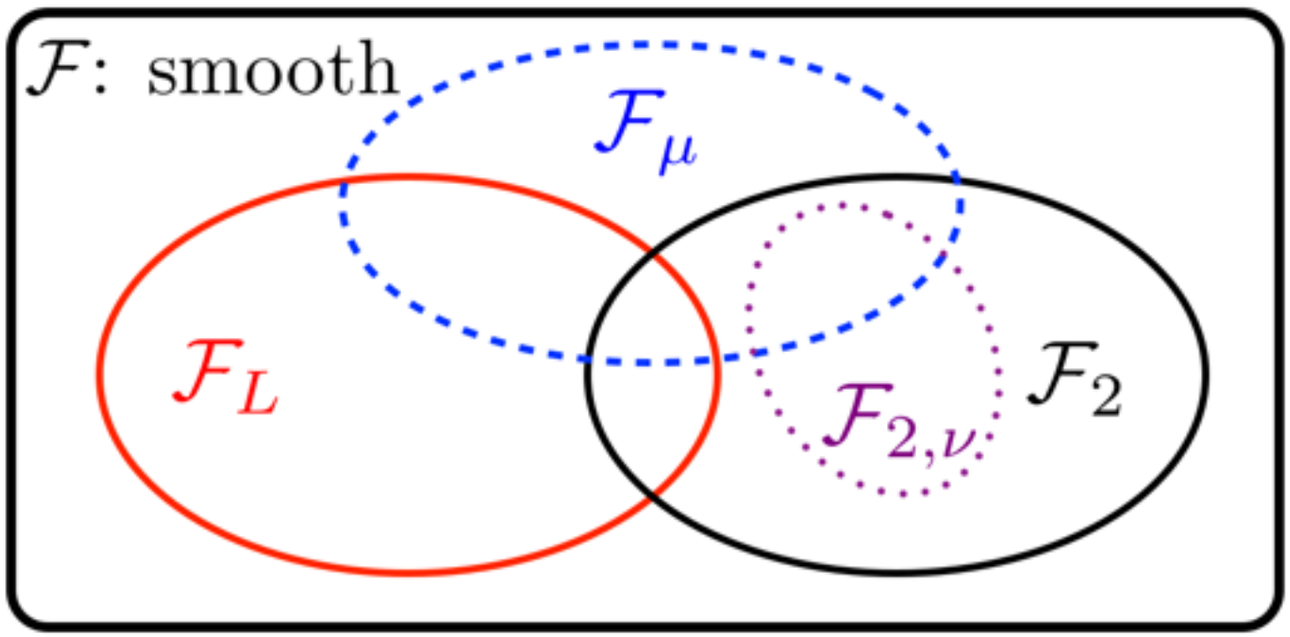


Barrier extensions

Constrained convex problems

$$g^* := \min_{\mathbf{x} \in \Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;



f is a ν-self-concordant barrier if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$ and $|\varphi'(t)| \leq \sqrt{\nu}\varphi''(t)^{1/2}$

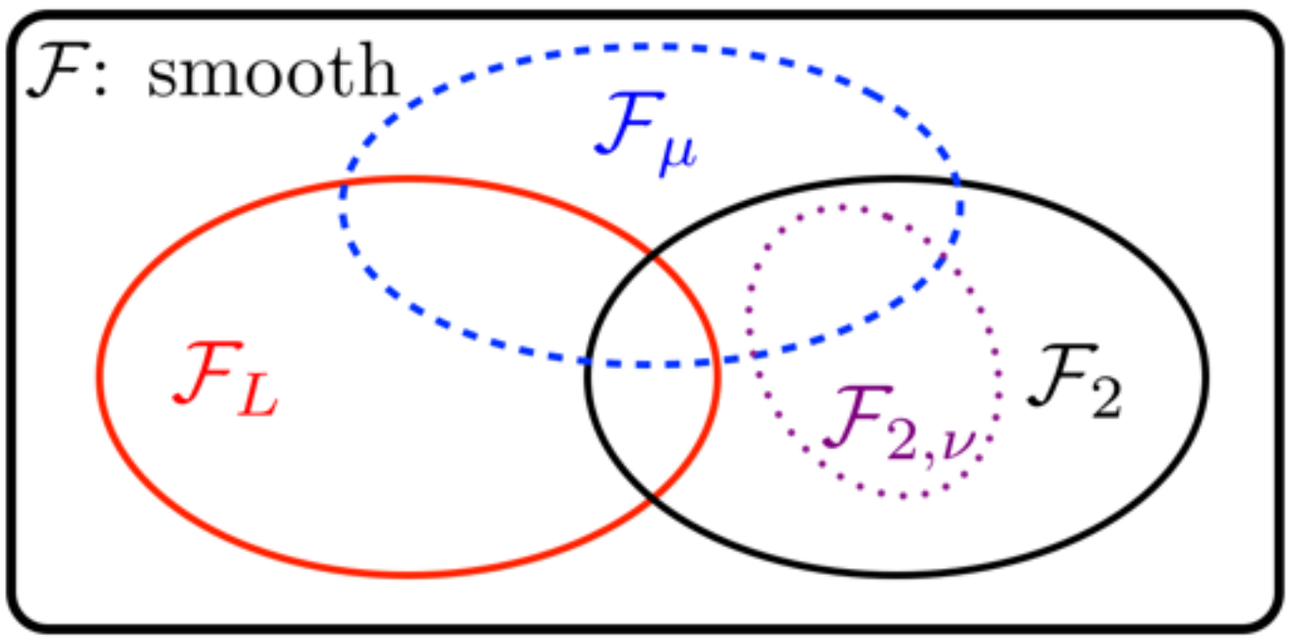
Constrained convex problems

$$g^* := \min_{\mathbf{x}\in\Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;

- Examples:

$$\begin{aligned}
\Omega : \mathbf{X} \succeq 0 \quad &\Rightarrow \quad f_\Omega(\mathbf{X}) = -\log\det(\mathbf{X}) \\
\Omega : \mathbf{a}^T\mathbf{x} \geq 0 \quad &\Rightarrow \quad f_\Omega(\mathbf{x}) = -\log(\mathbf{a}^T\mathbf{x}) \\
\Omega : \|\mathbf{A}\mathbf{x} - \mathbf{b}\| \leq \sigma \quad &\Rightarrow \quad f_\Omega(\mathbf{x}) = -\log(\sigma^2 - \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2)
\end{aligned}$$



f is a ν-self-concordant barrier if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$ and $|\varphi'(t)| \leq \sqrt{\nu}\varphi''(t)^{1/2}$

Constrained convex problems

$$g^* := \min_{\mathbf{x}\in\Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;

- Examples:

$$\begin{aligned}
\Omega : \mathbf{X} \succeq 0 \quad &\Rightarrow \quad f_\Omega(\mathbf{X}) = -\log\det(\mathbf{X}) \\
\Omega : \mathbf{a}^T\mathbf{x} \geq 0 \quad &\Rightarrow \quad f_\Omega(\mathbf{x}) = -\log(\mathbf{a}^T\mathbf{x}) \\
\Omega : \|\mathbf{A}\mathbf{x} - \mathbf{b}\| \leq \sigma \quad &\Rightarrow \quad f_\Omega(\mathbf{x}) = -\log(\sigma^2 - \|\mathbf{A}\mathbf{x} - \mathbf{b}\|^2)
\end{aligned}$$

- Main idea: ***solve a sequence of composite self-concordant problems***

$$\min_{\mathbf{x}\in\mathrm{int}(\Omega)} \{F(\mathbf{x}; t) := g(\mathbf{x}) + tf(\mathbf{x})\}$$

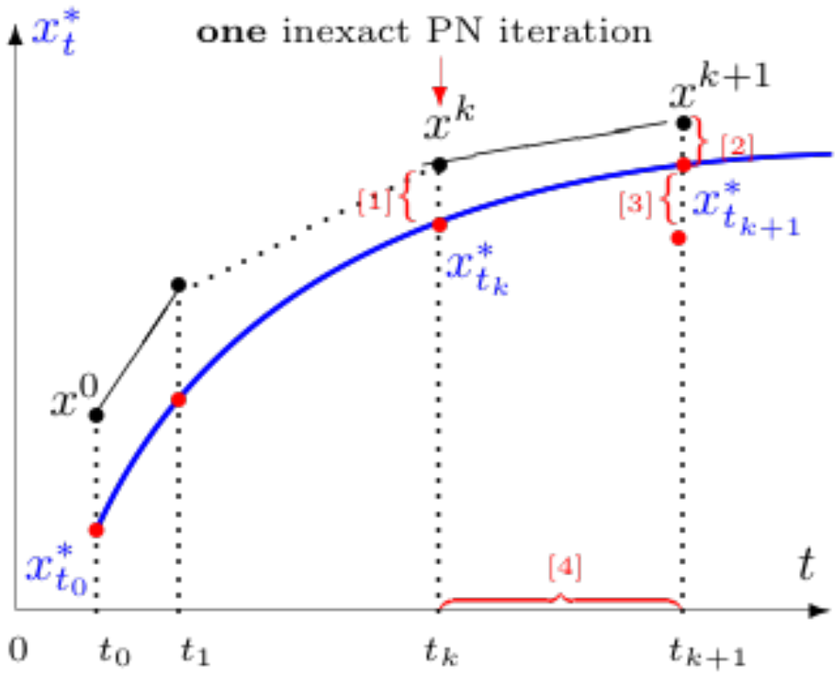
---

f is a ν-self-concordant barrier if $\varphi(t) := f(\mathbf{x} + t\mathbf{d})$ satisfies $|\varphi'''(t)| \leq 2\varphi''(t)^{3/2}$ and $|\varphi'(t)| \leq \sqrt{\nu}\varphi''(t)^{1/2}$

How does it work?

Main idea: solve a sequence of composite self-concordant problems as opposed to DCO

$$\min_{\mathbf{x}\in\mathrm{int}(\Omega)} \{F(\mathbf{x};t) := g(\mathbf{x}) + tf(\mathbf{x})\}$$



$[1] : \|x^k - x^*_{t_k}\|_{x^*_{t_k}}$
$[2] : \|x^{k+1} - x^*_{t_{k+1}}\|_{x^*_{t_{k+1}}}$
$[3] : \|x^*_{t_{k+1}} - x^*_{t_k}\|_{x^*_{t_k}}$
$[4] : \Delta t_k := |t_{k+1} - t_k|$

The true solution trajectory $x^*_{t_k}$

Approximate solution sequence $\{x^k\}$

One iteration **k** requires two updates **simultaneously**:

- Update the penalty parameter:

$$t_{k+1} := (1-\sigma_k)t_k, \quad \sigma_k \in [\underline{\sigma}, 1) \quad (\text{e.g.}, \underline{\sigma} = 0.0337/\sqrt{\nu}).$$

- Update the iterative vector (*can be solved approximately*):

$$\mathbf{x}^{k+1} := \mathrm{argmin}_{\mathbf{x}} \left\{ t_{k+1}\nabla f(\mathbf{x}^k)^T(\mathbf{x}-\mathbf{x}^k) + \frac{t_{k+1}}{2}(\mathbf{x}-\mathbf{x}^k)^T\nabla^2 f(\mathbf{x}^k)(\mathbf{x}-\mathbf{x}^k) + g(\mathbf{x}) \right\}$$

Details:** "An Inexact Proximal Path-Following Algorithm for Constrained Convex Minimization," **optimization-online and lions.epfl.ch/publications.

How does it converge?

The algorithm consists of **two** PHASES:

- **Phase I**: Find a **starting point** $\mathbf{x}_{p_2}^0$ such that: $\|\mathbf{x}_{p_2}^0 - \mathbf{x}^*(t_0)\|_{\mathbf{x}^*(t_0)} \leq 0.05$

- **Phase II**: Perform the ***path-following iterations***

Tracking properties on the penalty parameter and the iterative sequence in **Phase II**

- The penalty parameter t_k **decreases** at least with a factor $\tau := 1 - \dfrac{0.0337}{\sqrt{\nu}}$ at each iteration **k**

$$t_{k+1} = \tau t_k$$

- **Tracking error** of the iterative sequence:

$$\text{if } \boxed{\|\mathbf{x}^k - \mathbf{x}^*(t_k)\|_{\mathbf{x}^*(t_k)} \leq 0.05} \text{ then } \boxed{\|\mathbf{x}^{k+1} - \mathbf{x}^*(t_{k+1})\|_{\mathbf{x}^*(t_{k+1})} \leq 0.05}$$

Worst-case complexity:

- **Phase I**: Finding a starting point for **Phase II** requires at most

$$j_{\max} := \left\lfloor \frac{F(\mathbf{x}^0; t_0) - F(\mathbf{x}^*(t_0); t_0)}{0.0012} \right\rfloor + 1$$

- **Phase II**: The **worst-case complexity** to reach an ε**- solution** is at most:

$$\mathcal{O}\left(\sqrt{\nu} \log\left(\frac{Ct_0}{\varepsilon}\right)\right)$$

- **Note:** This worst-case complexity is as the **same** as in *standard path-following methods* [see Nesterov2004]

Proximal path-following

Upshot: no-heavy lifting!

Proximal path following for conic programming with rigorous guarantees

$$g^* := \min_{\mathbf{x}\in\Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;

Example: Low-rank SDP matrix approximation ...

$$\begin{array}{ll} \min\limits_{\mathbf{X}} & \rho\|\mathrm{vec}(\mathbf{X}-\mathbf{M})\|_1 + (1-\rho)\mathrm{tr}(\mathbf{X}) \\ \text{s.t.} & \mathbf{X}\succeq 0,\ \mathbf{L}_{ij}\le \mathbf{X}_{ij}\le \mathbf{U}_{ij},\ i,j=1,\ldots,n. \end{array}$$

#variables
#constraints

ρ is a regularization parameter in (0, 1), **M** is the given input matrix.

	Solver\n	80	100	120	140	160
Size	$[n_v; n_c]$	[16,200; 9,720]	[25,250; 15,150]	[36,300; 21,780]	[49,350; 29,610]	[64,400; 38,640]
Time (sec)	PFPN	15.738	24.046	24.817	25.326	36.531
	SDPT3	156.340	508.418	881.398	1742.502	2948.441
	SeDuMi	231.530	970.390	3820.828	9258.429	17096.580
$g(\mathbf{X}^*)$	PFPN	306.9159	497.6706	635.4304	842.4626	1096.6516
	SDPT3	306.9153	497.6754	635.4306	842.4644	1096.6540
	SeDuMi	306.9176	497.6821	635.4384	842.4776	1096.6695
[rank, sparsity]	PFPN	[20, 30.53%]	[26, 27.37%]	[30, 25.27%]	[35, 23.64%]	[40, 21.54%]
	SDPT3	[20, 41.02%]	[25, 36.99%]	[30, 51.61%]	[35, 45.03%]	[40, 49.07%]
	SeDuMi	[20, 45.23%]	[25, 64.20%]	[30, 54.83%]	[35, 60.87%]	[40, 59.24%]

Proximal path-following

Upshot: desired scaling!

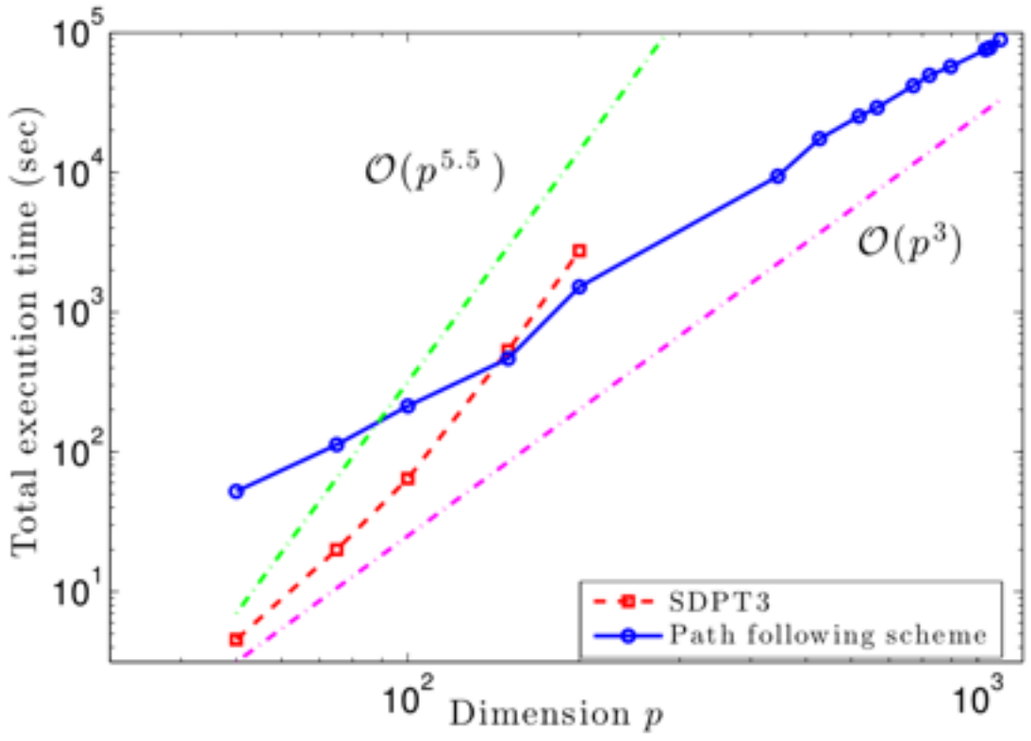
Proximal path following for conic programming with rigorous guarantees

$$g^* := \min_{\mathbf{x}\in\Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;

Example: Max-norm clustering

$$\begin{array}{ll} \min\limits_{\mathbf{L},\mathbf{R},\mathbf{K}} & \|\mathrm{vec}\,(\mathbf{K}-\mathbf{A})\|_1 \\ \text{s.t.} & \begin{bmatrix} \mathbf{L} & \mathbf{K} \\ \mathbf{K}^T & \mathbf{R} \end{bmatrix} \succ 0,\ \mathbf{L}_{ii} \le 1,\ \mathbf{R}_{ii} \le 1,\ i = 1,\ldots,p. \end{array}$$



DCO:

	SDPT3		PF scheme
p	variables	constraints	variables
50	15.1	2.6	10
75	33.9	5.8	22.5
100	60.2	10.2	40
150	135.3	22.8	90
200	240.4	40.4	160

	p	50	75	100	150	200
Time (sec)	PF	62.450	109.426	202.600	416.044	1573.881
	SDPT3	4.396	21.282	64.939	522.021	2588.721
	splitting	102.217	236.366	354.444	778.904	1420.844
$g(\mathbf{K}^*)$	PF	549.1567	1293.6727	2232.5897	5396.0485	9809.6066
	SDPT3	549.1860	1293.7890	2233.0747	5396.7305	9809.6934
	splitting	597.8825	1387.1379	2496.6535	5583.8605	9958.0974

Proximal path-following **Upshot: auto. regularization!**

Proximal path following for conic programming with rigorous guarantees

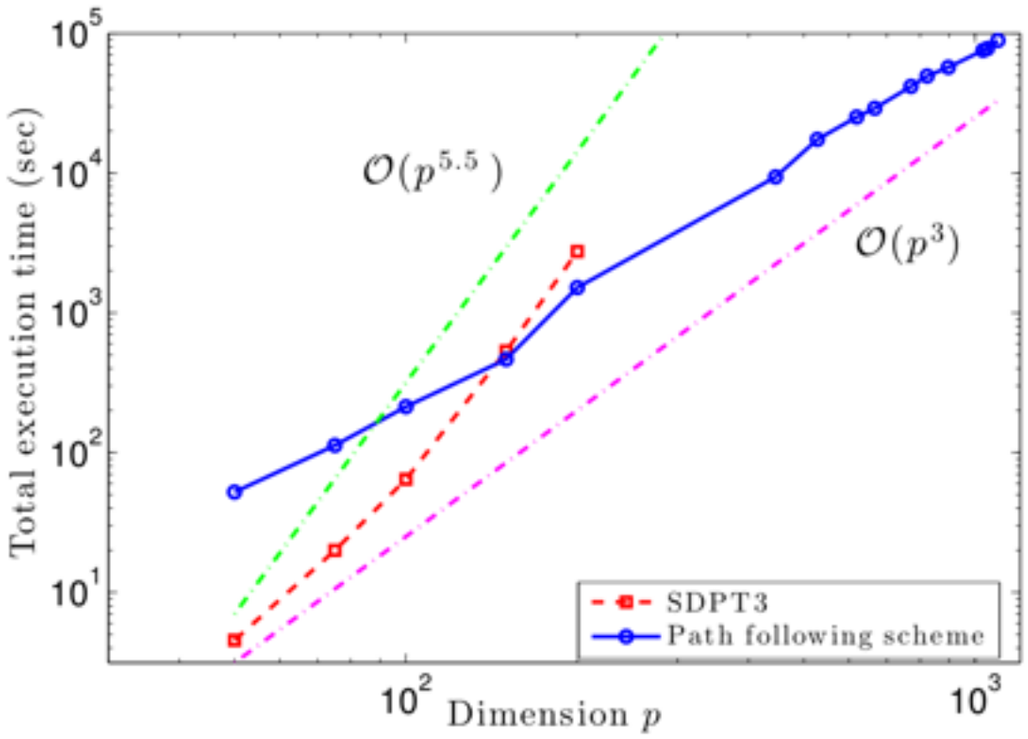
$$g^* := \min_{\mathbf{x}\in\Omega} g(\mathbf{x})$$

- Ω is endowed with a self-concordant barrier $f(x)$;

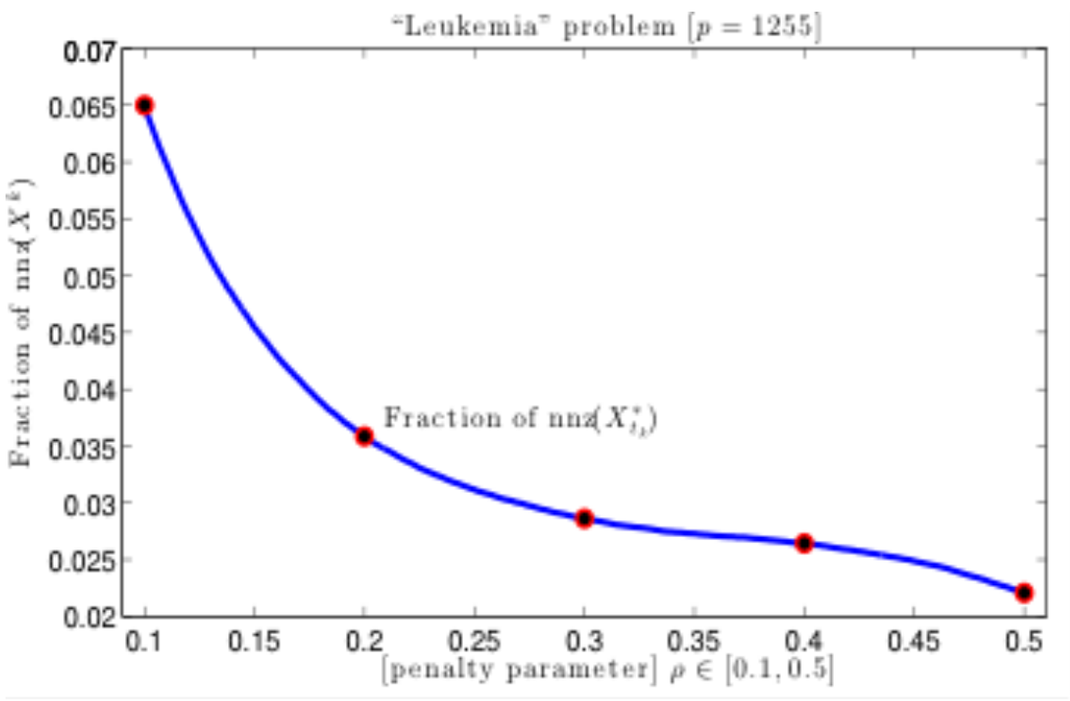
- Main idea: solve a sequence of composite self-concordant problems as opposed to DCO

$$\min_{\mathbf{x}\in\mathrm{int}(\Omega)} \{F(\mathbf{x};t) := g(\mathbf{x}) + tf(\mathbf{x})\}$$

Example: Max-norm clustering



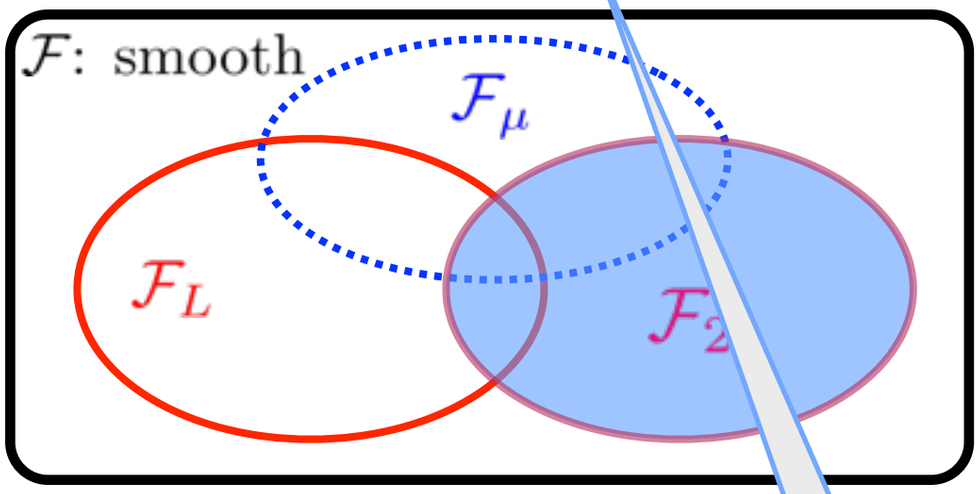
Example: Graph selection



Conclusions

Conclusions

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



- $\mathcal{F}_\mu$ - μ-strongly convex
- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_2$ - self-concordant

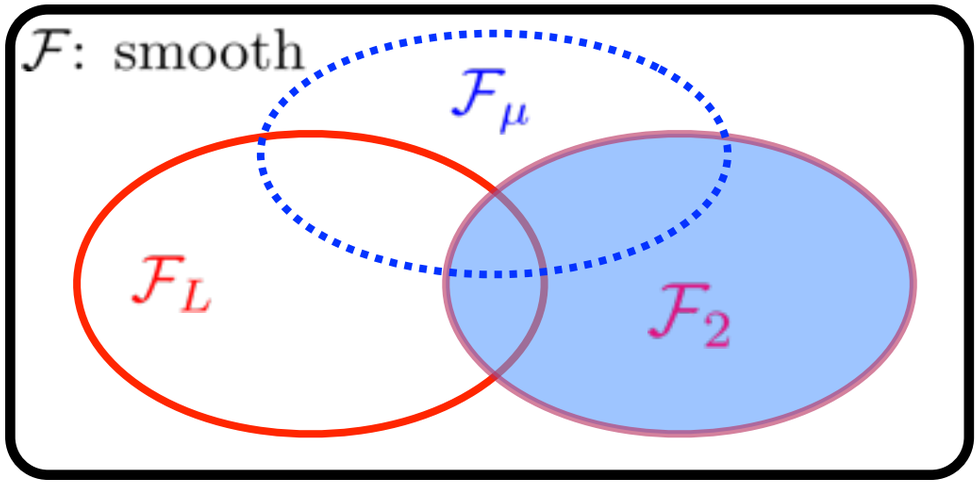
A new variable metric proximal-point framework for composite self-concordant minimization

+

Extensions

Conclusions

$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



- $\mathcal{F}_\mu$ - μ-strongly convex
- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_2$ - self-concordant

- Highlights

- **Globalization**: a new strategy for finding step-size **explicitly**

strategy

- **Search direction**: motivate "*forward-looking*" line-search

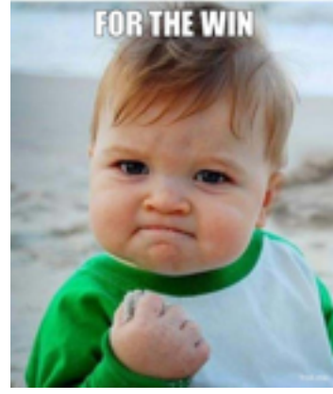
- **Local convergence**: efficient (strongly convex program)

Hessian

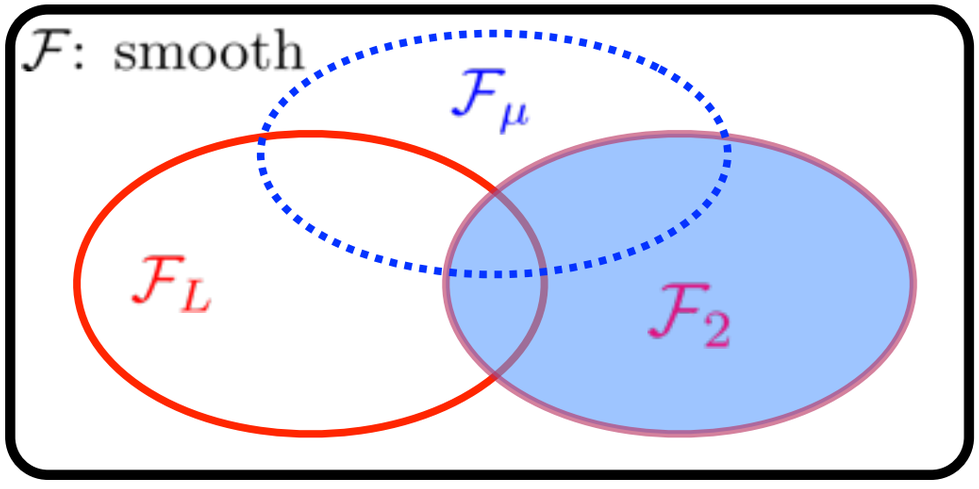
quadratic convergence *without boundedness of the*

analytic quadratic convergence region

Conclusions



$$\min_{\mathbf{x}\in\mathbb{R}^n} \{\phi(\mathbf{x}) := f(\mathbf{x}) + g(\mathbf{x})\}$$



- $\mathcal{F}_\mu$ - μ-strongly convex
- $\mathcal{F}_L$ - L-Lipschitz gradient
- $\mathcal{F}_2$ - self-concordant

- Highlights

- **Globalization**: a new strategy for finding step-size **explicitly**
- **Search direction**: motivate "*forward-looking*" line-search strategy
- **Local convergence**: efficient (strongly convex program) quadratic convergence *without boundedness of the Hessian*

analytic quadratic convergence region

- Practical contributions (this talk)

  - SCOPT package has quasi-Newton / first & second order methods @lions.epfl.ch/software
  - leverage fast proximal solvers for g(x) (structured norms etc.)
  - robust to subproblem solver accuracy

SCOPT FTW

- Postdoc & PhD positions @ LIONS / EPFL

contact: volkan.cevher@epfl.ch