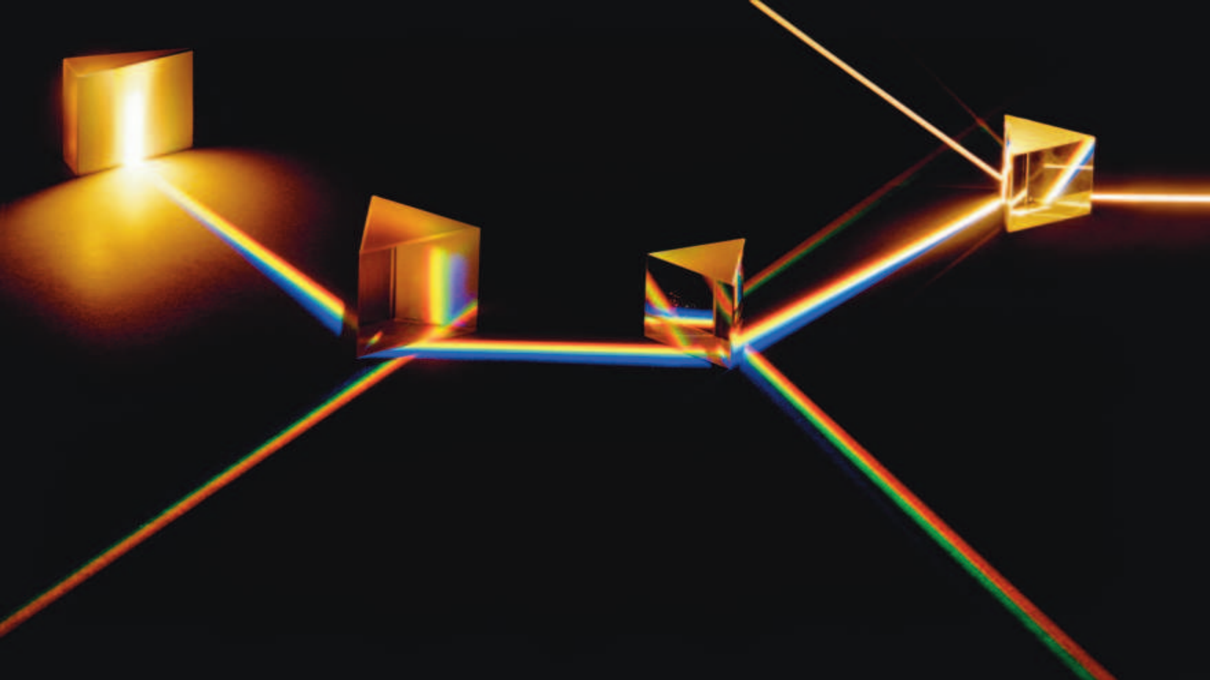
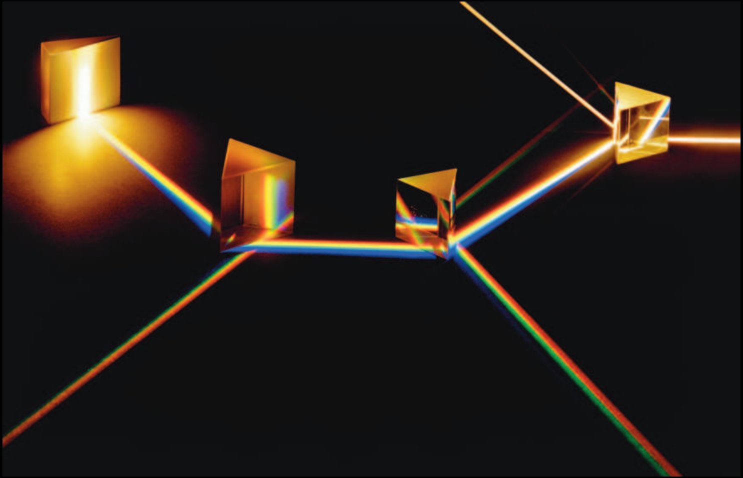
Matheon Workshop – December 12, 2013





Inverse Problems Regularized by Sparsity

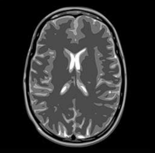
Martin Vetterli

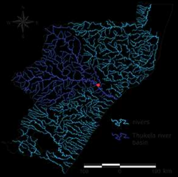
"Far better **an approximate answer to the right question**, *which is often vague, than* **an exact answer to the wrong question**, *which can always be made precise."*

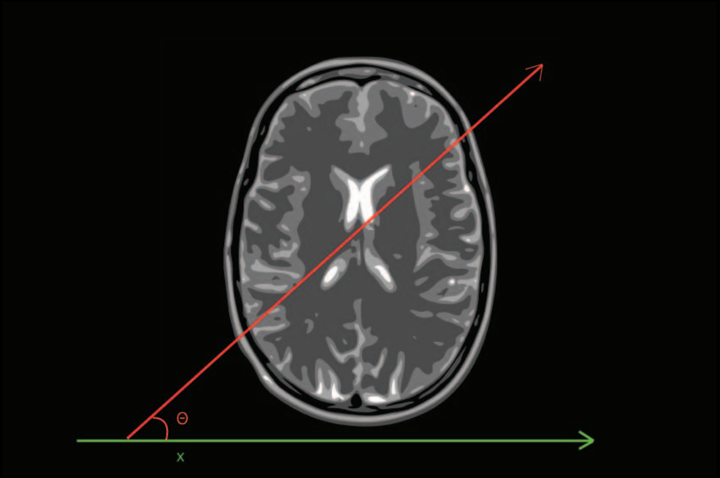
— John Tukey

Overview:

- Introduction: Sparsity is good for you!
- Can one hear the shape of a room?
- Can one localize inside a room?
- Can one put sensors optimally?
- Can one know the nuclear fallout of Fukushima?
- Conclusions
- Acknowledgements




rivers
Thukela river basin
100
0
100 km



Introduction: Sparsity is good for you!

Inverse problems

Have a long history.... Hadamard 1915!

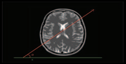
The inverse problem $\mathbf{y} = \mathbf{A}\mathbf{x}$

- is well posed when
  - Existence: $\forall y, \exists x$ s.t. $y = Ax$.
  - Uniqueness: $Ax_1 = Ax_2 \Rightarrow x_1 = x_2$
  - Stability: A^{-1} is continuous.
- otherwise... ill posed!

Finite dimensional, linear case

- Existence: Least squares
- Uniqueness: Minimum norm solution
- Stability: Condition number reasonable

Inverse problems: The tomography example



- Forward model

$$y(\alpha_0, \theta_0) = \int_{-\infty}^{\infty} x(\alpha + \alpha_0, \alpha \tan \theta_0)\ d\alpha$$

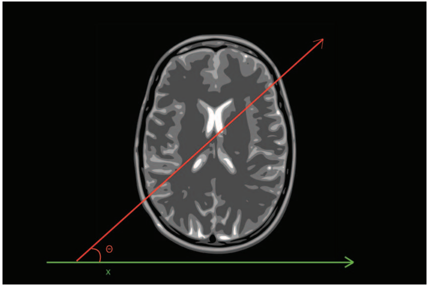
- Inverse

$$\hat{x}(\alpha, \beta) \quad \text{s.t.} \quad \hat{y}(\alpha, \beta) \simeq y(\alpha, \beta)$$

- Usually discretized, for some set $\{\alpha_n, \theta_m\}$

$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

- Noise, condition number, complexity etc
- Priors.... Sparsity!

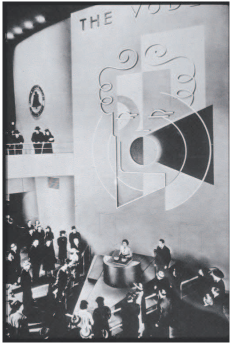


Tomography

Sparsity is good for you!

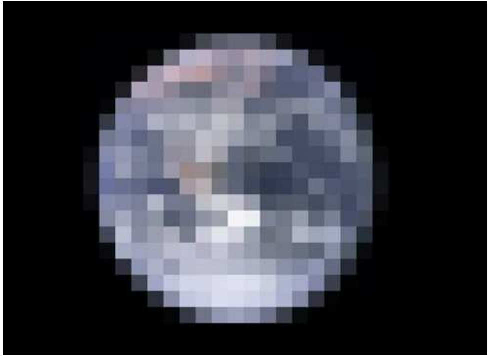
A bit of history on the topic

- Occam's razor
- Parametric signal processing
  - Sinusoidal retrieval
- Regularizations
  - Tikhonov: $\|Ax - y\|_2^2 + \lambda \|Ox\|_2^2$
  - ℓ_1 regularization
- Uncertainty principles:
  Sparse in one world, not in the other!



Pedro The Voder, 1939

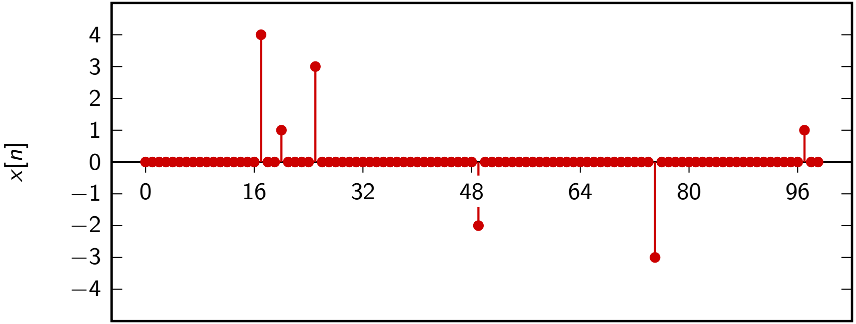
Modeling: Two views of the world



Discrete/digital

Analog/continuous

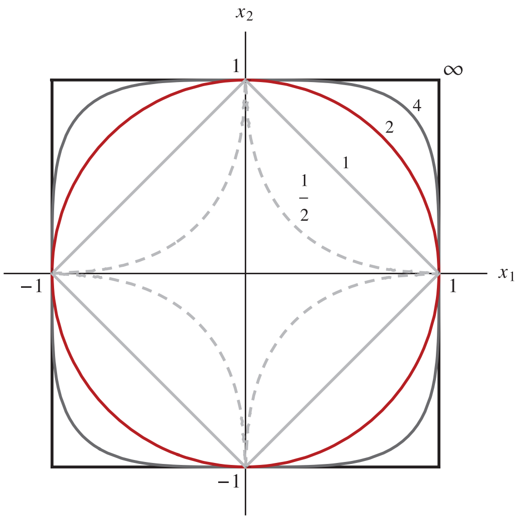
A sparse discrete-time signal



$\mathrm{N} = 100, \ \mathrm{k} = 6, \ \mathrm{M} = 20$

The world looks different using different norms!

Unit balls in different norms: quasinorm $\ell_{1/2}$, norms $\ell_1, \ell_2, \ell_4, \ell_\infty$



Solution of linear systems using different norms 1/2

- Consider an under-determined system of equations

$$\mathbf{x} = \mathbf{A}\alpha$$

where $\mathbf{x}$ is $N \times 1$, $\mathbf{A}$ is $N \times M$, α is $M \times 1$ and $N < M$.

- Expansion with respect to an overcomplete set of vectors is not unique.
- Example:

$$x = \frac{1}{5} \cdot [1\ 2] \cdot \begin{bmatrix} \alpha_0 \\ \alpha_1 \end{bmatrix}$$

$$\alpha' = \alpha + \alpha^{\perp} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 2 \\ -1 \end{bmatrix} \gamma,$$

This is a line with slope $-1/2$ in the $[\alpha_0, \alpha_1]$ plane.

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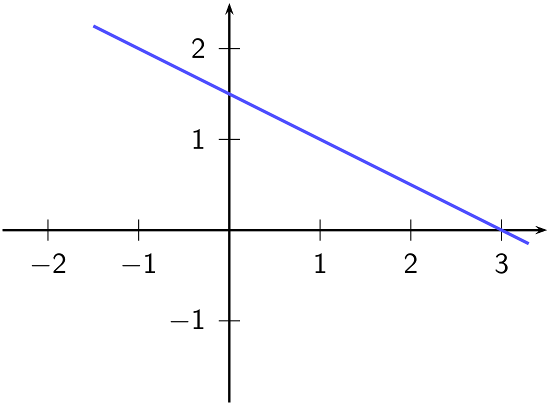
Different norm minimizations $\|\alpha\|_p$, $p \in \{0, 1, 2\}$ give different solutions (Ex: $x = 3/5$)



and one of them is sparse!

Solution of linear systems using different norms 2/2

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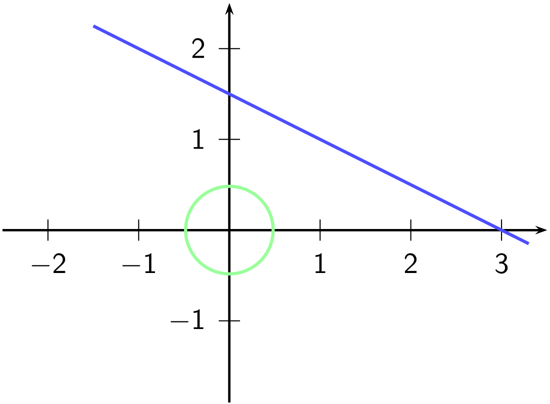

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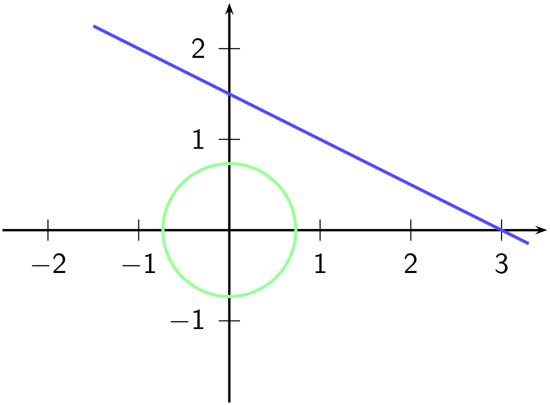

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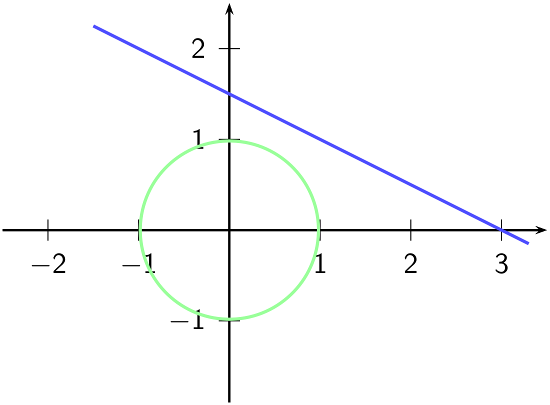

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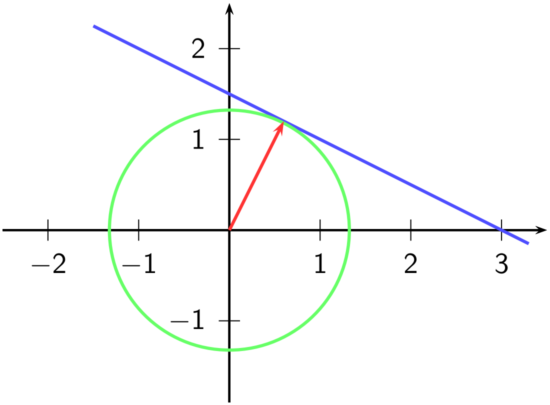

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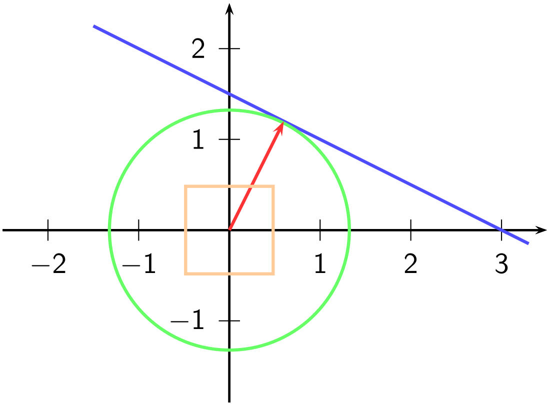

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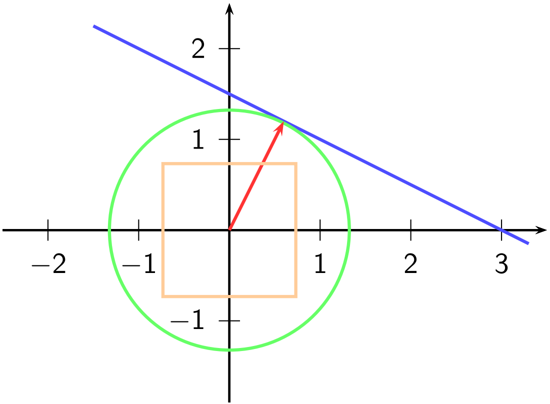

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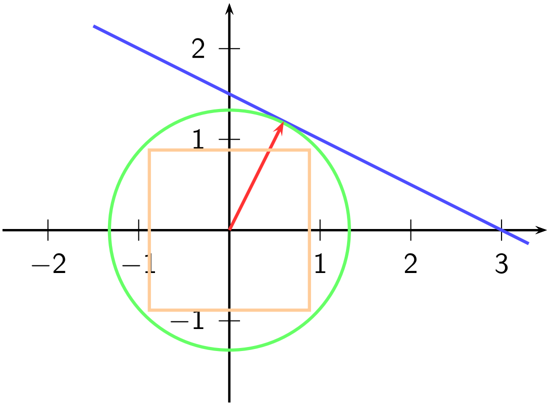

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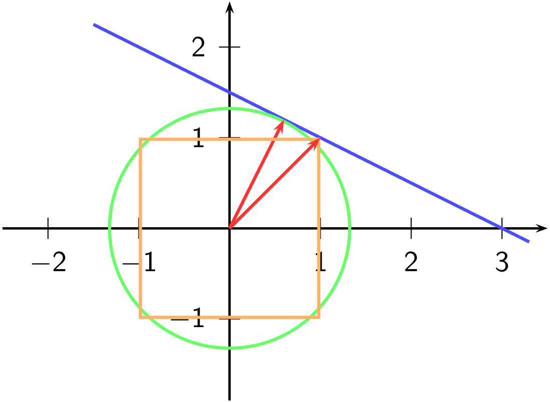

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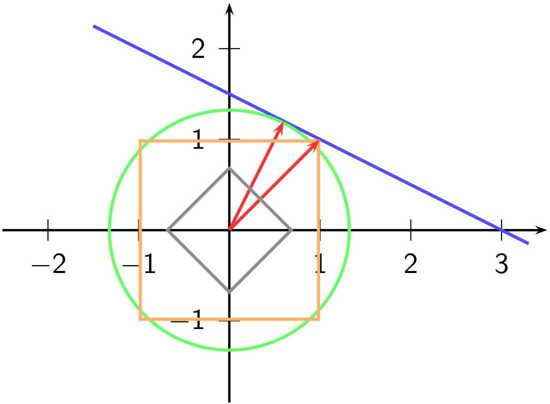

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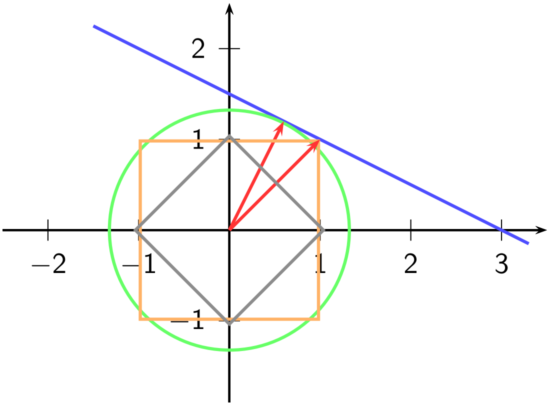

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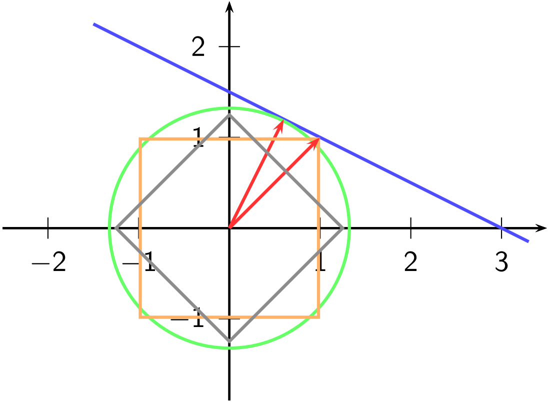

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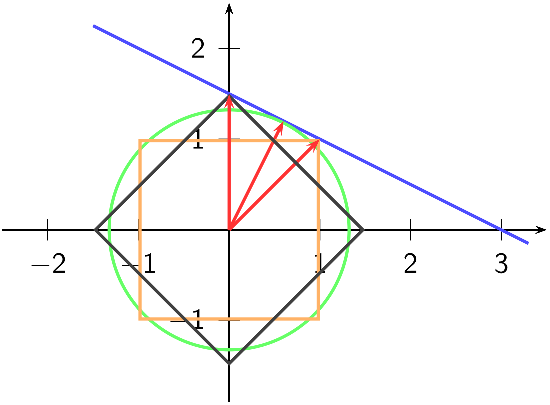

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There is ℓ_1 magic!

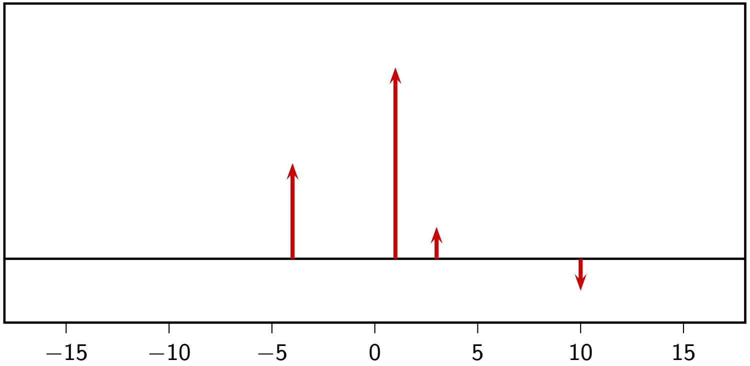
Many ways to skin that cat

- Classic solution: Tikhonov regularization $\|Ax - y\|_2^2 + \lambda\|x\|_2^2$
- Modern solutions: ℓ_1 regularization and convex optimization
  - Linear program: $\min\|x\|_1$ s.t. $Ax = y$
  - Lasso: $\min(\|Ax - y\|_2^2 + \lambda\|x\|_1)$
  - Equivalence of ℓ_0 and ℓ_1: Restricted isometry property, spark, $\mu(A)$
- Geometry: Norm conservation under K-sparsity

$$(1-\delta)\|x\|_2^2 \leq \|Ax\|_2^2 \leq (1+\delta)\|x\|_2^2 \qquad \text{for } \|x\|_0 \leq K$$

- Efficient, N^3 algorithm
- Performance guarantees, also in the presence of noise

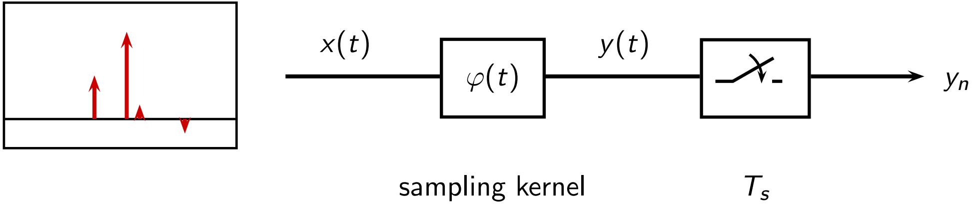
A sparse continuous-time signal



Finite rate of innovation (FRI) sampling

- Nyquist sufficient but not necessary!
- Sinusoidal retrieval ideas can be used for sampling sparse CT signals
- Sharp sampling results for FRI signals
- Efficient algorithms

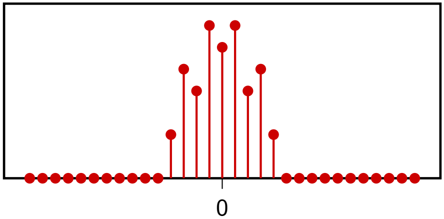
Gaspard de Prony (1755 - 1839)



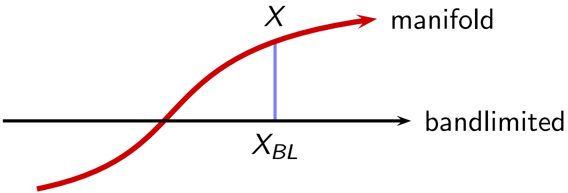
Finite rate of innovation (FRI) sampling

For a sparse input, e.g. weighted sum of Diracs

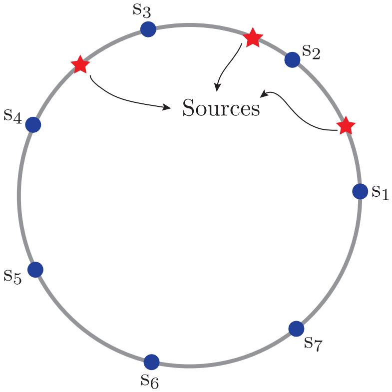
- One-to-one map $y_n \Longleftrightarrow x(t)$
- Efficient, complexity $O(K^3)$ algorithms, where K is sparsity
- Stable reconstruction
- Some robustness to noise
- Optimality of recovery (CRB)



FS coefficients



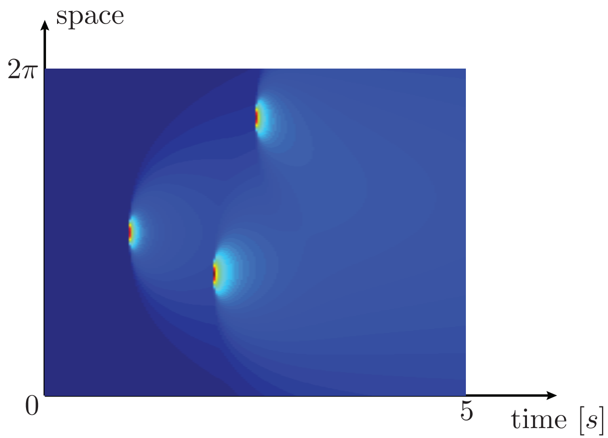
An example: Fourier's original problem, the copper ring!



- 3 heat sources
- 7 measurement points

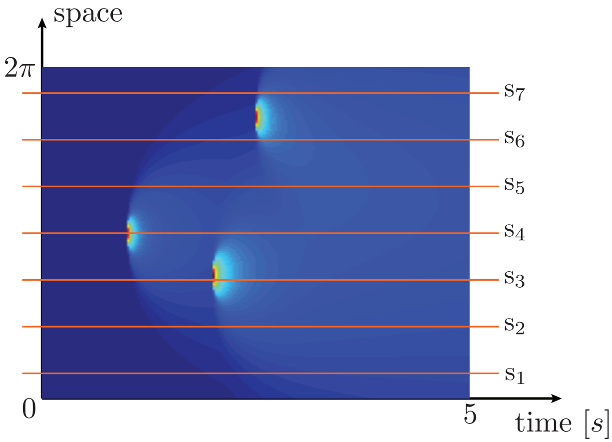
Temperature evolves according to the heat equation

The forward problem



- A space-time evolution of the temperature field.
- The 7 sensor measuring the field at fixed locations.

The forward problem



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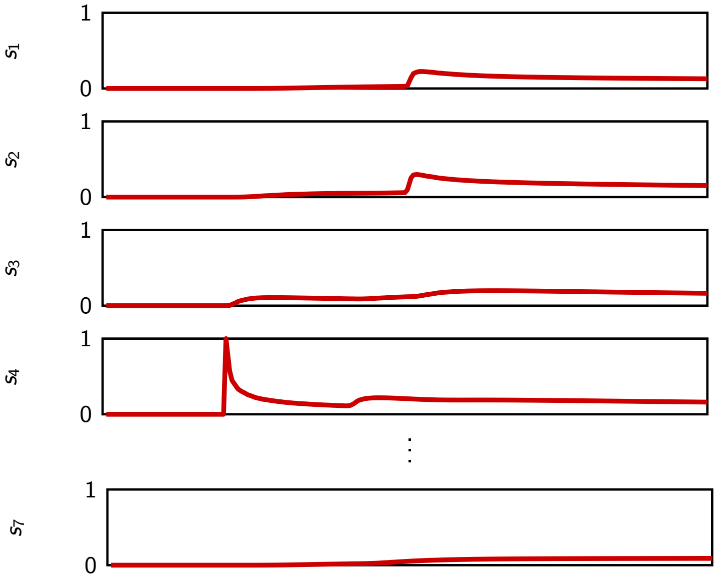
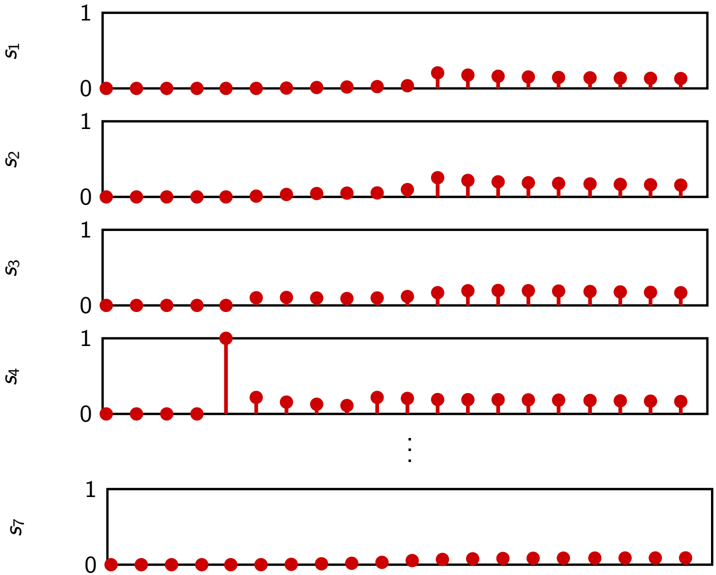
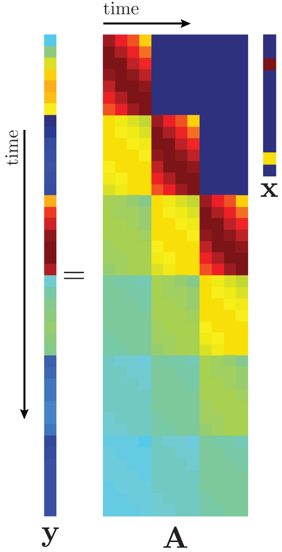
The measurements: Continuous time



The measurements: Discrete time



The compressed sensing approach

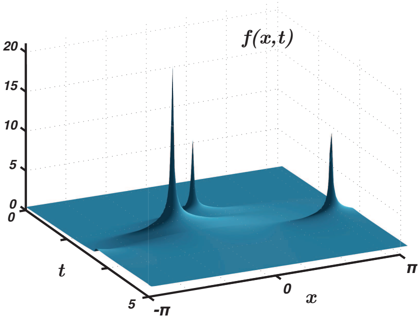


- Source locations are discretized
- Time is discretized
- Sources are sparse in time and space
- Recovery by ℓ_1 minimization

$$\min \quad \|\mathbf{x}\|_1 \quad \text{s.t. } \mathbf{y} = \mathbf{A}\mathbf{x}$$

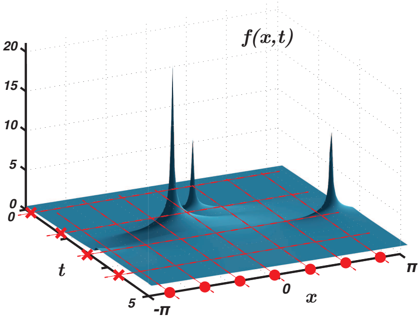
- **A** has a high coherence: no guarantees
- In practice, the quality depends on the resolution

The parametric (FRI) approach



- K unknown sources induce a field $f(x,t)$: $s(x,t) = \sum_{k=1}^{K} c_k \delta(x - x_k)\delta(t - t_k)$
- L sensors sample the diffusive field $f(x,t)$
- We locate the sources on $\mathbb{R}^2$ by FRI reconstruction

The parametric (FRI) approach



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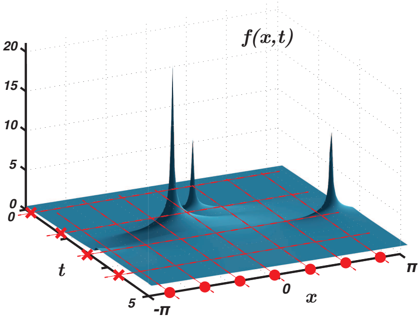

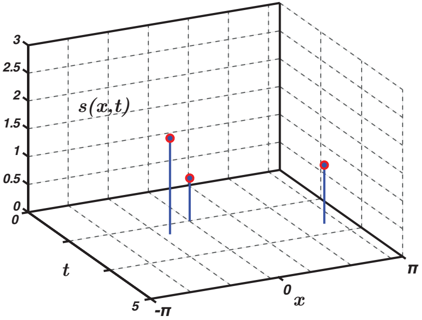

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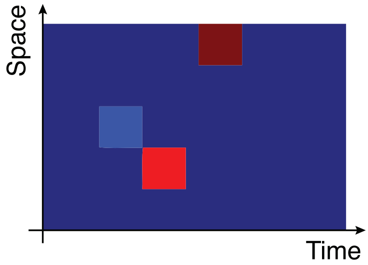
Comparison

Compressed sensing

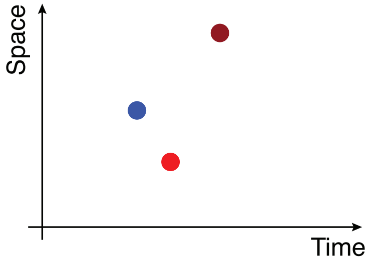
- Sources on discrete domain



- Solution by convex programming
- Resolution is computationally expensive
- When are we guaranteed to have a correct solution?

Parametric (FRI) approach

- Sources on continuous domain



- Solution by annihilation filter
- Numerical stability can be an issue
- How can we find reliably the time of appearance?

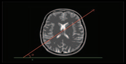
Our various problems involve different PDEs

The four questions have different forward models

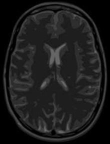
- Can one hear the shape of a room?
  - Location of acoustic sources in 3D space: Wave equation
- Can one locate a source on a graph?
  - Location of sources in graphs: Diffusion equation on graphs
- Can one put sensors optimally?
  - Sensors in space: Diffusion equation
- Can one know the nuclear fallout of Fukushima?
  - Sources of pollution: Transport equation

But the principles of sparsity apply to all of them!

Some problems are discrete, and some are not!



Question	Problem	Model	Algorithm
Shape of a room	Virtual sources	Continuous	Continuous
Source in a graph	Vertices	Discrete	Discrete
Sensor placement	Location in space	Discretized	Discrete
Pollution diffusion	Emission	Continuous/Discrete	Discrete






Can one hear the shape of a room?

Ivan Dokmanic

Play

Play

Play

Can one hear the shape of a drum?

The question...

Kac, 1966: "*Can one hear the shape of a drum?*"

...and the answer

Gordon, Webb & Wolpert, 1992: "*One cannot hear the shape of a drum*"

Can one hear the shape of a drum?

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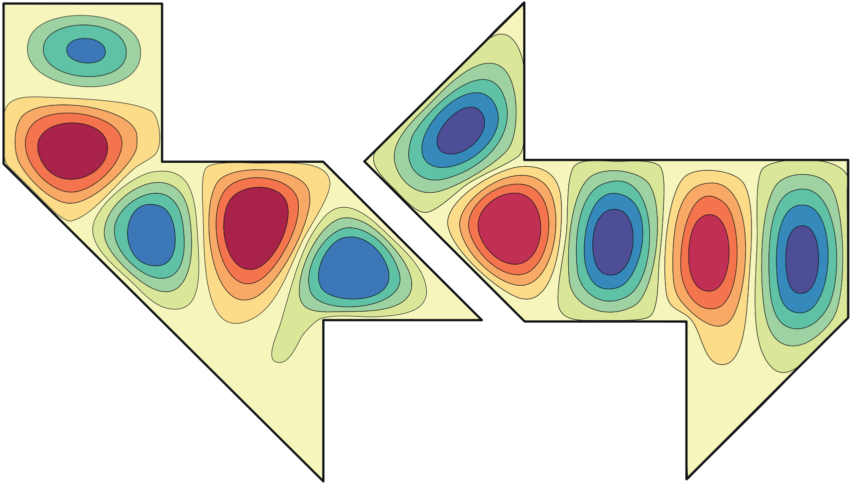
Can one hear the shape of a drum?



Can one hear the shape of a drum?

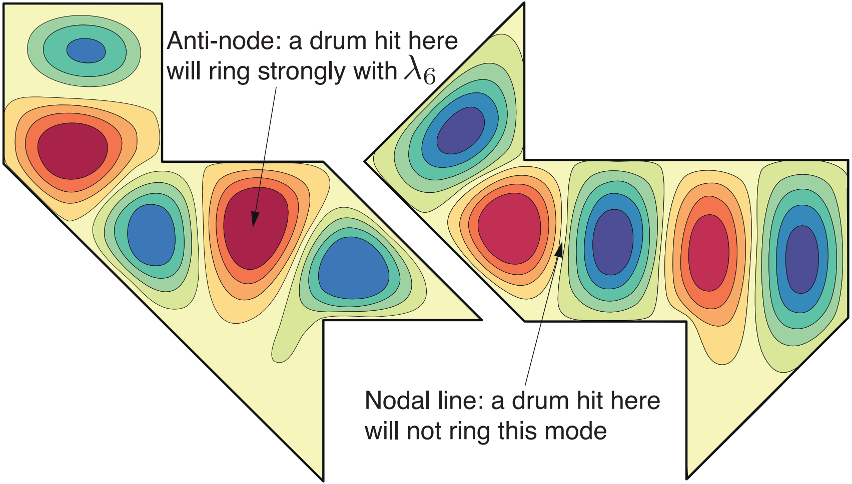


Image source model

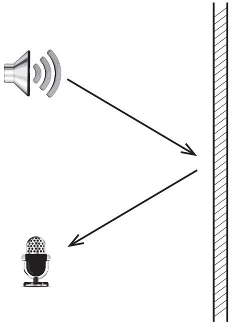
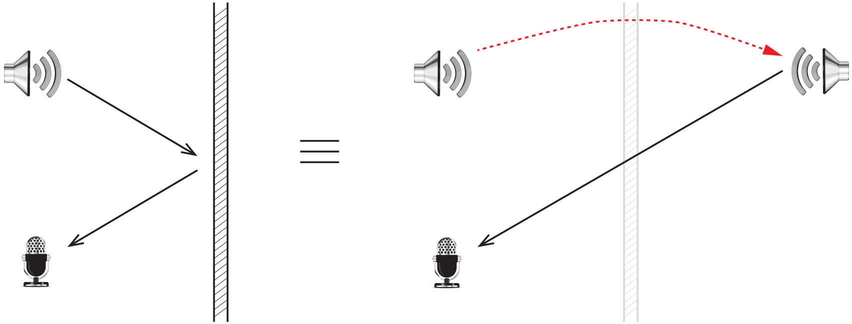


Image source model



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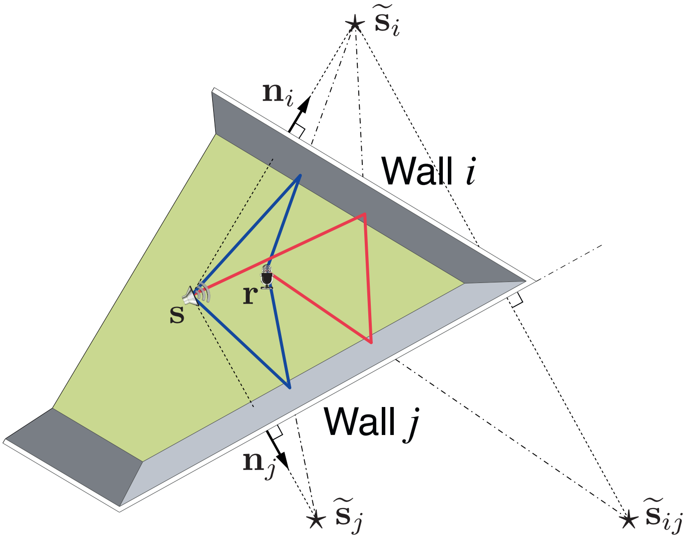
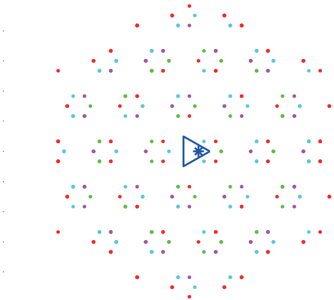
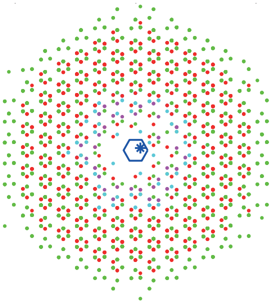


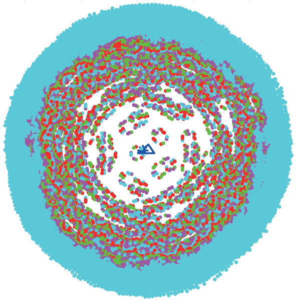
Image source model



Equilateral triangle



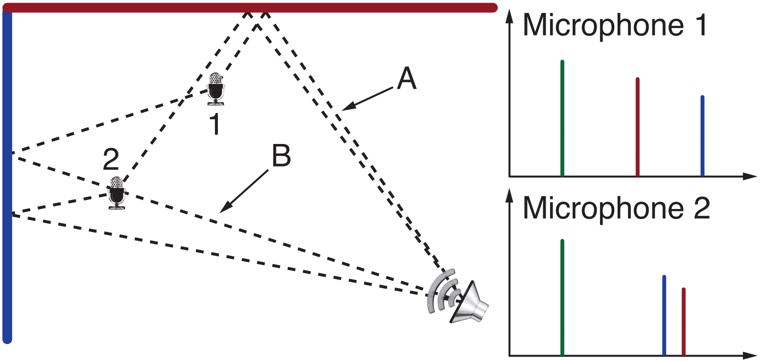
Regular hexagon



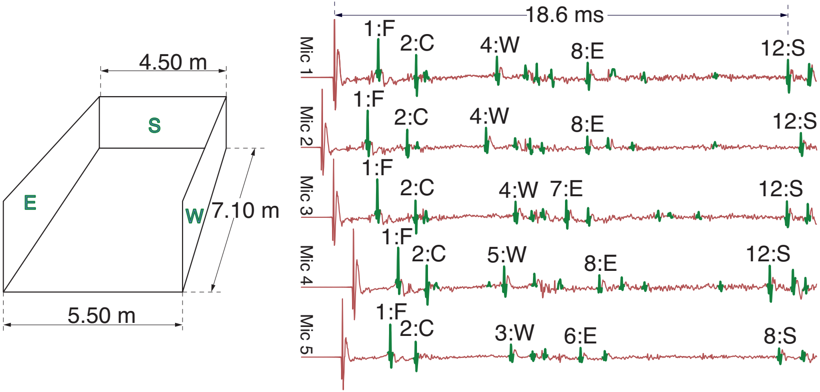
Irregular triangle

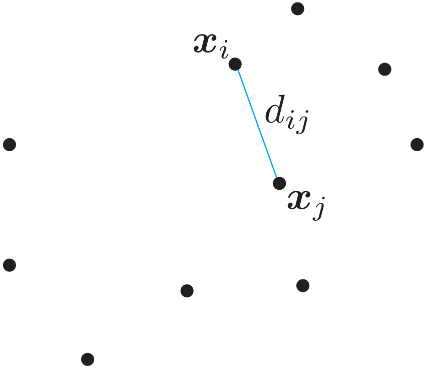
Echo sorting problem

- Echoes arrive at walls in different orders
- Need to label echoes: Echo sorting



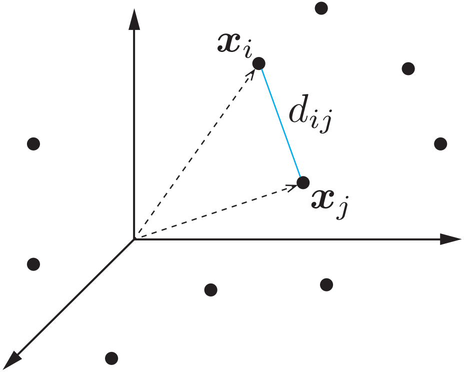
Actual measurements





- What can we say about $(d_{ij})_{i,j=1}^k$ when $\mathbf{x} \in \mathbb{R}^n$?

Rank testing of Euclidean distance matrix (EDM)



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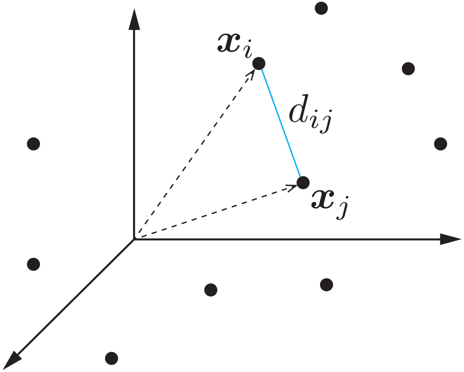
$$d_{ij}^2 = \langle \mathbf{x}_i - \mathbf{x}_j, \mathbf{x}_i - \mathbf{x}_j \rangle$$
$$= \langle \mathbf{x}_i, \mathbf{x}_i \rangle + \langle \mathbf{x}_j, \mathbf{x}_j \rangle - 2\langle \mathbf{x}_i, \mathbf{x}_j \rangle$$

$$g_{ij} \stackrel{\text{def}}{=} \langle \mathbf{x}_i, \mathbf{x}_j \rangle, \ \mathbf{G} \stackrel{\text{def}}{=} (g_{ij})$$

$$\mathbf{D} = -2\mathbf{G} + \mathbf{1}\,\mathrm{diag}(\mathbf{G})^T + \mathrm{diag}(\mathbf{G})\mathbf{1}^T$$

$$\Rightarrow \mathrm{rank}(\mathbf{D}) \le \mathrm{rank}(\mathbf{G}) + 1 + 1$$
$$= n + 2$$

Rank testing of Euclidean distance matrix (EDM)



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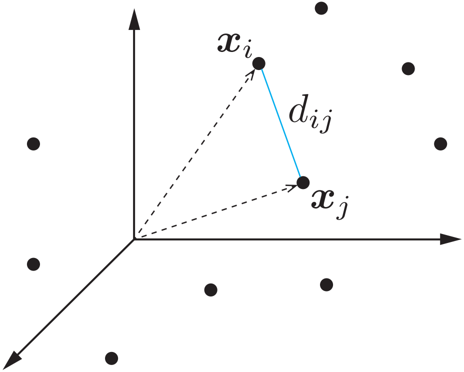

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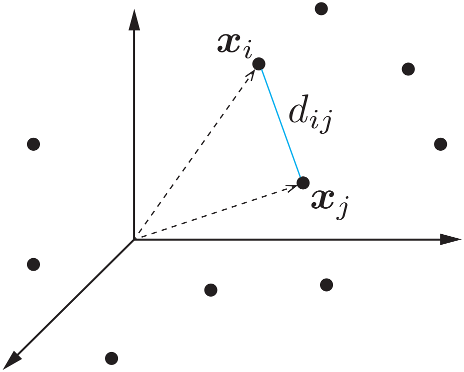

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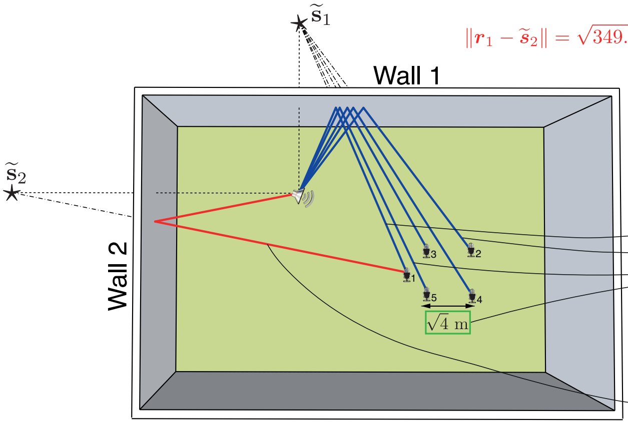
Echo sorting



$\|\boldsymbol{r}_1 - \widetilde{\boldsymbol{s}}_2\| = \sqrt{349.4}$ $\|\boldsymbol{r}_1 - \widetilde{\boldsymbol{s}}_1\| = \sqrt{161.1}$

D (Microphone distances)

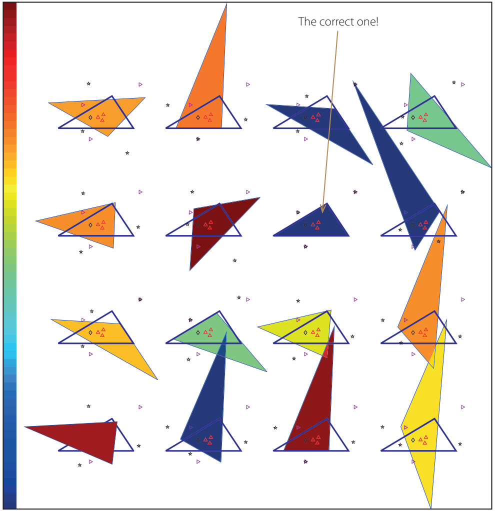
$$\mathbf{D}_{\mathrm{aug},1} = \begin{bmatrix} 0 & 11 & 2 & 10 & 2 & 161.1 \\ 11 & 0 & 5 & 5 & 9 & 178.8 \\ 2 & 5 & 0 & 8 & 4 & 149.8 \\ 10 & 5 & 8 & 0 & 4 & 224.4 \\ 2 & 9 & 4 & 4 & 0 & 196.4 \\ 161.1 & 178.8 & 149.8 & 224.4 & 196.4 & 0 \end{bmatrix}$$

$$\mathbf{D}_{\mathrm{aug},2} = \begin{bmatrix} 0 & 11 & 2 & 10 & 2 & 349.6 \\ 11 & 0 & 5 & 5 & 9 & 178.8 \\ 2 & 5 & 0 & 8 & 4 & 149.8 \\ 10 & 5 & 8 & 0 & \boxed{4} & 224.4 \\ 2 & 9 & 4 & \boxed{4} & 0 & 196.4 \\ 349.6 & 178.8 & 149.8 & 224.4 & 196.4 & 0 \end{bmatrix}$$

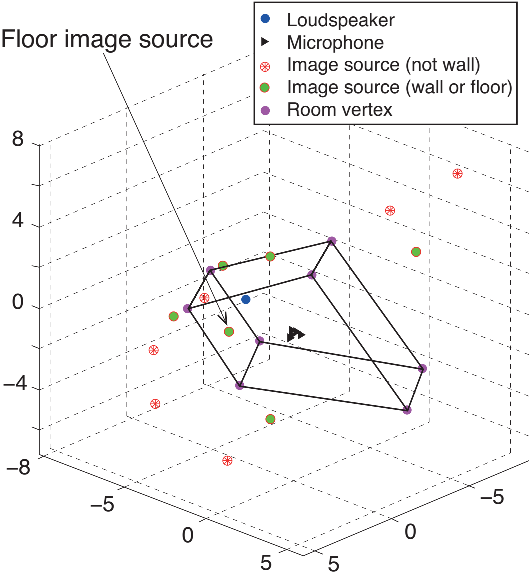
Theorem

Theorem

Consider a room with a loudspeaker and $M \geq 4$ microphones placed uniformly at random inside the feasible region. Then almost surely exactly one assignment of first-order echoes to walls describes a room.

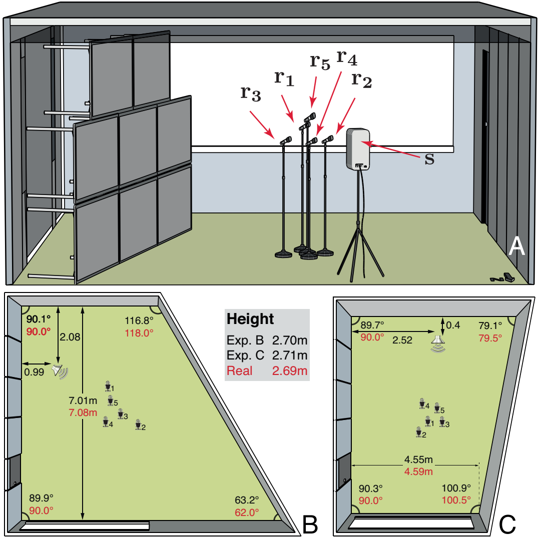


Practical algorithm

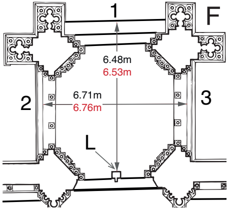


- Imperfect estimation of echo times
- Denoise using multidimensional scaling (MDS)
- Room from image sources?
- Use structure: Higher-order IS combinations of lower-order IS

Experiments



Experiments



Can one hear the shape of a room: Recap

- Continuous problem, continuous formulation
- Model: Convex polyhedron
- Result: Sparsity—(Virtual) sources sparse in space domain
- Key step—"Transformation" of the polyhedron (description using image sources)

2. Can one hear the shape of a room?

Publication:

- Ivan Dokmanic, Reza Parhizkar, Yue Lu, Andreas Walther, MV
- Acoustic echoes reveal room shape
- PNAS, June 17 2013, open access
- Data online `http://lcav.epfl.ch/eCathedral`

Ivan Dokmanic

Reza Parhizkar

Yue Lu

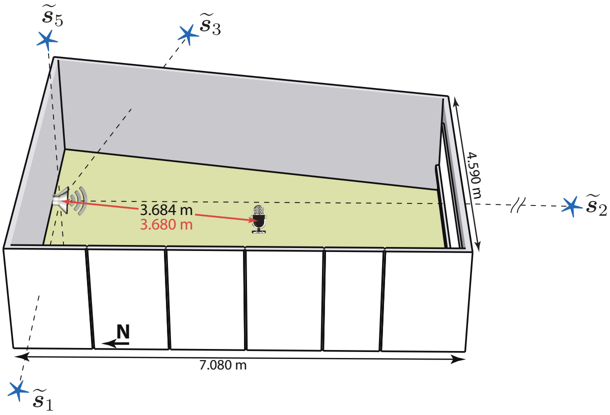
Andreas Walther

Can one localize inside a room?

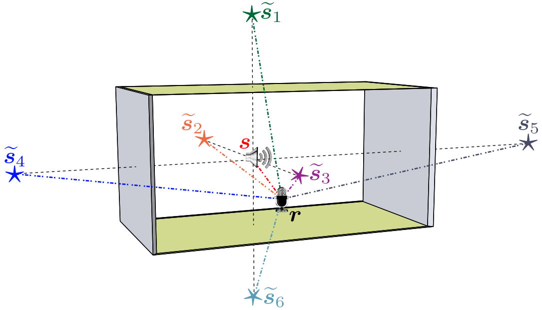
Ivan Dokmanic, Orhan Ocal, Reza Parhizkar

Can one localize inside a room?

- Echoes arrive at walls in different orders
- Need to label echoes: Echo sorting



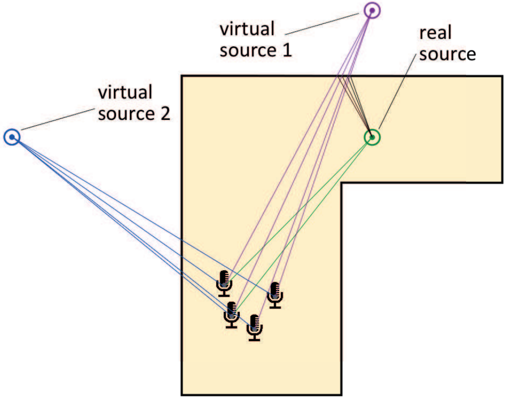
Known room, known source location



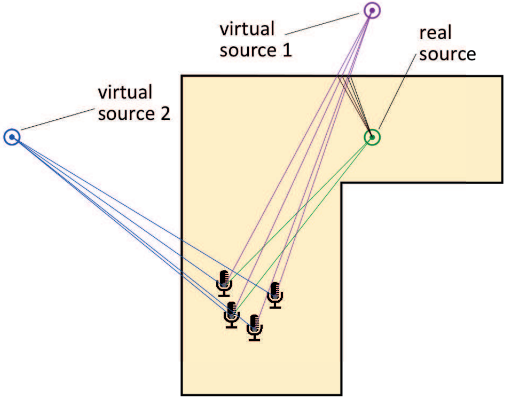
An exercise in echo sorting

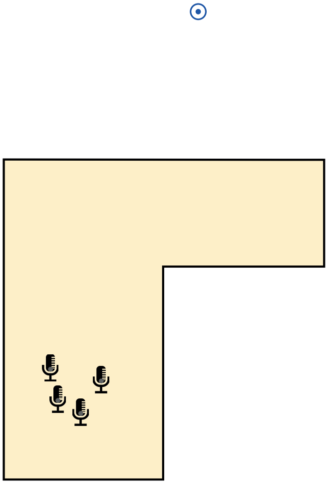
- Image sources = free multilateration measurements (after labeling!)

Localization 2: Nonconvex Rooms



Localization 2: Nonconvex Rooms





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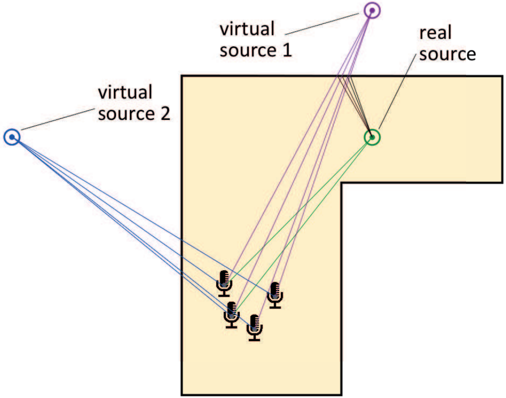

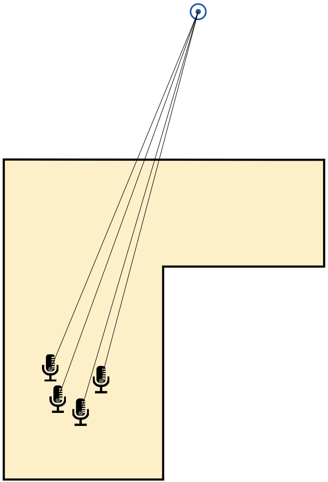

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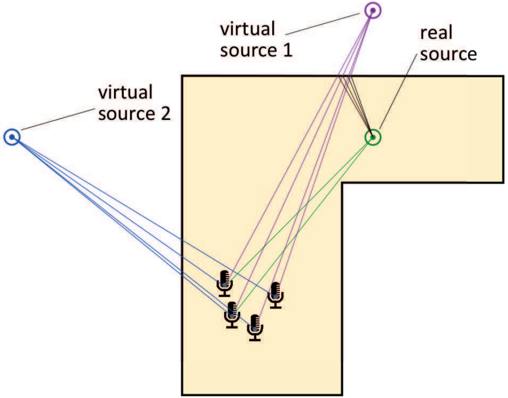

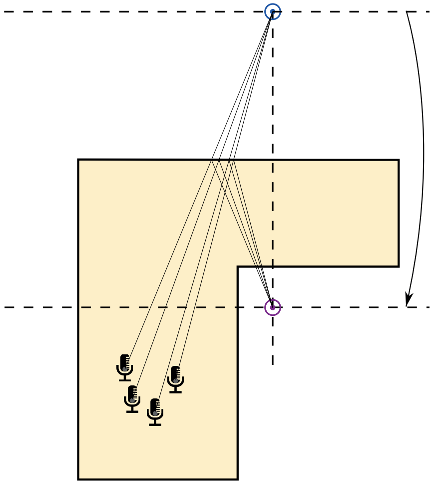

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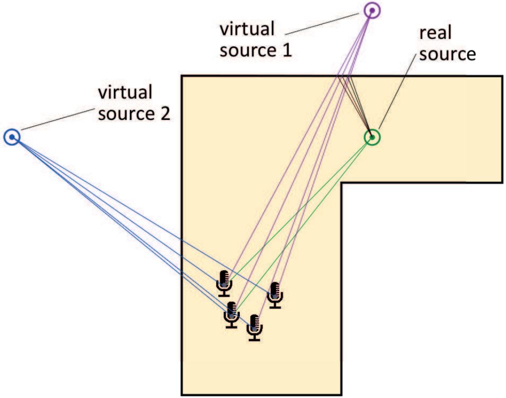
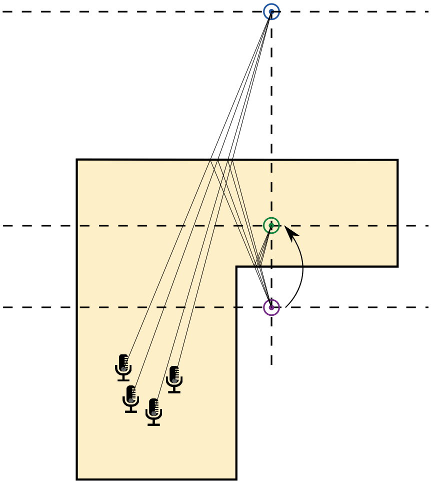




Human Echolocator: Daniel Kish

Blind man who taught himself to see

- Senses distance, shape and density of objects

Active sonar

- Insonifies surroundings by palatal clicks
- **Sees** and navigates by echoes

www.worldaccessfortheblind.org

- Facilitate self-directed achievement of the blind
- Increase public awareness

Can one localize in a room: Recap

- Continuous problem, continuous formulation
- Model: Image sources, images microphones
- Surprising results!

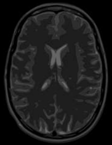
2. Can one localize in a room?

Publications:

- Reza Parhizkar, Ivan Dokmanic, MV
- Single-Channel Indoor Microphone Localization
- submitted, ICASSP 2014

and

- and
- Orhan Ocal, Ivan Dokmanic and MV
- Source Localization and Tracking in Non-convex Rooms
- submitted, ICASSP 2014

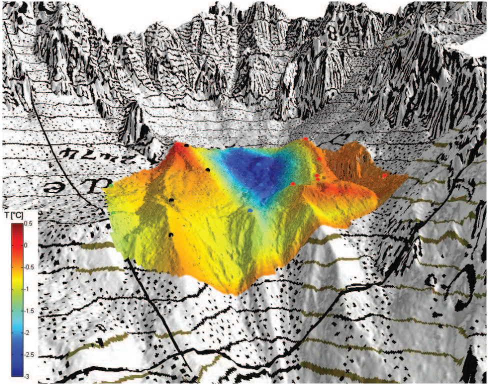





Can one put sensors optimally?

Juri Ranieri

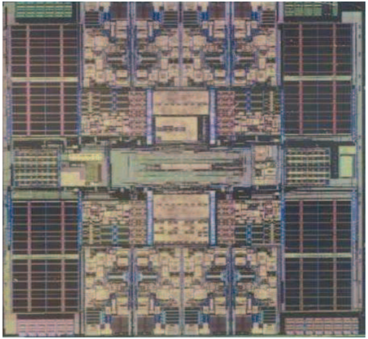
A sensor deployment in the swiss alps



SensorScope: a sensor network to study micro-climates

Sensing the temperature of a microprocessor

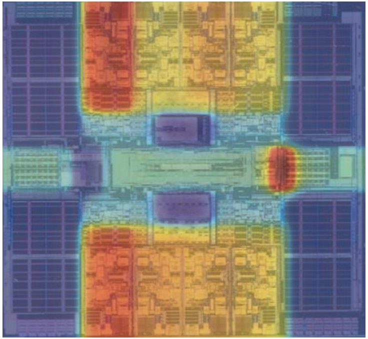
A modern 8 cores processor



- Thermal stress induces: failures, reduced performance, increased power consumption
- Temperature information is necessary to optimize the workload

Sensing the temperature of a microprocessor

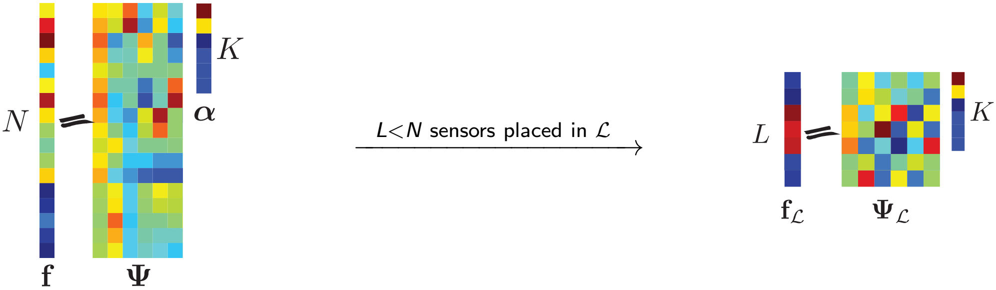
A modern 8 cores processor



- Thermal stress induces: failures, reduced performance, increased power consumption
- Temperature information is necessary to optimize the workload

Linear inverse problems of physical fields

- N: resolution of the physical field.
- K: number of parameters to estimate.
- L: number of sensors.

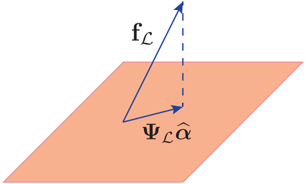


$$\mathbf{f} = \boldsymbol{\Psi}\boldsymbol{\alpha}$$

$$\mathbf{f}_{\mathcal{L}} = \boldsymbol{\Psi}_{\mathcal{L}}\boldsymbol{\alpha}$$

- Given $\mathcal{L}$ and $\mathbf{\Psi}_{\mathcal{L}}$, least square estimate of $\boldsymbol{\alpha}$:

$$\widehat{\boldsymbol{\alpha}} = \underbrace{(\mathbf{\Psi}_{\mathcal{L}}{}^{*}\mathbf{\Psi}_{\mathcal{L}})^{-1}\mathbf{\Psi}_{\mathcal{L}}{}^{*}}_{\text{Pseudoinverse}}\mathbf{f}_{\mathcal{L}}.$$



- Assume i.i.d. gaussian noise, the mean square error (MSE) is

$$\mathrm{E}\left[\|\widehat{\boldsymbol{\alpha}} - \boldsymbol{\alpha}\|^2\right] \propto \sum_{i=1}^{K} \frac{1}{\lambda_i}$$

where $\{\lambda_i\}_{i=1}^{K}$ are the eigenvalues of $\mathbf{T}_{\mathcal{L}} = \mathbf{\Psi}_{\mathcal{L}}{}^{*}\mathbf{\Psi}_{\mathcal{L}}$.

Sensor placement problem

- Given a linear inverse problem defined by a matrix $\boldsymbol{\Psi}$ and number of sensors L. Find the sensor allocation OPT that minimizes the MSE:

$$\mathrm{OPT} = \arg \min_{|\mathcal{L}|=L} \sum_{i=1}^{K} \frac{1}{\lambda_i}.$$

- Challenge 1: combinatorial problem!

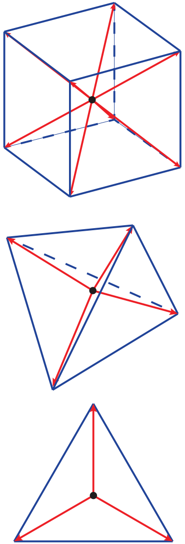
- Challenge 2: MSE hard to minimize, many local minima!

What if we can design completely $\mathbf{\Psi}_{\mathcal{L}}$?

- $\mathbf{\Psi}_{\text{OPT}}$ are minimizers of the Frame Potential (FP),

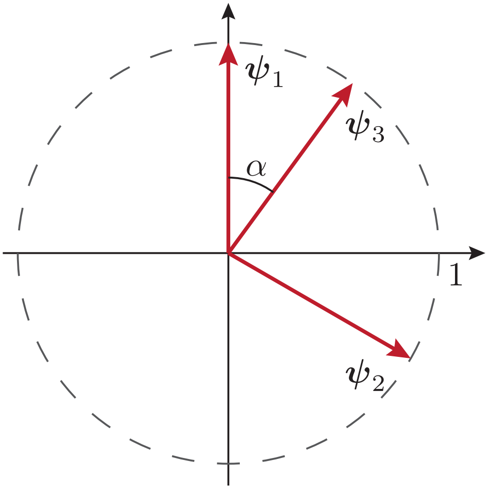
$$FP(\mathbf{\Psi}_{\mathcal{L}}) = \sum_{i,j \in \mathcal{L}} |\langle \psi_i, \psi_j \rangle|^2 = \sum_{i=1}^{K} |\lambda_i|^2,$$

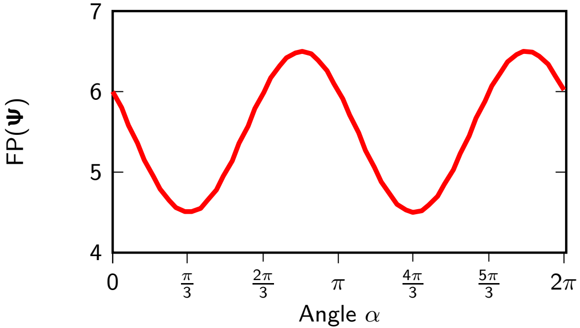
where ψ_i is the i-th row of $\mathbf{\Psi}$.



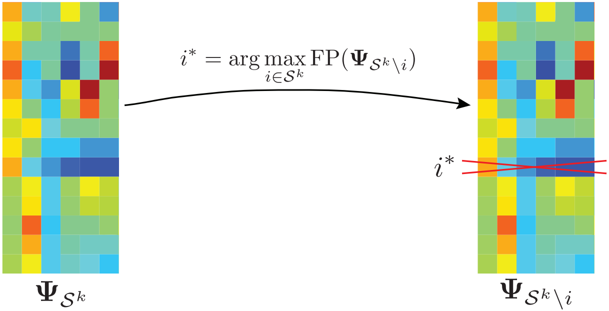
- Fix the sensing power: $P = \sum_{i=1}^{L} \|\psi_i\|^2 = \sum_{i=1}^{K} \lambda_i$
- If $L = K$: orthonormal matrix
- If $L > K$: tight frame [Goyal et al.]

Frame Potential





- Greedy *worst out* sensor selection
- At k-th iteration, we remove the row that maximizes the FP of $\boldsymbol{\Psi}_{\mathcal{S}^k}$.
- After $N - L$ iteration, we obtain the sensor placement as $\mathcal{L} = \mathcal{S}^{N-L}$



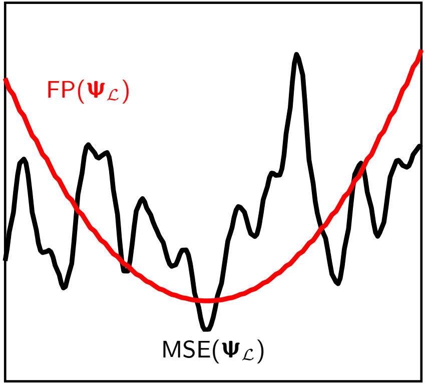
FrameSense is near-optimal w.r.t. MSE

Theorem

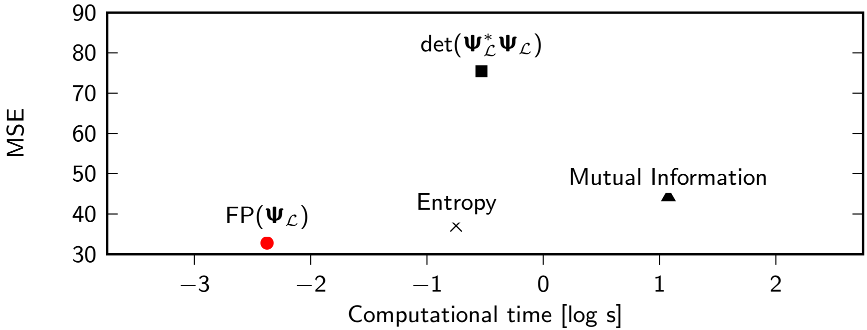
Assume the spectrum of $\boldsymbol{\Psi}$ satisfies some mild conditions and define the optimal allocation as $OPT = \arg\min_{|\mathcal{A}|=L} \mathrm{MSE}(\boldsymbol{\Psi}_{\mathcal{A}})$. Then the solution $\mathcal{L}$ of FrameSense is near-optimal w.r.t. MSE,

$$\mathrm{MSE}(\boldsymbol{\Psi}_{\mathcal{L}}) \leq \beta\, \mathrm{MSE}(\boldsymbol{\Psi}_{OPT})$$

where β is constant depending on the spectrum of $\boldsymbol{\Psi}$ and the norm of its rows.

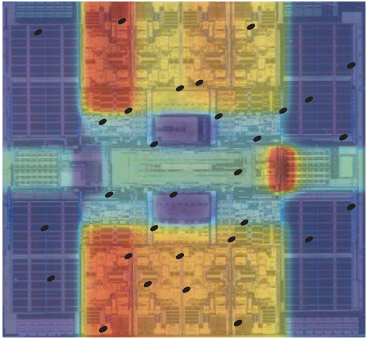


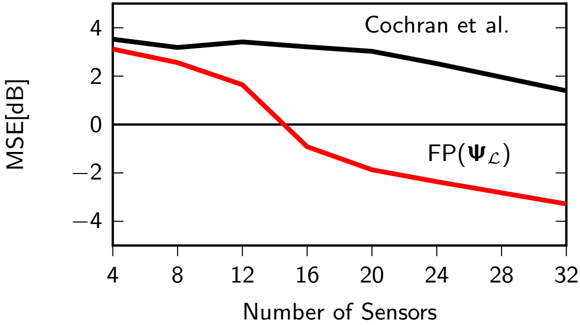
Performance vs Computational Time



- Random Tight Frames
- Locations $N = 100$, Parameters $K = 30$, Sensors $L = 50$.
- Mutual Information [Krause 2008], Entropy [Wang 2004], Determinant [Shamaiah 2010].

Sensor placement on multicore processors





- Case study: Niagara 8 core under typical workload
- $\boldsymbol{\Psi}$ learnt by PCA
- Cochran et al.: Sensor placed according to energy density.

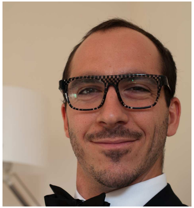
Can one put sensors optimally: Recap

- Combinatorial problem
- Greedy algorithm
- Guaranteed performance with respect to optimal problem
- Potential of frame potential!

4. Can one put sensors optimally?

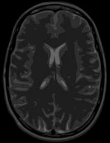
Publication:

- Juri Ranieri, Amina Chebira, MV
- Near-Optimal Sensor Placement for Linear Inverse Problems
- IEEE Tr. on Signal Processing, under revision, online at `http://arxiv.org/abs/1305.6292`,

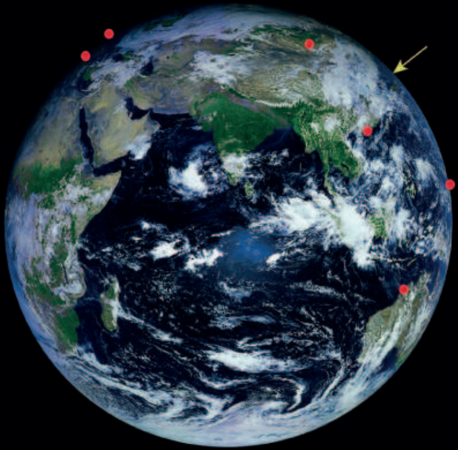


Juri Ranieri

Amina Chebira





Can one know the nuclear fallout of Fukushima?

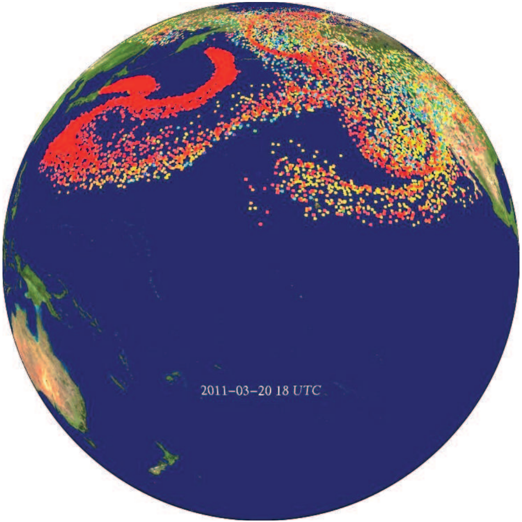
Marta Martinez-Camara

Nuclear fall out

	state	release	half life	deposition	measurements
Xe-133	gas	ventings, explosions	5.24 days	dry	air concentration
Cs-137	particle	leak,explosions	30 years	dry + wet	air conc. + soil deposition
Cs-134	particle	leak,explosions	2 years	dry + wet	air conc. + soil deposition
I-131	particle	leak,explosions	8.01 days	dry + wet	air conc. + soil deposition

Lagrangian dispersion models

Lagrangian dispersion models



$$y(\xi, t) = \int_0^t A(\xi, \tau)x(\tau)d\tau$$

Measurements

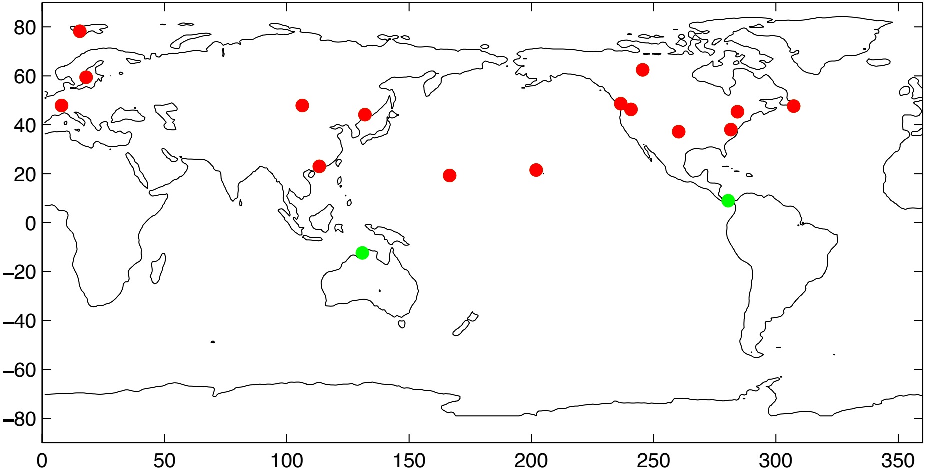
Weather

Emissions

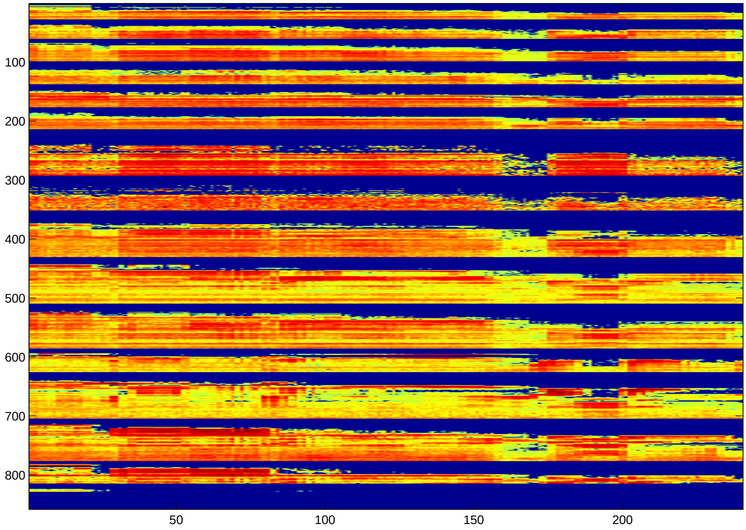
Known $\rightarrow$ $\mathbf{y} = \mathbf{A}\mathbf{x}$ $\leftarrow$ Unknown

Estimated

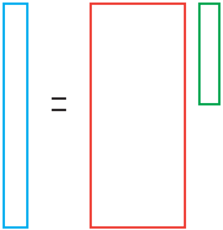
Sensors locations



Matrix describing the transport of Xenon



The Inverse problem



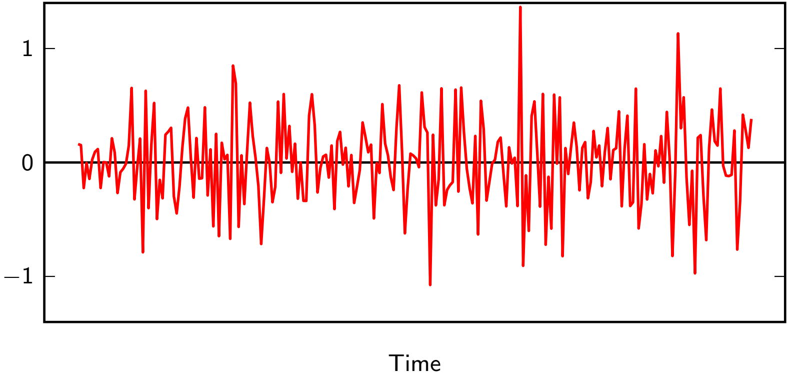
$$\mathbf{y} = \mathbf{A}\mathbf{x}$$

- What we know a priori:
  - Condition number very large
  - Most of the unknowns are equal to zero
  - Positive solution
- Possible regularizations:
  - Pseudoinverse solution: $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$
  - Tikhonov regularization: $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_2^2$
  - A priori solution: $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x} - \mathbf{x}_a\|_2^2$
  - Sparse regularization: $\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_1$

Pseudo-inverse solution

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2$$

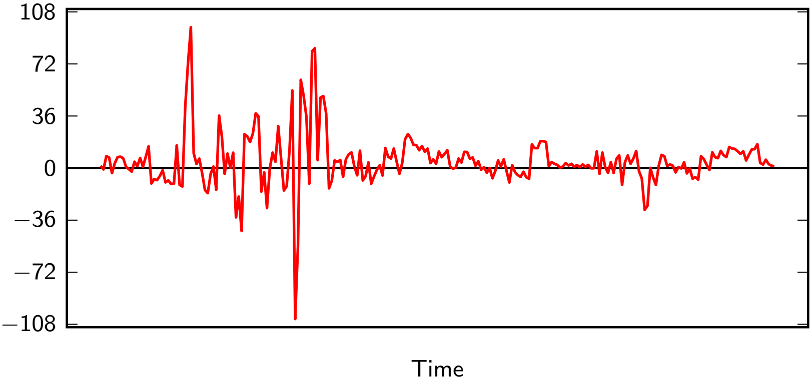
$\mathrm{cond}(\mathbf{A})$ large $\rightarrow$ solution very sensitive to noise



Tikhonov solution

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_2^2$$

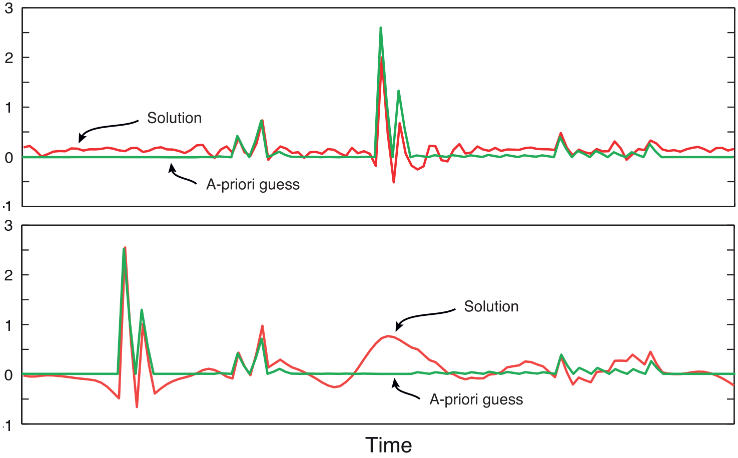
Non-sparse and non-positive...



Tikhonov with a priori solution - State of the Art

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x} - \mathbf{x}_a\|_2^2$$

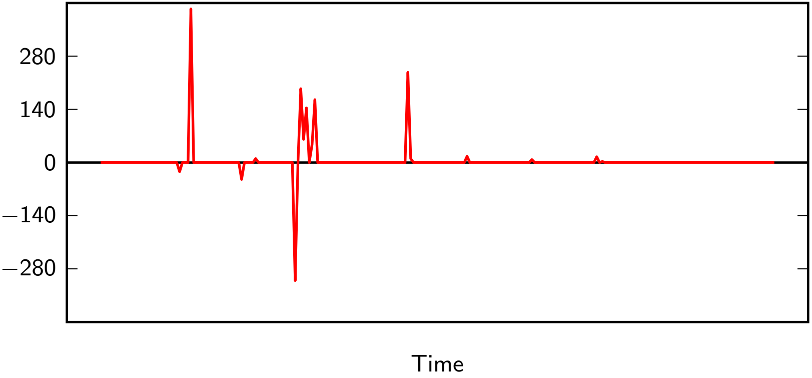
Solution overly biased by prior $\rightarrow$ disregards measurements



Sparse solution

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{x}\|_1$$

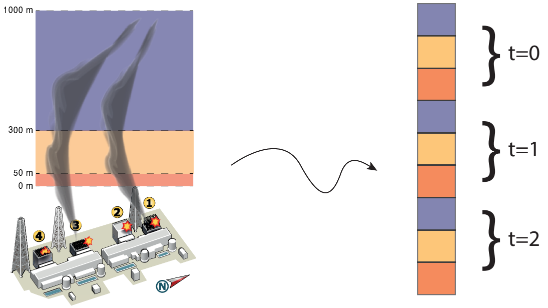
Sparse, non-biased $\rightarrow$ we still need positivity



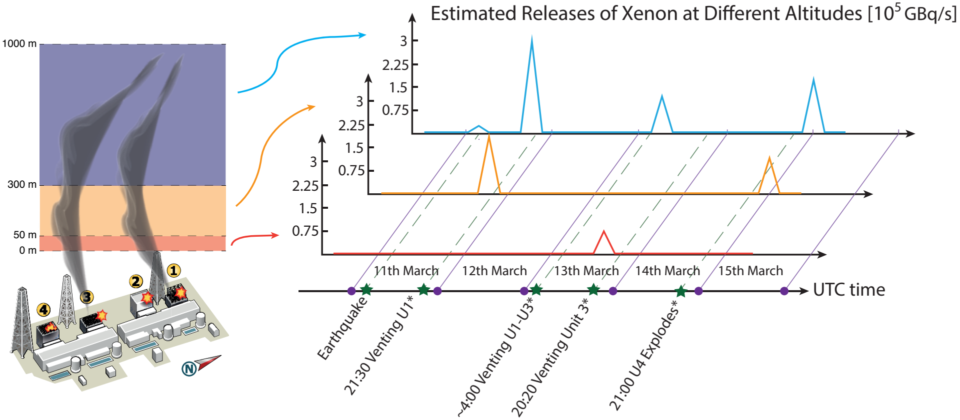
Our proposed solution: Sparse with positivity constraint

$$\min_{\mathbf{x}} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{x}\|_1 \quad \text{s.t } \mathbf{x} \geq 0$$

- The solution will be sparse, unbiased and positive
- The problem is more complicated - we have three heights that play into the dispersion



Results



1000 m
300 m
50 m
0 m
1
2
3
4
N
Estimated Releases of Xenon at Different Altitudes [10^5 GBq/s]
3
2.25
1.5
0.75
11th March
12th March
13th March
14th March
15th March
UTC time
Earthquake
21:30 Venting U1*
~4:00 Venting U1-U3*
20:20 Venting Unit 3*
21:00 U4 Explodes*

Can one know the nuclear fallout of Fukushima: Recap

- We have a solution which achieves the a priori constraints:
  - Sparse, positive
  - Robust against noise
- We recover a realistic source:
  - The total emitted quantity matches the inventory
  - The emissions can be related with the known events (venting, explosions)
- Difficult inverse problem
  - Gives a verifiable solution for Xenon
  - Cesium an open problem

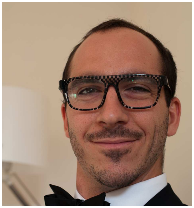
5. Can one know the nuclear fallout of Fukushima?

Publication:

- Marta Martinez-Camara, Ivan Dokmanic, Juri Ranieri, Robin Scheibler, MV and Andreas Stohl,
- The Fukushima Inverse Problem, ICASSP 2013
- Data online `http://rr.epfl.ch/44/`

Marta Martinez-Camara

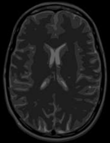
Ivan Dokmanic



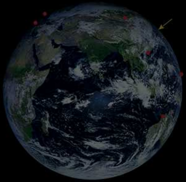
Juri Ranieri

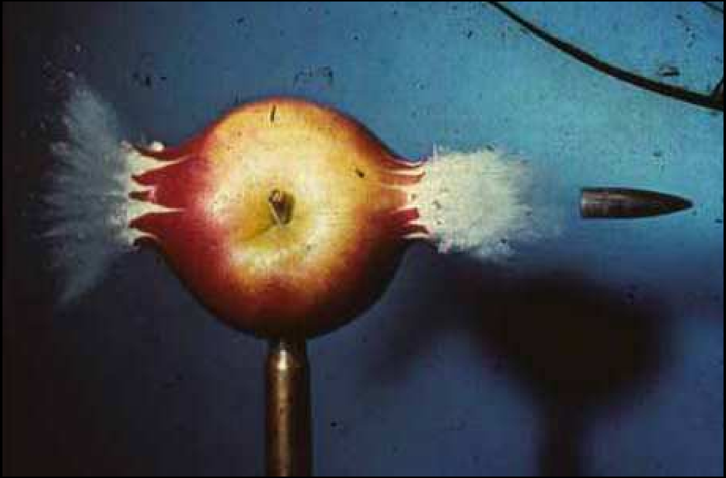
Robin Scheibler

Andreas Stohl







Conclusions

Four exemplary inverse problems

Question	Solution	Quality	Complexity
Shape of room	Image sources	Exact	$O(NK^{(M-1)})$
Source in graph	Source vertex	Approximate	$O(N^3)$
Sensor placement	Subset selection	Approximate (bd error)	$O(KN^2)$
Pollution	Source characterization	Approximate	$O(N^3)$

Are these problems important?

Problems can be cute, relevant, hard etc

- Hearing the shape of a room has many implications
  - Indoor localization
- Finding a source quickly helps
  - Spending on eradication of polio: 1 B $ per year
- Placing sensors smartly
  - Size of carbon market: 142 B $ (2011)
- Knowing pollution or radioactivity
  - Will make the difference if people can return to their villages...

On problems and tools

Sophisticated tools abound....

- Pick your tool given the problem, not the other way around!

Nature versus methods

- Nature picks $\mathbf{A}$, and it is not always well conditioned...

Sparse signal processing and applications

- Sparsity is a basic principle of modeling
- Efficient algorithms and approximations
- Connection to some fundamental theory questions

Acknowledgments

To those who made it possible

- The organizers !
- Prof. Andrea Rinaldo (EPFL) and his team for access to the Natal data
- Prof. David Atienza (EPFL) and his team for data on multicore processors
- Prof. Andreas Stohl (Norwegian Institute for Air Research) for forward models and data

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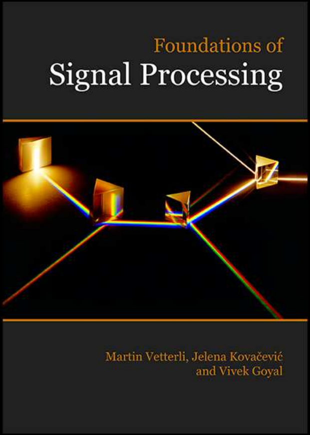
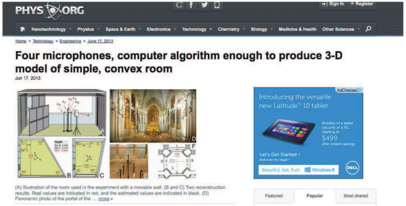
- Bill and Melinda Gates Foundation, Award OPP1070273


Foundations of
Signal Processing
Martin Vetterli, Jelena Kovačević
and Vivek Goyal

Media echo...



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Four microphones, computer algorithm enough to produce 3-D model of simple, convex room

Jun 17, 2013

(A) Illustration of the room used in the experiment with a movable wall. (B and C) Two reconstruction results. Real values are indicated in red, and the estimated values are indicated in black. (D) Panoramic photo of the portal of the ... more

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Shape of a room 'heard' by acoustic echoes

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by Sarah C. P. Williams on 17 June 2013, 3:10 PM | 2 Comments

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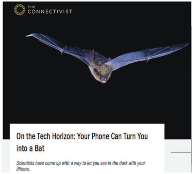
If you stand in the middle of Lausanne Cathedral in Switzerland and snap your fingers, the sound waves will project outward until they bounce off the stone walls, arches, altars, and statues of the Gothic church, and return, in a series of echoes, to your ears. From the noise, your brain might be able to infer the large size and openness of the sanctuary. But with a few microphones and a newly developed mathematical algorithm, scientists can now get more much information than that: They can determine the precise shape of the room.

ENLARGE IMAGE



THE CONNECTIVIST

On the Tech Horizon: Your Phone Can Turn You into a Bat

Scientists have come up with a way to let you see in the dark with your iPhone.

Cientistas criam tecnologia que imita sonar de Batman

A- A+

No filme *Batman — O Cavaleiro das Trevas*, o herói usa uma tecnologia extremamente avançada para caçar

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Le Point.fr - Publié le 10/08/2012 à 07:56

L'algorithme miracle a-t-il été découvert ?

Un chercheur affirme avoir mis au point un système mathématique permettant d'identifier l'origine de tout événement.

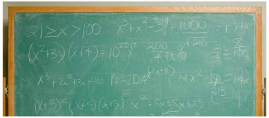

Photo d'illustration. © AFP

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SCIENCE : PHYSICS : TECH : NANO : NEWS

Math algorithm tracks crime, rumours, epidemics to source

August 10th, 2012 in Other Sciences / Mathematics

Enlarge

© Photos.com

(Phys.org) -- A team of EPFL scientists has developed an algorithm that can identify the source of an epidemic or information circulating within a network, a method that could also be used to help with criminal investigations.

technology review
Published by MIT

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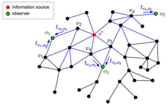
Network Theory Breakthrough Reveals The Origin Of Outbreaks

Researchers solve the seemingly impossible problem of using only a few measurements to find the first victim of a disease

THE PHYSICS ARXIV BLOG
Wednesday, August 15, 2012



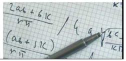
Público.es

Las matemáticas pueden ayudar a desenmascarar una terrorista

Volver a la noticia

Un algoritmo desarrollado por Escuela Politécnica de Lausana permite rehacer a la inversa el camino que ha seguido la información a través de internet hasta llegar a la fuente primaria

Arantza Valls (EFE) | Madrid | 10/08/2012 13:49 | Actualizado: 10/08/2012 13:58 |



Un algoritmo desarrollado en la Escuela Politécnica de Lausana (EPFL) permite dar respuesta a preguntas de difícil respuesta como: **¿quiénes participan en la planificación de un atentado terrorista?**, ¿cómo se propaga una epidemia de cólera? o ¿quién inició ese grosero rumor sobre mí en facebook?

Para encontrar las respuestas basta con disponer de fragmentos de información de algunas de las personas implicadas en la transmisión de los mensajes y conocer el momento exacto en el que estos fueron enviados.