# Hints for solving the exercises in Chapter 11

**Hints to Exercise 11.1** For some orthonormal basis of $\mathcal{K}_{n_*}(A, b)$ consider the corresponding transformation of the coordinates $T : \mathbb{R}^{n_*} \to \mathcal{K}_{n_*}(A, b)$. Show that the matrix $B \in \mathbb{R}^{n_* \times n_*}$ which corresponds to the mapping $T^{-1}AT : \mathbb{R}^{n_*} \to \mathbb{R}^{n_*}$ (with respect to the canonical basis of $\mathbb{R}^{n_*}$) has a complete system of eigenvectors.

**Hints to Exercise 11.3** The statement of part (b) can be obtained using the same technique as in the proof of Theorem 11.5 applied to the space $\mathcal{D}_n = \mathcal{K}_n(A, b)$, where the special basis $b$, $Ab, \ldots, A^{n-1}b$ should be considered here.

**Hints to Exercise 11.4** The second of the two first identities in the first line can be obtained by mathematical induction. From that together with the mutual orthogonality of the vectors $r_0, r_1, \ldots$ the identity $(*)$ $\| d_n \|_2^2 = \| r_n \|_2^4 \sum_{j=0}^{n} \| r_j \|_2^{-2}$ stated in the exercise is obtained immediately. The representation for $(**)$ $\langle d_n , d_j \rangle_2$ stated in the same line of this exercise can be obtained by fixing the index $j$ and using mathematical induction with respect to $n = j, j+1, \ldots$ . Use the representation (11.17) for the step sizes $\alpha_n$. The estimate $\| x_1 \|_2 \le \| x_2 \|_2 \le \ldots$ can be obtained by using $x_{n+1} = x_n + \alpha_n d_n$. Additionally one has to verify that $\alpha_n \langle x_n , d_n \rangle_2 > 0$ holds ( use $(*)$ for this). For the verification of the last inequality of the exercise, represent $r_n$ as a linear combination of $d_n$ and $d_{n-1}$ and additionally apply $(**)$.

**Hints to Exercise 11.5** (a) One has to find the concrete form of $\mathcal{K}_n(A, b)$ and $A(\mathcal{K}_n(A, b))$.

(b) Show that $x_*$ is an eigenvector of $A^\top A$. For this one has to compute first $Ax_*$ and then $A^\top A x_*$.