Hints for solving the exercises in Chapter 2

Hints to Exercise 2.1 The local representation

$$s(x) = a_k + b_k(x - x_k) + c_k(x - x_k)^2 \qquad \text{for} \quad x \in [x_k, x_{k+1}], \qquad k = 0, 1, \ldots, N-1,$$

and the differentiability conditions as well as the interpolation conditions lead to a linear system of N equations for the $N+1$ coefficients $b_0, b_1, \ldots, b_{N-1}, b_N$ with the auxiliary variable $b_N = 2h_{N-1}c_{N-1} + b_{N-1}$. The additionally given condition in part (a) of this exercise results in a linear system of $N+1$ equations for the $N+1$ unknown coefficients $b_0, b_1, \ldots, b_N$, and the additionally given condition in part (b) of the exercise results in a linear system of N equations for the N unknown coefficients $b_1, b_2, \ldots, b_N$. Consider then the system matrices corresponding to the given linear systems of equations, respectively. In part (b), determine the determinant of this matrix by an expansion along the first row.

Hints to Exercise 2.3 Use the same procedure as in the estimate of the error for cubic spline interpolation (w.r.t. to the third derivative).