Thursday, December 14th

9:00 – 9:15 *Opening*

Chair: Aad Dijksma

9:15 – 9:40 **Alexander Markus**
On convexity of ranges of quadratic forms on various sets

9:45 – 10:10 **Daniel Alpay**
Generalized Nevanlinna functions in the Banach space setting

10:15 – 10:40 **Mark Malamud**
Completeness of root vectors for arbitrary boundary value problems for Sturm-Liouville equations

10:45 – 12:00 *Refund of travel expenses (MA 674)*

& *Coffee break (DFG Lounge MA 315)*

Thursday, December 14th

Chair: Vadim Adamyan

12:00 – 12:25 **Harald Woracek**
Admissible majorants for de Branges spaces of entire functions

12:30 – 12:55 **Annemarie Luger**
On non-canonical extensions with finitely many negative squares

13:00 – 13:25 **Pavel Kurasov**
Operator models for singular ordinary differential expressions

13:30 – 15:30 *Lunch break*

Thursday, December 14th

Chair: Branko Curgus

15:30 – 15:55 **Hagen Neidhardt**
Wigner's R-matrix and Weyl functions

16:00 – 16:25 **Yury Arlinskii**
Matrix representations of contractions with rank one defects and inverse spectral problems

16:30 – 16:55 **Rostyslav Hryniv**
What spectra can non-self-adjoint Sturm-Liouville operators have?

17:00 – 17:45 *Coffee break (DFG Lounge MA 315)*

Thursday, December 14th

Chair: Vladimir Derkach

17:45 – 18:10 **Uwe Günther**
A toy model of PT-symmetric quantum mechanics, the squire equation and UV-IR-duality

18:15 – 18:40 **Michal Wojtylak**
Domination in Krein spaces

18:45 – 19:10 **Vladimir Strauss**
A test for commutative J-symmetric families of D_κ^+-class

Friday, December 15th

Chair: Heinz Langer

9:00 – 9:25	**Marinus Kaashoek** The inverse problem for Krein orthogonal entire matrix functions
9:30 – 9:55	**Paul Binding** Darboux transformations and commutation in Pontryagin space
10:00 – 10:25	**Aurelian Gheondea** A Krein space interpretation of Dirac operators
10:30 – 11:30	*Conference photo* & *Coffee break (DFG Lounge MA 315)*

Friday, December 15th

Chair: Henk de Snoo

11:30 – 11:55	**Andrei Shkalikov** Remarks on global stability for inverse Sturm-Liouville problems
12:00 – 12:25	**Matej Tusek** Influence of the curvature on the impurity spectrum in a quantum dot
12:30 – 12:55	**Niels Hartanto** Block numerical ranges of nonnegative matrices
13:00 – 15:15	*Lunch break*

Friday, December 15th

Chair: Andreas Lasarow

15:15 – 15:40 **Leiba Rodman**
On factorization of matrix functions in the Wiener algebra

15:45 – 16:10 **Andre C.M. Ran**
On the relation between $XX^{[*]}$ and $X^{[*]}X$ in an indefinite inner product space

16:15 – 16:40 **Ekaterina Lopushanskaya**
On the representation of generalized Caratheodory functions in the area Ω_ν

16:45 – 17:30 *Coffee break (DFG Lounge MA 315)*

Friday, December 15th

Chair: Kresimir Veselic

17:30 – 17:55 **Henrik Winkler**
Kato decompositions for quasi-Fredholm relations

18:00 – 18:25 **Marina Chugunova**
Diagonalization of the coupled-mode system

18:30 – 18:55 **Victor Khatskevich**
On the structure of semigroups of operators acting in spaces with indefinite metric

19:30 *Bus shuttle to restaurant "Savoie Rire"*

Bus leaves in front of the math department,
Straße des 17. Juni 136, 10623 Berlin

20:00 *Conference dinner*

Savoie Rire, Tegeler Weg 104, 10589 Berlin

Saturday, December 16th

Chair: Matthias Langer

9:00 – 9:25 **Vadim Adamyan**
Non-negative perturbations of non-negative selfadjoint operators

9:30 – 9:55 **Aad Dijksma**
The Schur transformation for generalized Nevanlinna functions centered at a point in $\mathbb{C}^+$

10:00 – 10:25 **Vyacheslav Pivovarchik**
On spectra of a certain class of quadratic operator pencils with one-dimensional linear part

10:30 – 11:15 *Coffee break (DFG Lounge MA 315)*

Saturday, December 16th

Chair: Daniel Alpay

11:15 – 11:40 **Kresimir Veselic**
On eigenvalue bounds for indefinite selfadjoint operators

11:45 – 12:10 **Birgit Jacob**
Location of the spectrum of operator matrices which are associated to second order equations

12:15 – 12:40 **Maxim Derevyagin**
A criterion for the resolvent set of generalized Jacobi operators acting in Krein spaces

12:45 – 15:00 *Lunch break*

Saturday, December 16th

Chair: Paul Binding

15:00 – 15:25 **Henk de Snoo**
A Lebesgue decomposition for nonnegative forms

15:30 – 15:55 **Branko Curgus**
Positive operators in Krein spaces similar to self-adjoint operators in Hilbert spaces

16:00 – 16:25 **Elena Andreisheva**
Approximation of generalized Schur functions

16:30 – 17:15 *Coffee Break (DFG Lounge MA 315)*

Saturday, December 16th

Chair: Mark Malamud

17:15 – 17:40 **Franciszek H. Szafraniec**
Rows versus columns

17:45 – 18:10 **Adrian Sandovici**
Composition of Dirac structures for infinite-dimensional systems

18:15 – 18:40 **Igor Sheipak**
Indefinite Sturm-Liouville operators with singular self-similar weights

Sunday, December 17th

Chair: Alexander Markus

9:00 – 9:25 **Vladimir Derkach**
Coupling method in the theory of generalized resolvents of symmetric operators

9:30 – 9:55 **Tomas Azizov**
Pontryagin theorem and an analysis of spectral stability of solitons

10:00 – 10:25 **Illya Karabash**
The abstract kinetic equation on a half-line

10:30 – 11:15 *Coffee break (DFG Lounge MA 315)*

Sunday, December 17th

Chair: Leiba Rodman

11:15 – 11:40 **Yuri Shondin**
On approximation of a boundary value problem at 'regular singular' endpoint

11:45 – 12:10 **Aleksey Kostenko**
Indefinite Sturm-Liouville operators with the singular critical point zero

12:15 – 12:40 **Lyudmila I. Sukhocheva**
On linearizators of hyperbolic type pencils

12:45 – 14:30 *Lunch break*

Sunday, December 17th

Chair: Marinus Kaashoek

14:30 – 14:55	**Mikhail Denisov** Spectral function for some product of selfadjoint operators
15:00 – 15:25	**Carsten Trunk** Normal and hyponormal matrices in inner product spaces
15:30 – 15:55	**Jussi Behrndt** On the eigenvalues of non-canonical self-adjoint extensions
16:00	*Closing*

Non-Negative Perturbations of Non-Negative Selfadjoint Operators

V. Adamyan

Let A be a non-negative selfadjoint operator in a Hilbert or Krein space $\mathcal{H}$ and let A_0 be some densely defined closed restriction of A_0, $A_0 \subseteq A \neq A_0$. It is of interest to know whether A is the unique non-negative selfadjoint extensions of A_0 in $\mathcal{H}$. We give a natural criterion that this is the case and if it fails, we describe all non-negative extensions of A_0. The obtained results are applied to the investigation of non-negative singular point perturbations of some elliptic differential operators in $\mathbb{L}_2(\mathbb{R}_n)$.

Generalized Nevanlinna Functions in the Banach Space Setting

D. Alpay

joint work with D. Volok and O. Timoshenko

We define Nevanlinna and generalized Nevanlinna functions in the setting of analytic functions which take values from a Banach space into its conjugate dual space. Two kind of representations are given; the first is based on Helly's theorem while the second uses ideas of of Krein and Langer and is based on relations in Pontryagin spaces.

The manuscript is available on the Arxiv server at:

http://arxiv.org/pdf/math.FA/0607283

Approximation of Generalized Schur functions

E.N. Andreisheva

In [1] M.G. Krein and H. Langer investigated questions concerning the approximation of Nevanlinna functions. Our purpose is to get such result for Schur functions.

Theorem 1 *Let* $s(\lambda) = \lambda^k s_k(\lambda)$, $s_k(0) \neq 0$, $k \leqslant n$. *Then we have assertions*
1. $s \in S_\varkappa$, *where* $S_\varkappa$ *is the generalized Schur class;*
2. for some integer $n > 0$ *there exist real numbers* $c_1, c_2, \ldots, c_{2n}$ *such that*

$$s(\lambda) = 1 - \sum_{\nu=1}^{2n} c_\nu (\lambda - 1)^\nu + O((\lambda - 1)^{2n+1}), \ \ \lambda \to 1, \lambda \in \Lambda_\theta$$

if and only if there exist a Pontryagin space $\Pi_\varkappa$, *a contraction* T *in* $\Pi_\varkappa$ *and a generative element* $u \in \operatorname{dom}(I - T)^{-(n+1)}$ *for the operator* T *such that:*

$$s(\lambda) = \lambda^k - \frac{1}{s_k(0)} \lambda^k (\lambda - 1)[(I - \lambda T)^{-1}(I - T)^{-1} T^{k+1} u, T^k u]$$

for $\lambda \in \mathbb{D}$, $\frac{1}{\lambda} \notin \sigma_p(T)$. *In this case:*

$$c_\nu = \begin{cases} \dfrac{1}{s_k(0)} \displaystyle\sum_{i=1}^{\nu} C_{k-i}^{\nu-i} [(I - T)^{-(i+1)} T^{k+1} u, T^k u] - C_k^\nu, & 1 \leqslant \nu < k + 1; \\ \dfrac{1}{s_k(0)} [(I - T)^{-(\nu+1)} T^\nu u, T^k u], \quad k + 1 \leqslant \nu \leqslant n; & \\ \dfrac{1}{s_k(0)} [(I - T)^{-(n+1)} T^n u, (I - T^c)^{-(\nu-n)} T^{c(\nu-n)} T^k u], \quad n + 1 \leqslant \nu \leqslant 2n; & \end{cases}$$

[1] Krein M. G., H. Langer, "Über einige Fortsetzungsprobleme, die eng mit der Theorie hermitescher Operatoren im Raume Π_κ zusammenhängen. I. Einige Funktionenklassen und ihre Darstellungen", *Math. Nachr.* *77* (1977), 187-236.

This research is supported by the RFBR grant 05-01-00203.

Matrix Representations of Contractions with Rank One Defects and Inverse Spectral Problems

Yu. Arlinskiĭ

joint work with L. Golinskiĭ and E. Tsekanovskiĭ

For a completely nonunitary contraction with rank one defect operators acting in a separable Hilbert space we establish a new model given by a five-diagonal matrix. For such matrices we develop the direct and inverse spectral analysis.

Pontryagin Theorem and an Analysis of the Spectral Stability of Solitons

T.Ya. Azizov

joint work with M.V. Chugunova

In the analysis of the spectral stability of localized solutions (solitons) of Hamiltonian systems one uses a linearization method in a neighborhood of these solutions. A study of the spectrum of a linearized system is reduced to the eigenvalue problem $L_-L_+u = \gamma u$, where L_- and L_+ are differential operators of a special type. We show how one can apply a well-known Pontryagin theorem in the stability problem.

This research is supported by the RFBR grant 05-01-00203-a.

On the Eigenvalues of Non-Canonical Self-Adjoint Extensions

J. Behrndt

joint work with A. Luger

Let $\widetilde{A}$ be a self-adjoint extension in $\tilde{\mathcal{K}}$ of a fixed symmetric operator A in $\mathcal{K} \subseteq \tilde{\mathcal{K}}$. An analytic characterization of the eigenvalues of $\widetilde{A}$ is given in terms of the Q-function and the parameter function in the Krein-Naimark formula.

[1] J. Behrndt, A. Luger: *An analytic characterization of the eigenvalues of self-adjoint extensions*, J. Funct. Anal. **242** (2007), 607–640.

Darboux Transformations and Commutation in Pontryagin Space

P. Binding

Darboux transformations have become popular for generating hierarchies of differential equations, and are frequently used in mathematical physics. They are intimately connected with von Neumann's theorem on products of operators and their adjoints, with square roots and (restricted) commutation properties of these products, and with polar decompositions of operators, in Hilbert space. Motivated by Sturm-Liouville problems with eigenvalue dependent boundary conditions, we shall discuss corresponding ideas in Pontryagin space. In this case one has to tread more carefully, and for example some of the above constructions need not even exist.

Diagonalization of the Coupled-Mode System

M. Chugunova

joint work with D. Pelinovsky

We consider the Hamiltonian coupled-mode system that occur in nonlinear optics, photonics, and atomic physics. Spectral stability of gap solitons is determined by eigenvalues of the linearized coupled-mode system. In the special class of symmetric nonlinear potentials, we construct a block-diagonal representation of the linearized coupled-mode equations, when the spectral problem reduces to two coupled two-by-two Dirac systems.

Positive Operators in Krein Spaces Similar to Self-Adjoint Operators in Hilbert Spaces

B. Ćurgus

I will present several results about positive operators in Krein spaces which are similar to self-adjoint operators in Hilbert spaces. Additive perturbations of such operators will also be considered. These results will be applied to differential operators.

Spectral Function for some Product of Selfadjoint Operators

M. Denisov

Let $\mathcal{H}$ be a Hilbert space with a scalar product $(\cdot,\cdot)$. Let $A : \mathcal{H} \to \mathcal{H}$ be a linear continuous operator, such that $A = A^*$ and $A \geq 0$, and let $G : \mathcal{H} \to \mathcal{H}$ be a linear continuous operator, $G = G^*$ and $0 \notin \sigma_p(G)$.

Consider the form $[\cdot,\cdot] := (G\cdot,\cdot)$; a Hilbert space $\mathcal{H}$ equipped with such a form $[\cdot,\cdot]$ is named G-space.

It is a well-known result of H. Langer that J-nonnegative operators possess a spectral function. Our aim is to construct the spectral function for a G-nonnegative operator AG. In the construction of the spectral function we follow J.Bognar. We give an example of a G-nonnegative operator which has no spectral function.

[1] T. Ja. Azizov and I.S. Iohvidov, Theory linear operators in the space with indefinite metric.- M.: Nauka, 1986.
[2] J.Bognar, A proof of the spectral theorem for J-positive operators. Acta sci. math.,1983, 15, 1-2, p.75-80.
[3] Langer.H, Spektralfunktionen einer Klasse J-selbstadjungierter Operatoren. — Math. Nachr., 1967, 33, 1-2, p. 107-120.

This research is supported by the $RFBR$ grant 05-01-00203.

A Criterion for the Resolvent Set of Generalized Jacobi Operators acting in Krein Spaces

M. Derevyagin

Let us consider a generalized Jacobi matrix, i.e. the following tridiagonal block matrix

$$\begin{pmatrix} A_0 & \widetilde{B}_0 & & \mathbf{0} \\ B_0 & A_1 & \widetilde{B}_1 & \\ & B_1 & A_2 & \ddots \\ \mathbf{0} & & \ddots & \ddots \end{pmatrix}.$$

Under some assumptions, this matrix generates a bounded self-adjoint operator in a Krein space. A characterization of the resolvent set of such operators via associated polynomials is given. In particular, we obtain an explicit description of the spectrum of operators generated by periodic generalized Jacobi matrices, that is generalized Jacobi matrices with the following property

$$A_{js+k} = A_k, \quad \widetilde{B}_{js+k} = \widetilde{B}_k, \quad B_{js+k} = B_k,$$

where $s \in \mathbb{N}$, $j \in \mathbb{N} \cup \{0\}$, and $k \in \{0, \dots, s-1\}$.

Coupling Method in the Theory of Generalized Resolvents of Symmetric Operators

V. Derkach

joint work with S. Hassi, M.M. Malamud and H.S.V. de Snoo

The Kreĭn-Naimark formula provides a parametrization of all selfadjoint exit space extensions $\widetilde{A}$ of a, not necessarily densely defined, symmetric operator A_1, in terms of the so-called Nevanlinna families. The new notion of a boundary relation introduced in [1] makes it possible to treat every Nevanlinna family as a Weyl family of some symmetric operator A_2 corresponding to the boundary relation. We construct the selfadjoint exit space extension $\widetilde{A}$ of A_1 as a coupling of two symmetric operators A_1 and A_2. Then the Kreĭn-Naimark formula for the generalized resolvents of the operator A_1 is derived from the formula for canonical resolvents of the symmetric operator $A_1 \oplus A_2$.

[1] V. Derkach, S. Hassi, M. Malamud and H. de Snoo, Boundary relations and their Weyl families, Trans. Amer. Math. Soc. 358 (2006), 5351-5400.

The Schur Transformation for Generalized Nevanlinna Functions centered at a Point in $\mathbb{C}^+$

A. Dijksma

joint work with D. Alpay, H. Langer and Yu. Shondin

We define the Schur transformation for generalized Nevanlinna functions at a point z_1 in $\mathbb{C}^+$ and discuss its effect on the self-adjoint operator or relation realization of these functions. With Tomas Azizov (Voronezh) and Gerald Wanjala (Mbarara) we have studied similar questions for generalized Schur functions.

A Krein Space Interpretation of Dirac Operators

A. Gheondea

Following the notion of Krein spaces induced by symmetric operators we give an interpretation of the Dirac operators as indefinite energetic spaces. We also discuss the existence and uniqueness of these spaces, with some relevant examples.

A Toy Model of $\mathcal{PT}-$Symmetric Quantum Mechanics, the Squire Equation and UV-IR-Duality

Uwe Günther

joint work with Frank Stefani and Miloslav Znojil

Some facts about the spectrum of a $\mathcal{PT}-$symmetric quantum mechanical (PTSQM) toy model with potential

$$V(x) = Gx^2(ix)^\nu$$

in a box $x \in [-L, L]$ are presented for the parameter region $\nu \in [-2, 0]$. The corresponding Hamiltonian is selfadjoint in an appropriately chosen Krein space and for $\nu = -1$ the spectral problem maps into that of the hydrodynamic Squire equation. It is shown that in the limit $L \to \infty$ a spectral singularity occurs and that the PTSQM $\rightleftarrows$ Squire mapping can be interpreted as a special type of strong-coupling—weak-coupling (UV-IR) duality. Finally, the system behavior in the vicinity of a spectral triple point is sketched.

partially based on:
J. Math. Phys. **46** (2005) 063504, math-ph/0501069.
Czech. J. Phys. **55** (2005) 1099-1106, math-ph/0506021.

Block Numerical Ranges of Nonnegative Matrices

N. Hartanto

joint work with K.-H. Förster

Let A be a $\ell \times \ell$ square matrix. For a block partition

$$A = (A_{rs})_{r,s=1,\dots,n}$$

we define the block numerical range as

$$W^n(A_{rs}) = \big\{\lambda \in \mathbb{C} \,|\exists\ (x_1, \dots, x_n) \in \mathbb{C}^\ell, \|x_j\| = 1\ : \\ \lambda \text{ is an eigenvalue of } \big(\langle A_{rs}x_s, x_r\rangle\big)_{r,s=1,\dots,n}\big\}$$

and the block numerical radius

$$w^n(A_{rs}) = \max\{|\lambda| \mid \lambda \in W^n(A_{rs})\}.$$

The main result of the talk is: Let A be an (entrywise) nonnegative irreducible matrix with index of imprimitivity m. Then we get the following Perron-Frobenius type result

$$\{\lambda \in W^n(A_{rs}) \mid |\lambda| = w^n(A_{rs})\} = \\ = \{w^n(A_{rs})e^{2k\pi i/m}, k = 0, 1, \dots, m-1\}.$$

For $n = 1$ this is the result of J.N. Issos on the usual numerical range and for $n = \ell$ it is the theorem of Perron-Frobenius on the peripheral spectrum of nonnegative irreducible matrices.

What Spectra can Non-Self-Adjoint Sturm-Liouville Operators have?

R. Hryniv

joint work with S. Albeverio and Ya. Mykytyuk

We address the question, what spectra non-self-adjoint Sturm-Liouville operators on a finite interval can have. Although in the self-adjoint case the question is completely understood, the non-self-adjoint case is more difficult due to possibility of nonsimple and/or nonreal eigenvalues. We solve the inverse spectral problem of reconstructing the complex-valued potential of a Sturm-Liouville operator from two spectra or from a spectrum and the sequence of suitably defined norming constants. We also establish a criterion on solubility of the inverse spectral problem and thus answer the question posed in the title.

Location of the Spectrum of Operator Matrices which are Associated to Second Order Equations

B. Jacob

joint work with C. Trunk

We study second order equations of the form

$$\ddot{z}(t) + A_0 z(t) + D\dot{z}(t) = 0.$$

Here the stiffness operator A_0 is a possibly unbounded positive operator on a Hilbert space H, which is assumed to be boundedly invertible, and D, the damping operator, is an unbounded operator, such that $A_0^{-1/2} D A_0^{-1/2}$ is a bounded non negative operator on H. This second order equation is equivalent to the standard first-order equation $\dot{x}(t) = Ax(t)$, where $A : \mathrm{dom}(A) \subset \mathrm{dom}(A_0^{1/2}) \times H \to \mathrm{dom}(A_0^{1/2}) \times H$, is given by

$$A = \begin{bmatrix} 0 & I \\ -A_0 & -D \end{bmatrix},$$

$$\mathrm{dom}(A) = \left\{ \left[\begin{smallmatrix} z \\ w \end{smallmatrix}\right] \in \mathrm{dom}(A_0^{1/2}) \times \mathrm{dom}(A_0^{1/2}) \mid A_0 z + Dw \in H \right\}.$$

This block operator matrix has been studied in the literature for more than 20 years.

It is well-known that A generates a C_0-semigroup of contraction, and thus the spectrum of A is located in the closed left half plane.

We are interested in a more detailed study of the location of the spectrum of A in the left half plane. In general the (essential) spectrum of A can be quite arbitrary in the closed left half plane. Under various conditions on the damping operator D we describe the location of the spectrum and the essential spectrum of A. Further, we show that under various conditions there is even a Riesz basis of $\mathrm{dom}(A_0^{1/2}) \times H$ consisting of generalized eigenvectors of A.

The Inverse Problem for Kreĭn Orthogonal Entire Matrix Functions

M.A. Kaashoek

joint work with I. Gohberg and L. Lerer

Let $k \in L_1^{n\times n}[-\omega,\omega]$, and let k be hermitian, that is, $k(t)^* = k(-t)$ for $-\omega \le t \le \omega$. Assume the equation

$$\varphi(t) - \int_0^\omega k(t-s)\varphi(s)\,ds = k(t), \quad 0 \le t \le \omega, \tag{1}$$

has a solution $\varphi \in L_1^{n\times n}[0,\omega]$, and consider the associated entire $n\times n$ matrix function

$$\Phi(\lambda) = I + \int_0^\omega e^{i\lambda t}\varphi(t)\,dt, \quad \lambda \in \mathbb{C}. \tag{2}$$

For the scalar case ($n=1$) functions of type (2) determined by (1) have been introduced by M.G. Kreĭn as continuous analogues of the classical Szegö orthogonal polynomials with respect to the unit circle. For this reason Φ is referred to as a *Kreĭn orthogonal function* generated by k. Kreĭn in the fifties and Kreĭn and Langer in the eighties proved a number of remarkable results (still for $n=1$). One of these results is the solution of the inverse problem: For a scalar function Φ of the form (2) there exists a hermitian $k \in L_1[-\omega,\omega]$ such that (1) holds if and only if Φ has no real zeroes and no conjugate pairs of zeroes. Many of the Kreĭn-Langer results have been extended to matrix-valued functions. The following theorem, which seems to be new, gives the solution to the inverse problem for matrix-valued functions.

Theorem 1 *For a function Φ defined by (2) there exists a hermitian k in $L_1^{n\times n}[-\omega,\omega]$ such that (1) holds if and only if $\det\Phi(\lambda)$ has no real zero and for any symmetric pair of zeros $\lambda_0, \bar\lambda_0$ of $\det\Phi(\lambda)$ we have*

$$\sum_{j=0}^{k}\langle\varphi_{k-j},\psi_j\rangle_{\mathbb{C}^n} = 0, \quad k = 0,1,\ldots,\min\{p,q\}-1,$$

where $\varphi_0,\varphi_1,\ldots,\varphi_{p-1}$ and $\psi_0,\psi_1,\ldots,\psi_{q-1}$ are arbitrary Jordan chains for Φ at λ_0 and $\bar\lambda_0$, respectively.

The proof is more involved than the one for the scalar counterpart. New techniques based on recent results from the theory of continuous analogues of resultant and Bezout matrices are required. Also solutions to certain equations in entire matrix functions enter into the proof. In the talk the various steps in the proof will be reviewed.

The Abstract Kinetic Equation on a Half-Line

I. Karabash

Consider the boundary value problem

$$w(\mu)\frac{\partial\psi}{\partial x}(x,\mu) = \frac{\partial^2\psi}{\partial\mu^2}(x,\mu) - q(\mu)\psi(x,\mu),$$

where $0 < x < \infty$, $\mu \in \mathbb{R}$, and

$$\psi(0,\mu) = \phi_+(\mu) \text{ if } w(\mu) > 0,$$
$$\int_{\mathbb{R}} |\psi(x,\mu)|^2 |w(\mu)| d\mu = O(1) \text{ as } x \to \infty.$$

Here the weight function w changes its sign on $\mathbb{R}$. We assume that the operator $L : y \mapsto |w|^{-1}(-y'' + qy)$ is a self-adjoint operator in the Hilbert space $L^2(\mathbb{R}, |w(x)|dx)$. Boundary value problems of this type arise as various kinetic equations (e.g. [1,2] and references).

We consider the abstract kinetic equation in the following form:

$$\frac{d\psi}{dx} = -JL\psi(x) \qquad (0 < x < \infty), \tag{1}$$

where J and L are operators in the abstract Hilbert space H such that $J = J^{-1} = J^*$ is an *signature operator* in H and L is a self-adjoint operator. By $P_\pm$ we denote the orthogonal projections onto $H_\pm := \ker(J \mp I)$. The problem is to find a continuous function $\psi : [0, +\infty) \to H$ which is H-differentiable on $(0, \infty)$ and satisfies Eq. (1) with the following boundary conditions

$$P_+\psi(0) = \phi_+, \tag{2}$$
$$\|\psi(x)\|_H = O(1) \qquad (x \to +\infty), \tag{3}$$

where $\phi_+ \in H_+$ is a given vector.

The case when L is nonnegative and has discrete spectrum has been described in great detail in papers of Beals, Kaper, Protopopescu, van der Mee, Pyatkov and other authors.

First we eliminate the assumption that the spectrum of L is discrete.

Theorem 1 *Assume that* $L = L^* \geq 0$, $\ker L = 0$, *and* J-*nonegative operator* $A := JL$ *is definitizable. Assume that neither* 0 *nor* ∞ *are singular critical points of* A. *Then for each* $\phi_+ \in H_+$, *there is a unique solution* ψ *of* (1)-(3).

Secondly, the case when the negative spectrum of L is nonempty is considered.

Let us introduce the following boundary condition at ∞

$$(E^\alpha): \qquad \|\psi(x)\|_H = O(e^{\alpha x}) \qquad (x \to +\infty).$$

Theorem 2 *Suppose that* L *is bounded below, that* $A := JL$ *is a definitizable operator, and* ∞ *is not a singular critical point of* A. *Then there exist* $\alpha \in \mathbb{R}$ *such that problem* (1)-(2)-(E^α) *has solutions for each* $\phi_+ \in H_+$.

[1] *van der Mee, C.V.M., Ran, A.C.M., Rodman, L.* Stability of stationary transport equation with accretive collision operators. J. Funct. Anal. **174** (2000), 478–512.

[2] *Pyatkov, S.G.* Operator Theory. Nonclassical Problems. Utrecht, Boston, Köln, Tokyo, VSP 2002.

On the Structure of Semigroups of Operators Acting in Spaces with Indefinite Metric

V. Khatskevich

We consider one parameter semigroups of plus-operators in spaces with indefinite metrics. We study some basic properties of such semigroups, in particular, we show that under some general conditions all the members of such semigroups are bistrict plus-operators.

Indefinite Sturm-Liouville Operators with the Singular Critical Point Zero

A. Kostenko

joint work with I. Karabash

We present a new necessary condition for similarity of indefinite Sturm-Liouville operators to self-adjoint operators. This condition is formulated in terms of Weyl-Titchmarsh m-functions. Also we obtain necessary conditions for regularity of the critical points 0 and ∞ of J-nonnegative Sturm-Liouville operators. Using this result, we construct several examples of operators with the singular critical point zero. In particular, it is shown that 0 is a singular critical point of the operator

$$-\frac{(\operatorname{sgn} x)}{(3|x|+1)^{-4/3}}\frac{d^2}{dx^2}$$

acting in the Hilbert space $L^2(\mathbb{R},(3|x|+1)^{-4/3}dx)$ and therefore this operator is not similar to a self-adjoint one. Also we construct a J-nonnegative Sturm-Liouville operator of type $(\operatorname{sgn} x)(-d^2/dx^2+q(x))$ with the same properties.

Operator Models for Singular Ordinary Differential Expressions

P. Kurasov

joint work with A. Luger

The following singular Sturm-Liouville differential expression

$$l(y) = -y''(x) + \frac{q_0+q_1x}{x^2}y(x) \quad \text{for } x \in (0,\infty),$$

with $q_0 > \frac{3}{4}$ and $q_1 \in \mathbb{R}$ is investigated. We introduce a certain singular perturbation with support at the origin and discuss its operator model. The spectral properties of this model are described by a certain generalized Nevanlinna function. The main topic is to explain the relationship between this generalized Q-function and the (generalized) Titchmarsh-Weyl function, recently introduced for this differential expression.

On the Representation of Generalized Caratheodory Functions in the Area Ω_ν

E. Lopushanskaya

The representation of a generalized Nevanlinna function g in some area $W_\nu = \{\alpha \in C_0, |\arg\alpha - \frac{\pi}{2}| \leq \nu\}$, where $0 \leq \nu < \frac{\pi}{2}$, was found in the paper by Krein M.G. and Langer H., [1]. They also solved the approximation problem for the generalized Nevanlinna function g in this area W_ν. Generalized Caratheodory functions are connected with generalized Nevanlinna functions through the Cayley-Neumann transformation. In this work, the results about the representation and approximation of the generalized Caratheodory functions are submitted. They are based on the following lemma.

It is known ([2, Chapter V]) that the function f belongs to the set C_κ if and only if there exist a Pontryagin space Π_κ, a unitary operator $V : \Pi_\kappa \to \Pi_\kappa$ and a generating element $v \in \Pi_\kappa$ such, that:

$$f(\lambda) = f(0) + 2\lambda[(V-\lambda)^{-1}v, v], \qquad (\lambda \in \Omega_\nu \setminus \sigma_p(V)). \qquad (1)$$

Lemma 1 *The function f satisfies the following conditions*

$$1.\ f(\lambda) \in C_\kappa, \quad 2.\ \overline{\lim_{\substack{\lambda\to 1\\ \lambda\in\Omega_\nu}}} \frac{Ref(\lambda)}{|1-\lambda|} < \infty, \quad \lim_{\substack{\lambda\to 1\\ \lambda\in\Omega_\nu}} f(\lambda) = 0,$$

if and only if, the generating element $v \in \Pi_\kappa$ in the representation (1) belongs to $dom(V-I)^{-1}$ and the representation (1) has the form

$$f(\lambda) = -2(\lambda-1)[(V-\lambda)^{-1}v, (V-I)^{-1}v] \qquad (\lambda \in \Omega_\nu \setminus \sigma_p(V)).$$

[1] Krein M.G., Langer H.K. Über einige Forsetzungsprobleme die eng mit der Theorie hermitescher Operatoren in Raume Π_κ zusammenhängen.-Math. Nachr., 1977, 77, S. 193-206.

[2] Azizov T.Ya., Iohvidov I.S. Foundations of the theory of linear operators in space with an indefinite metric, Nauka, Moscow 1986.

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On Non-Canonical Extensions with Finitely Many Negative Squares

A. Luger

joint work with J. Behrndt and C. Trunk

Self-adjoint operators with finitely many negative squares appear e.g. in connection with Sturm-Liouville problems with certain indefinite weight functions. For particular eigenvalue dependent boundary value problems the linearization turns out to be an operator with also finitely many negative squares (in a larger space). In this talk we are going to discuss the following issue:

Let S be a symmetric operator in a Krein space $\mathcal{K}$ with defect one and assume that it has a self-adjoint extension A_0 which has κ negative squares. The latter is equivalent to the fact that the corresponding Q-function m belongs to the so-called $\mathcal{D}_\kappa$-class. Then Kreins formula

$$\mathcal{P}_{\mathcal{K}}\big(\widetilde{A}-\lambda\big)^{-1}\,|_{\mathcal{K}}=(A_0-\lambda)^{-1}-\frac{(\,\cdot\,,\varphi(\overline{\lambda}))}{m(\lambda)+\tau(\lambda)}\varphi(\lambda)$$

establishes a one-to-one correspondence between all $\mathcal{K}$-minimal self-adjoint extensions $\widetilde{A}$ which have finitely many negative squares and the parameters τ which belong to $\bigcup_{\kappa\geq 0}\mathcal{D}_\kappa$.

In particular, we show that the actual number of negative squares of $\widetilde{A}$ can be counted precisely as the sum of the indexes of m and τ with a possible correction depending on the functions local behaviour at 0 at ∞.

Completeness of Root Vectors for arbitrary Boundary Value Problems for Sturm-Liouville Equations

M. Malamud

Completeness of root vectors of nonregular boundary value problems for Sturm-Liouville operators are investigated. Some sufficient conditions on completeness in terms of a potential will be discussed.

On Convexity of Ranges of Quadratic Forms on Various Sets

A. Markus

joint work with I. Feldman and N. Krupnik

Let H be a complex inner product space. By the famous Töplitz - Hausdorff theorem, the range of any quadratic form on the unit sphere S of the space H is convex. We consider the following question: which subsets of H besides S have this property (*Töplitz-Hausdorff property*)? One of the obtained results follows.

Theorem. *Let* $H = \boldsymbol{C}^n$, $0 < p < \infty$ *and* $0 < r \leq R < \infty$. *The set*

$$\{z \in \boldsymbol{C}^n : r^p \leq \sum_{k=1}^{n} |z_k|^p \leq R^p\}$$

has the Töplitz - Hausdorff property if and only if $p = 2$.

Corollary. *The unit sphere in* l^p*-norm has the Töplitz - Hausdorff property if and only if* $p = 2$.

Wigner's R-Matrix and Weyl Functions

H. Neidhardt

joint work with J. Behrndt, E. Racec, P. Racec and U. Wulf

The goal of the talk is to give a rigorous treatment of Wigner's and Eisenbud's R-matrix method for the scattering matrix. We consider a complete scattering system consisting of two different self-adjoint extensions of the same symmetric operator with finite deficiency indices. In the framework of boundary triplets an abstract R-matrix method for the scattering matrix is developed which generalizes the approach of Wigner and Eisenbud in a natural manner. The results are applied to an ordinary Schrödinger operator on the real axis.

On Spectra of a certain Class of Quadratic Operator Pencils with One-Dimensional Linear Part

V. Pivovarchik

Pioneering results on direct and inverse problems of small transversal vibrations of an inhomogeneous string with pointwise damping were obtained by M.G. Krein, A.A. Nudelman [1] and D.Z. Arov [2]. In these papers necessary and sufficient conditions were obtained for a sequence of complex numbers to be the spectrum of a string whose density belongs to the class of so-called S-strings. One of the general approaches to abstract versions of such problems is to use the theory of entire functions. The spectra of strings are identified with sets of zeros of functions of Hermite-Biehler class or generalized Hermite-Biehler class. For compressed beam vibrations (see [3]) one needs to use so-called shifted Hermite-Biehler functions (see [4]). Another approach is to use the theory of quadratic operator pencils. Here an important step was done in their famous paper by M.G. Krein and H. Langer [5].

We review abstract results on quadratic operator pencils associated with boundary problems which have eigenvalues in both half-planes. The considered operator pencils are of the form

$$L(\lambda) = \lambda^2 M - i\lambda K - A$$

with $M \geq 0$, $K \geq 0$, $A = A^* \geq \beta I$, $-\infty < \beta < 0$. In particular, we are interested in the case of a one-dimensional operator K. The obtained results are applied to spectral problems which occur in physics.

[1] M.G. Krein, A.A. Nudelman. J. Operator Theory, **22** (1989), 369-395. (in Russian).
[2] D.Z.Arov. Sibirian Math. J., **16** (1975), 440-463 (in Russian).
[3] M. Möller, V. Pivovarchik. Zeitschrift für Analysis und ihre Anwendungen, **25**, No. 3 (2006), 341-366.
[4] V. Pivovarchik, H. Woracek. Integral Equations and Operator Theory (to appear).
[5] M.G.Krein, H.Langer. Integral Equations and Operator Theory, **1** (1978), 364-399, 539-566.

On the Relation between $XX^{[*]}$ and $X^{[*]}X$ in an Indefinite Inner Product Space

A. Ran

joint work with J.S. Kes

On $\mathbb{C}^n$ we consider the indefinite inner product given by a Hermitian invertible matrix H. For an $n \times n$ matrix X we define by $X^{[*]}$ the adjoint in this indefinite inner product, that is, $X^{[*]} = H^{-1}X^*H$. Obviously, both $X^{[*]}X$ and $XX^{[*]}$ are H-selfadjoint. We discuss the relations between the canonical forms for the pairs $(XX^{[*]}, H)$ and $(X^{[*]}X, H)$. For some specific cases which are obtained by imposing restrictions on rank$X$ or rank$X^{[*]}X$ or both, the relations between these canonical forms can be found explicitly.

More precisely, given the canonical form of the pair $(X^{[*]}X, H)$, we shall describe the canonical form of $(XX^{[*]}, H)$ in the case where rank$X^{[*]}X =$ rank$X$, as well as in the case where $X^{[*]}X = 0$. Note that these two cases can be seen as being opposite extremes. Several other cases are discussed as well.

On Factorization of Matrix Functions in the Wiener Algebra

L. Rodman

joint work with I.M. Spitkovsky

Let G be a (multiplicative) connected compact abelian group, let Γ be its (additive) discrete character group, and let $\preceq$ be a fixed linear order such that $(\Gamma, \preceq)$ is an ordered group.

Given $a = \{a_j\}_{j\in\Gamma} \in \ell^1(\Gamma)$, the *symbol* of a is the complex-valued continuous function $\hat{a}$ on G defined by

$$\hat{a}(g) = \sum_{j\in\Gamma} a_j \langle j, g\rangle, \qquad g \in G,$$

where $\langle j, g\rangle$ stands for the action of the character $j \in \Gamma$ on the group element $g \in G$ (thus, $\langle j, g\rangle$ is a unimodular complex number). The set of all symbols of elements $a \in \ell^1(\Gamma)$ forms the *Wiener algebra* $W(G)$ of continuous functions on G (with pointwise multiplication and addition). Denote by $W(G)_+$ (resp., $W(G)_-$) the algebra of symbols of elements in $\ell^1(\Gamma_+)$ (resp., $\ell^1(\Gamma_-)$), where Γ_+, resp. Γ_-, is the set of nonnegative, resp. nonpositive, elements of Γ with respect to $\preceq$.

A (*left*) *factorization* of matrix function $A \in (W(G))^{n\times n}$ is a representation of the form

$$A(g) = A_+(g)\Lambda(g)A_-(g), \quad g \in G,$$

where A_+ and its inverse belong to $(W(G)_+)^{n\times n}$, A_- and its inverse belong to $(W(G)_-)^{n\times n}$, $\Lambda = \operatorname{diag}(\langle j_1, \cdot\rangle, \dots, \langle j_n, \cdot\rangle)$, and *the indices* $j_1, \dots, j_n \in \Gamma$.

Main result:

Theorem. *Let Γ' be a subgroup of Γ, let G and G' be the character groups of Γ and Γ', respectively. Assume that $A \in W(G')^{n\times n}$ admits a Γ-factorization. Then A admits a Γ'-factorization, necessarily with the same indices.*

Composition of Dirac Structures for Infinite-Dimensional Systems

A. Sandovici

joint work with O. Iftime

Dirac structures are used to formalize the power–conserving interconnection structure of physical systems. For finite–dimensional systems it is known that the composition or interconnection is again a Dirac structure. For infinite–dimensional systems this is not always the case. Necessary and sufficient conditions for preserving the Dirac structure under the composition are provided.

Indefinite Sturm-Liouville Operators with Singular Self-Similar Weights

I.A. Sheipak

joint work with A.A. Vladimirov

We study the asymptotics of the spectrum for the boundary eigenvalue problem

$$-y'' - \lambda\rho y = 0,$$
$$y(0) = y(1) = 0,$$

where $\rho \in \overset{\circ}{W}{}_2^{-1}[0,1]$ is the generalized derivative of fractal (self-similar) function $P \in L_2[0,1]$.

We prove that the eigenvalues of positive and negative type accumulate to $\pm\infty$ and their counting functions have representation

$$N_\pm(\lambda) = |\lambda|^{D/2} \cdot (s_\pm(\ln|\lambda|) + o(1)), \quad |\lambda| \to \infty \qquad (*),$$

where $D \in (0,2)$ is the fractal dimension of the function P and $s_\pm$ are some periodic functions. We study three cases of self-similarity of P which induce different behaviour of functions $s_\pm$.

The particular case of definite ρ, when ρ is a self-similar measure, was studied by M. Solomyak and E. Verbitsky. (A paper of M. Levitin and D. Vassiliev is also related to the topic). They obtained the asymptotic formula $(*)$ with $s_- \equiv 0$ and $D \in (0,1]$.

We pay attention that fractal dimension D can be greater than 1 only in the case when ρ is indefinite.

[1]*Self-similar functions in $L_2[0,1]$ and Sturm–Liouville problem with Singular Indefinite Weight* Math. Sbornik, 2006, **197** (11), p.13-30 (http://www.arxiv.org/math.FA/0405410)
[2] *Indefinite Sturm–Liouville problem for Some Classes of Self-similar Singular Weights* Proc. Steklov Institute of Math., 2006, **255**, p.88-98 (http://www.arxiv.org/math.FA/0507017).

Remarks on Global Stability for Inverse Sturm-Liouville Problems

A.A. Shkalikov

joint work with A. Savchuk

We study inverse problems for the Sturm-Liouville operator

$$Ly = -y'' + q(x)y$$

on the finite interval $[0, \pi]$. The main attention is paid to the reconstruction of the potential $q(x)$ from given two spectra $\{\lambda_k\}_1^\infty$ and $\{\mu_k\}_1^\infty$ of the operators L_D and L_{DN} generated by L with Dirichlet ($y(0) = y(\pi) = 0$) and Dirichlet-Neumann ($y(0) = y'(\pi)$) conditions, respectively. We introduce the special spaces $\hat{l}_2^\theta$ which are finite dimensional dilations of the usual weighted l_2-spaces and give the complete characterization for the sequences $\{\lambda_k\}_1^\infty$ and $\{\mu_k\}_1^\infty$ (in terms of these spaces) to be the spectra L_D and L_{DN} with the potential $q(x)$ belonging to the Sobolev space $W_2^{\theta-1}[0, \pi]$, provided that $\theta \geqslant 0$. The case $\theta = 1$ gives the classical result due to Borg, Marchenko and Levitan.

Then, we prove the estimates (characterizing the global stability)

$$\|q^0(x) - q^1(x)\|_{W^{\theta-1}} \leqslant C(\|\{\sqrt{\lambda_k^0} - \sqrt{\lambda_k^1}\}\|_{\hat{l}^\theta} + \|\{\sqrt{\mu_k^0} - \sqrt{\mu_k^1}\}\|_{\hat{l}^\theta})$$

provided that $\{\lambda_k^j\}_1^\infty$ and $\{\mu_k^j\}_1^\infty$, j=0,1, lie inside some "natural" convex sets, and the constant C depends only on the parameters characterizing these sets. Estimates of this type are new for all $\theta \geqslant 0$ including the classical case $\theta = 1$.

On Approximation of a Boundary Value Problem at 'Regular Singular' Endpoint

Y. Shondin

joint work with A. Dijksma and A. Luger

We consider Sturm-Liouville expressions which have a 'regular singular' point, say at zero, e.g. the Bessel expression, and for which in L^2 setting the 'limit point' case at zero prevails. In this case there is a Pontryagin space realization of the minimal operator, which is symmetric and has nonzero defect indices, and the 'limit circle' case at zero is reproduced. We discuss the approximation of associated boundary value problems by appropriate boundary value problems close to zero regular points.

A Lebesgue Decomposition for Nonnegative Forms

H. de Snoo

joint work with S. Hassi and Z. Sebestyén

A nonnegative form $\mathfrak{t}$ on a complex linear space is decomposed with respect to another nonnegative form $\mathfrak{w}$ into an almost dominated part and a singular part. The almost dominated part is the largest form majorized by $\mathfrak{t}$ which is closable. This decomposition addresses a problem posed by Simon. The Lebesgue decomposition of a pair of finite measures corresponds to the present decomposition of the forms which are induced by the measures. Ando's decomposition of a nonnegative bounded linear operator in a Hilbert space with respect to another nonnegative bounded linear operator is a consequence. An important ingredient in the present paper is the parallel sum of forms.

A Test for Commutative J-Symmetric Families of D_κ^+-Class

V. Strauss

A goal of this report is a study of relations between a commutative J-symmetric operator family of so-called D_κ^+-class and its system of eigenvectors and rootvectors.

On Linearizators of Hyperbolic Type Pencils

L. Sukhocheva

An operator class of Krein space selfadjoint operators which elements are linearizators of hyperbolic type pencils is described.

This research is supported by the RFBR grant 05-01-00203.

Rows versus Columns

F.H. Szafraniec

joint work with M. Möller

The anatomy of matrices of unbounded operators will be presented in some detail.

Influence of the Curvature on the Impurity Spectrum in a Quantum Dot

M. Tusek

joint work with V. Geyler and P. Stovicek

We consider an explicitly solvable model of quantum dots with a short-range impurity; the dot is displaced in a flat two-dimensional nanostructure or in a curved nanostructure with constant negative curvature. For the Hamiltonian of the dot without impurity we choose the Laplace-Beltrami operator; the confinement is taken into account with the help of the harmonic oscillator potential. The impurity is modelled by a point potential. Using the operator extension theory we give an explicit form for the Green function of the full Hamiltonian. The equation for eigenvalues is obtained in terms of hypergeometric, spheroidal and Bessel functions. Analytic and numerical analysis of the dependence of eigenvalues on parameters of the problem (curvature, scattering length, size of the dot) is performed.

Normal and Hyponormal Matrices in Inner Product Spaces

C. Trunk

joint work with C. Mehl

Complex matrices that are structured with respect to a possibly degenerate indefinite inner product are studied. Based on the theory of linear relations, the notion of an adjoint is introduced: the adjoint of a matrix is defined as a linear relation which is a matrix if and only if the inner product is nondegenerate.

This notion is then used to give a new definition for normal matrices which allows the generalization of an extension result for positive invariant subspaces from the case of nondegenerate inner products to the case of degenerate inner products.

Moreover, we will introduce the notion of hyponormal matrices in inner product spaces and discuss some of their properties.

On Eigenvalue Bounds for Indefinite Selfadjoint Operators

K. Veselić

We derive tight eigenvalue bounds for perturbations of selfadjoint operators which are not semibounded. Instead of variational principles we use analyticity and monotonicity properties. We apply our abstract result to operators factorised as

$$T = GJG^*$$

where $G^{-1}, J = J^*, J^{-1}$ are ewerywhere defined and bounded and G is perturbed into $\tilde{G} = G + \delta G$ with

$$\|\delta G^* \psi\| \leq \beta \|G^* \psi\|, \beta < 1.$$

It turns up that the perturbation bounds depend on the regularity of the positive J-selfadjoint operator $S = JG^*G$. Some apparently new and calculable criteria for the regularity are derived. Applications include various matrix operators defined as quadratic forms.

Kato Decompositions for Quasi-Fredholm Relations

H. Winkler

joint work with J.-Ph. Labrousse, A. Sandovici, H.S.V. de Snoo

A closed linear operator A in a Hilbert space $\mathfrak{H}$ is said to be semi-Fredholm if $\operatorname{ran} A$ is closed and $\ker A$ or $\mathfrak{H}/\operatorname{ran} A$ is finite-dimensional. T. Kato has shown that these operators allow an algebraic decomposition. The more general class of quasi-Fredholm operators was investigated by J.-Ph. Labrousse. A range space relation A in a Hilbert space $\mathfrak{H}$ is said to be quasi-Fredholm of degree $d \in \mathbb{N} \cup \{0\}$ if

1. $\operatorname{ran} A^n \cap \ker A = \operatorname{ran} A^d \cap \ker A$ for all $n \geq d$;
2. $\ker A \cap \operatorname{ran} A^d$ is closed in $\mathfrak{H}$;
3. $\operatorname{ran} A + \ker A^d$ is closed in $\mathfrak{H}$.

Quasi-Fredholm relations of degree d are characterized by an Kato-like decomposition into a quasi-Fredholm relation of degree 0 and a nilpotent operator. The adjoint of a quasi-Fredholm relation of degree d is a quasi-Fredholm relation of degree d.

Domination in Krein Spaces

M. Wojtylak

Let the operator A be symmetric in a Krein space $\mathcal{K}$, and let $(S_n)_{n=0}^{\infty} \subseteq \mathbf{B}(\mathcal{K})$ be a sequence tending to $I_{\mathcal{K}}$ in the weak operator topology. Our main result says that if the operators $S_nA - AS_n$ and AS_n^+ are densely defined for $n \in \mathbb{N}$ and

$$\sup_{n\in\mathbb{N}} \|AS_n - S_nA\| < +\infty$$

then A is selfadjoint in $\mathcal{K}$. We consider various examples of the sequence $(S_n)_{n=0}^{\infty}$. For instance, we take $S_n = (-z_n)^m(S-z_m)^{-m}$, where S is a selfadjoint operator such that S^m dominates A, i.e. $\mathcal{D}(S^m) \subseteq \mathcal{D}(A)$. In this way we generalize some results from [1] onto Krein spaces and also obtain a new criteria for selfadjointness in a Hilbert spaces.

[1] D. Cichoń, J. Stochel, F.H. Szafraniec, Noncommuting domination, *Oper. Theory Adv. Appl.* 154 (2004), 19-33 .
[2] M. Wojtylak, Noncommuting domination in Krein spaces via commutators of block operator matrices, (to appear).

Admissible Majorants for de Branges Spaces of Entire Functions

H. Woracek

joint work with A. Baranov

The notion of admissible majorants for shift-coinvariant subspaces of the Hardy space $H^2(\mathbb{C}^+)$ was recently introduced in a series of papers by V. Havin and J. Mashreghi. It applies in particular to de Branges spaces of entire functions.

Let $\mathcal{H}(E)$ be a de Branges space and let ω be a nonnegative function on $\mathbb{R}$. Then we investigate the subspace

$$\mathcal{R}_\omega(E) = \mathrm{clos}_{\mathcal{H}(E)}\big\{F \in \mathcal{H}(E) : \exists C > 0 : |E^{-1}F| \le C\omega \text{ on } \mathbb{R}\big\}$$

of $\mathcal{H}(E)$.

We show that $\mathcal{R}_\omega(E)$ is a de Branges subspace and describe all subspaces of $\mathcal{H}(E)$ which can be represented in this form. We study those majorants ω such that $\mathcal{R}_\omega(E) = \mathcal{H}(E)$ and give a criterion for the existence of positive minimal majorants. Note that the condition $\mathcal{R}_\omega(E) = \mathcal{H}(E)$ means that all elements of $\mathcal{H}(E)$ can be approximated by functions which are in a sense small on the real line. To illustrate the general situation we give some examples, which are obtained from the Beurling-Malliavin Theorem and from the theory of canonical systems of differential equations.