Approximation of Generalized Schur functions

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In [1] M.G. Krein and H. Langer investigated questions concerning the approximation of Nevanlinna functions. Our purpose is to get such result for Schur functions.

Theorem 1 *Let $s(\lambda) = \lambda^k s_k(\lambda),\ s_k(0) \neq 0$, $k \leqslant n$. Then we have assertions*

1. *$s \in S_\varkappa$, where $S_\varkappa-$ generalised Schur class;*
2. *for some integer $n > 0$ there exist $2n$ real numbers $c_1, c_2, \ldots, c_{2n}$ such that*

$$s(\lambda) = 1 - \sum_{\nu=1}^{2n} c_\nu (\lambda - 1)^\nu + O((\lambda - 1)^{2n+1}), \quad \lambda \to 1,\ \lambda \in \Lambda_\theta \tag{1}$$

if and only if when exist a Pontryagin space $\Pi_\varkappa$, a contraction operator T in $\Pi_\varkappa$ and the generative element $u \in dom(I - T)^{-(n+1)}$ for operator T such that:

$$s(\lambda) = \lambda^k - \frac{1}{s_k(0)} \lambda^k (\lambda - 1)[(I - \lambda T)^{-1}(I - T)^{-1} T^{k+1} u, T^k u],\ \lambda \in \mathbb{D},\ \frac{1}{\lambda} \notin \sigma_p(T) \tag{2}$$

In this case:

$$c_\nu = \begin{cases} \dfrac{1}{s_k(0)} \displaystyle\sum_{i=1}^{\nu} C_{k-i}^{\nu-i} [(I - T)^{-(i+1)} T^{k+1} u, T^k u] - C_k^\nu, & 1 \leqslant \nu < k + 1; \\ \dfrac{1}{s_k(0)} [(I - T)^{-(\nu+1)} T^\nu u, T^k u], & k + 1 \leqslant \nu \leqslant n; \\ \dfrac{1}{s_k(0)} [(I - T)^{-(n+1)} T^n u, (I - T^c)^{-(\nu-n)} T^{c(\nu-n)} T^k u], & n + 1 \leqslant \nu \leqslant 2n; \end{cases}$$

References

[1] Krein M. G., H. Langer, "Über einige Fortsetzungsprobleme, die eng mit der Theorie hermitescher Operatoren im Raume Π_κ zusammenhängen. I. Einige Funktionenklassen und ihre Darstellungen", *Math. Nachr. 77* (1977), 187-236.