Darboux Transformations and Commutation in Pontryagin Space

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Darboux transformations have become popular for generating hierarchies of differential equations, and are frequently used in mathematical physics. They are intimately connected with von Neumann's theorem on products of operators and their adjoints, with square roots and (restricted) commutation properties of these products, and with polar decompositions of operators, in Hilbert space. Motivated by Sturm-Liouville problems with eigenvalue dependent boundary conditions, we shall discuss corresponding ideas in Pontryagin space. In this case one has to tread more carefully, and for example some of the above constructions need not even exist.