A Criterion for the Resolvent Set of Generalized Jacobi Operators Acting in Krein Spaces

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Let us consider a generalized Jacobi matrix, i.e. the following tridiagonal block matrix

$$\begin{pmatrix} A_0 & \widetilde{B}_0 & & \mathbf{0} \\ B_0 & A_1 & \widetilde{B}_1 & \\ & B_1 & A_2 & \ddots \\ \mathbf{0} & & \ddots & \ddots \end{pmatrix}.$$

Under some assumptions, this matrix generates a bounded self-adjoint operator in a Krein space. A characterization of the resolvent set of such operators via associated polynomials is given. In particular, we obtain an explicit description of the spectrum of operators generated by periodic generalized Jacobi matrices, that is generalized Jacobi matrices with the following property

$$A_{js+k} = A_k, \quad \widetilde{B}_{js+k} = \widetilde{B}_k, \quad B_{js+k} = B_k,$$

where $s \in \mathbb{N}$, $j \in \mathbb{N} \cup \{0\}$, and $k \in \{0, \ldots, s-1\}$.