Block Numerical Ranges of Nonnegative Matrices

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joint work with K.-H. Förster

Let A be a $\ell \times \ell$ square matrix. For a block partition $A = (A_{rs})_{r,s=1,\ldots,n}$ we define the block numerical range as

$$W^n(A_{rs}) = \big\{\lambda \in \mathbb{C} \mid \exists\ (x_1, \ldots, x_n) \in \mathbb{C}^\ell, \|x_j\| = 1\ : \\ \lambda \text{ is an eigenvalue of } \big(\langle A_{rs}x_s, x_r\rangle\big)_{r,s=1,\ldots,n}\big\}$$

and the block numerical radius

$$w^n(A_{rs}) = \max\{|\lambda| \mid \lambda \in W^n(A_{rs})\}.$$

The main result of the talk is: Let A be an (entrywise) nonnegative irreducible matrix with index of imprimitivity m. Then we get the following Perron-Frobenius type result

$$\{\lambda \in W^n(A_{rs}) \mid |\lambda| = w^n(A_{rs})\} = \{w^n(A_{rs})e^{2k\pi i/m}, k = 0, 1, \ldots, m-1\}.$$

For $n = 1$ this is the result of J.N. Issos on the usual numerical range and for $n = \ell$ it is the theorem of Perron-Frobenius on the peripheral spectrum of nonnegative irreducible matrices.