Location of the Spectrum of Operator Matrices which are Associated to Second Order Equations

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We study second order equations of the form

$$\ddot{z}(t) + A_0 z(t) + D\dot{z}(t) = 0. \tag{1}$$

Here the stiffness operator A_0 is a possibly unbounded positive operator on a Hilbert space H, which is assumed to be boundedly invertible, and D, the damping operator, is an unbounded operator, such that $A_0^{-1/2} D A_0^{-1/2}$ is a bounded non negative operator on H. This second order equation is equivalent to the standard first-order equation $\dot{x}(t) = Ax(t)$, where $A : \mathcal{D}(A) \subset \mathcal{D}(A_0^{1/2}) \times H \to \mathcal{D}(A_0^{1/2}) \times H$, is given by

$$A = \begin{bmatrix} 0 & I \\ -A_0 & -D \end{bmatrix},$$

$$\mathcal{D}(A) = \left\{ \left[\begin{smallmatrix} z \\ w \end{smallmatrix}\right] \in \mathcal{D}(A_0^{1/2}) \times \mathcal{D}(A_0^{1/2}) \mid A_0 z + Dw \in H \right\}.$$

This block operator matrix has been studied in the literature for more than 20 years.

It is well-known that A generates a C_0-semigroup of contraction, and thus the spectrum of A is located in the closed left half plane.

We are interested in a more detailed study of the location of the spectrum of A in the left half plane. In general the (essential) spectrum of A can be quite arbitrary in the closed left half plane. Under various conditions on the damping operator D we describe the location of the spectrum and the essential spectrum of A. Further, we show that under various conditions there is even a Riesz basis of $\mathcal{D}(A_0^{1/2}) \times H$ consisting of generalized eigenvectors of A.