On the Representation of Generalized Caratheodory Functions in the Area Ω_ν

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The representation of the generalized Nevanlinna function g in some area $W_\nu = \{\alpha \in C_0, |arg\alpha - \frac{\pi}{2}| \leq \nu\}$, where $0 \leq \nu < \frac{\pi}{2}$, was found in the paper by Krein M.G. and Langer H. [1] . They also solved the approximation problem of the generalized Nevanlinna function g in this area W_ν. Generalized Caratheodory functions are connected with generalized Nevanlinna functions through the Cayley- Neumann transformation. In this work, the results about the representation and approximation of the generalized Caratheodory functions are submitted. They are based on the following lemma.

It is known ([2, chapter V,§3]) that the function f belongs to the set C_κ if and only if there exist Pontryagin space Π_κ, unitary operator $V : \Pi_\kappa \to \Pi_\kappa$ and generating element $v \in \Pi_\kappa$ such, that:

$$f(\lambda) = f(0) + 2\lambda[(V-\lambda)^{-1}v, v], \qquad (\lambda \in \Omega_\nu \setminus \sigma_p(V)). \tag{1}$$

Lemma 1 *The function f satisfies the following conditions*

$$1. f(\lambda) \in C_\kappa, \quad 2. \overline{\lim_{\substack{\lambda\to 1\\ \lambda\in\Omega_\nu}}} \frac{Ref(\lambda)}{|1-\lambda|} < \infty, \quad \lim_{\substack{\lambda\to 1\\ \lambda\in\Omega_\nu}} f(\lambda) = 0$$

if and only if, the generating element $v \in \Pi_\kappa$ in the representation (1) belongs to $dom(V-I)^{-1}$ and the representation (1) looks like:

$$f(\lambda) = -2(\lambda-1)[(V-\lambda)^{-1}v, (V-I)^{-1}v] \qquad (\lambda \in \Omega_\nu \setminus \sigma_p(V)). \tag{2}$$

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References

[1] Krein M.G., Langer H.K. Uber einige Forsetzungsprobleme die eng mit der Theorie hermitescher Operatoren in Raume Π_κ zusammenhängen.- Math. Nachr., 1977, 77, S. 193-206.

[2] Azizov T.Ya., Iohvidov I.S. Foundations of the theory of linear operators in space with an indefinite metric, Nauka, Moscow 1986.