On Non-Canonical Extensions with Finitely Many Negative Squares

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Self-adjoint operators with finitely many negative squares appear e.g. in connection with Sturm-Liouville problems with certain indefinite weight functions. For particular eigenvalue dependent boundary value problems the linearization turns out to be an operator with also finitely many negative squares (in a larger space). In this talk we are going to discuss the following issue:

Let S be a symmetric operator in a Krein space $\mathcal{K}$ with defect 1 and assume that it has a self-adjoint extension A_0 which has κ negative squares. The latter is equivalent to the fact that the corresponding Q-function m belongs to the so-called $\mathcal{D}_\kappa$-class. Then Kreins formula

$$\mathcal{P}_{\mathcal{K}}\big(\widetilde{A}-\lambda\big)^{-1}|_{\mathcal{K}} = (A_0-\lambda)^{-1} - \frac{(\,\cdot\,,\varphi(\overline{\lambda}))}{m(\lambda)+\tau(\lambda)}\varphi(\lambda)$$

establishes a one-to-one correspondence between all $\mathcal{K}$-minimal self-adjoint extensions $\widetilde{A}$ which have finitely many negative squares and the parameters τ which belong to $\bigcup_{\kappa\geq 0}\mathcal{D}_\kappa$.

In particular, we show that the actual number of negative squares of $\widetilde{A}$ can be counted precisely as the sum of the indexes of m and τ with a possible correction depending on the functions local behaviour at 0 at ∞.