

On Convexity of Ranges of Quadratic Forms on Various Sets

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Let H be a complex inner product space. By the famous Töplitz - Hausdorff theorem, the range of any quadratic form on the unit sphere S of the space H is convex. We consider the following question: which subsets of H besides S have this property (*Töplitz - Hausdorff property*)? One of the obtained results follows.

Theorem. *Let $H = \mathbf{C}^n$, $0 < p < \infty$ and $0 < r \leq R < \infty$. The set*

$$\{z \in \mathbf{C}^n : r^p \leq \sum_{k=1}^n |z_k|^p \leq R^p\}$$

has the Töplitz - Hausdorff property if and only if $p = 2$.

Corollary. *The unit sphere in l^p -norm has the Töplitz - Hausdorff property if and only if $p = 2$.*