

# On Spectra of a Certain Class of Quadratic Operator Pencils with One-Dimensional Linear Part

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Pioneering results on direct and inverse problems of small transversal vibrations of an inhomogeneous string with point-wise damping were obtained by M.G. Krein, A.A. Nudelman [1] and D.Z. Arov [2]. In these papers conditions were obtained necessary and sufficient for a sequence of complex numbers to be the spectrum of a string whose density belongs to the class of so-called S-strings. One of the general approaches to abstract version of such problems is to use the theory of entire functions. The spectra of strings are identified with sets of zeros of function of Hermite-Biehler class or generalized Hermite-Biehler class. For compressed beam vibrations (see [3]) one needs to use so-called shifted Hermite-Biehler functions (see [4]). Another approach is to use the theory of quadratic operator pencils. Here an important step was done in their famous paper by M.G. Krein and H. Langer [5].

We review abstract results on quadratic operator pencils associated with boundary problems which have eigenvalues in both half-planes. The considered operator pencils are of the form

$$L(\lambda) = \lambda^2 M - i\lambda K - A$$

with  $M \geq 0$ ,  $K \geq 0$ ,  $A = A^* \geq \beta I$ ,  $-\infty < \beta < 0$ . In particular, we are interested in the case of one-dimensional operator  $K$ . The obtained results are applied to spectral problems which occur in physics.

## References

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