

On the relation between $XX^{[*]}$ and $X^{[*]}X$ in an indefinite inner product space

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On \mathbb{C}^n we consider the indefinite inner product given by a Hermitian invertible matrix H . For an $n \times n$ matrix X we define by $X^{[*]}$ the adjoint in this indefinite inner product, that is, $X^{[*]} = H^{-1}X^*H$. Obviously, both $X^{[*]}X$ and $XX^{[*]}$ are H -selfadjoint. We discuss the relations between the canonical forms for the pairs $(XX^{[*]}, H)$ and $(X^{[*]}X, H)$. For some specific cases which are obtained by imposing restrictions on $\text{rank}X$ or $\text{rank}X^{[*]}X$ or both, the relations between these canonical forms can be found explicitly.

More precisely, given the canonical form of the pair $(X^{[*]}X, H)$, we shall describe the canonical form of $(XX^{[*]}, H)$ in the case where $\text{rank}X^{[*]}X = \text{rank}X$, as well as in the case where $X^{[*]}X = 0$. Note that these two cases can be seen as being opposite extremes. Several other cases are discussed as well.