On Factorization of Matrix Functions in the Wiener Algebra

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Let G be a (multiplicative) connected compact abelian group, let Γ be its (additive) discrete character group, and let $\preceq$ be a fixed linear order such that $(\Gamma, \preceq)$ is an ordered group.

Given $a = \{a_j\}_{j \in \Gamma} \in \ell^1(\Gamma)$, the *symbol* of a is the complex-valued continuous function $\hat{a}$ on G defined by

$$\hat{a}(g) = \sum_{j \in \Gamma} a_j \langle j, g \rangle, \qquad g \in G,$$

where $\langle j, g \rangle$ stands for the action of the character $j \in \Gamma$ on the group element $g \in G$ (thus, $\langle j, g \rangle$ is a unimodular complex number). The set of all symbols of elements $a \in \ell^1(\Gamma)$ forms the *Wiener algebra* $W(G)$ of continuous functions on G (with pointwise multiplication and addition). Denote by $W(G)_+$ (resp., $W(G)_-$) the algebra of symbols of elements in $\ell^1(\Gamma_+)$ (resp., $\ell^1(\Gamma_-)$), where Γ_+, resp. Γ_-, is the set of nonnegative, resp. nonpositive, elements of Γ with respect to $\preceq$.

A (*left*) *factorization* of matrix function $A \in (W(G))^{n \times n}$ is a representation of the form

$$A(g) = A_+(g) \Lambda(g) A_-(g), \quad g \in G, \tag{1}$$

where A_+ and its inverse belong to $(W(G)_+)^{n \times n}$, A_- and its inverse belong to $(W(G)_-)^{n \times n}$, $\Lambda = \operatorname{diag}(\langle j_1, \cdot \rangle, \ldots, \langle j_n, \cdot \rangle)$, and *the indices* $j_1, \ldots, j_n \in \Gamma$.

Main result:

Theorem. *Let Γ' be a subgroup of Γ, let G and G' be the character groups of Γ and Γ', respectively. Assume that $A \in W(G')^{n \times n}$ admits a Γ-factorization. Then A admits a Γ'-factorization, necessarily with the same indices.*