Indefinite Sturm-Liouville Operators with Singular Self-Similar Weights

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We study the asymptotics of the spectrum for the boundary eigenvalue problem

$$-y'' - \lambda \rho y = 0,$$
$$y(0) = y(1) = 0,$$

where $\rho \in \overset{\circ}{W}{}_2^{-1}[0,1]$ is the generalized derivative of fractal (self-similar) function $P \in L_2[0,1]$.

We prove that the eigenvalues of positive and negative type accumulate to $\pm\infty$ and their counting functions have representation

$$N_{\pm}(\lambda) = |\lambda|^{D/2} \cdot \left(s_{\pm}(\ln|\lambda|) + o(1)\right), \quad |\lambda| \to \infty \qquad (*)$$

where $D \in (0,2)$ is the fractal dimension of the function P and $s_{\pm}$ are some periodic functions. We study three cases of self-similarity of P which induce different behaviour of functions $s_{\pm}$.

The particular case of definite ρ, when ρ is a self-similar measure, was studied by M. Solomyak and E. Verbitsky. (A paper of M. Levitin and D. Vassiliev is also related to the topic). They obtained the asymptotic formula $(*)$ with $s_{-} \equiv 0$ and $D \in (0,1]$.

We pay attention that fractal dimension D can be greater than 1 only in the case when ρ is indefinite.

The talk is based on the joint works with A.A. Vladimirov:

1. *Self-similar functions in* $L_2[0,1]$ *and Sturm–Liouville problem with Singular Indefinite Weight* Math. Sbornik, 2006, **197** (11), p.13-30 (http://www.arxiv.org/math.FA/0405410) and

2. *Indefinite Sturm–Liouville problem for Some Classes of Self-similar Singular Weights* Proc. Steklov Institute of Math., 2006, **255**, p.88-98 (http://www.arxiv.org/math.FA/0507017).