Remarks on Global Stability for Inverse Sturm-Liouville Problems

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We study inverse problems for the Sturm-Liouville operator

$$Ly = -y'' + q(x)y$$

on the finite interval $[0, \pi]$. The main attention is paid to the reconstruction of the potential $q(x)$ from given two spectra $\{\lambda_k\}_1^\infty$ and $\{\mu_k\}_1^\infty$ of the operators L_D and L_{DN} generated by L with Dirichlet ($y(0) = y(\pi) = 0$) and Dirichlet-Neumann ($y(0) = y'(\pi)$) conditions, respectively. We introduce the special spaces $\hat{l}_2^\theta$ which are finite dimensional dilations of the usual weighted l_2-spaces and give the complete characterization for the sequences $\{\lambda_k\}_1^\infty$ and $\{\mu_k\}_1^\infty$ (in terms of these spaces) to be the spectra L_D and L_{DN} with the potential $q(x)$ belonging to the Sobolev space $W_2^{\theta-1}[0, \pi]$, provided that $\theta \geqslant 0$. The case $\theta = 1$ gives the classical result due to Borg, Marchenko and Levitan.

Then, we prove the estimates (characterizing the global stability)

$$\|q^0(x) - q^1(x)\|_{W^{\theta-1}} \leqslant C(\|\{\sqrt{\lambda_k^0} - \sqrt{\lambda_k^1}\}\|_{\hat{l}^\theta} + \|\{\sqrt{\mu_k^0} - \sqrt{\mu_k^1}\}\|_{\hat{l}^\theta})$$

provided that $\{\lambda_k^j\}_1^\infty$ and $\{\mu_k^j\}_1^\infty$, j=0,1, lie inside some "natural" convex sets, and the constant C depends only on the parameters characterizing these sets. Estimates of this type are new for all $\theta \geqslant 0$ including the classical case $\theta = 1$.