On Eigenvalue Bounds for Indefinite Selfadjoint Operators

K. Veselic

We derive tight eigenvalue bounds for perturbations of selfadjoint operators which are not semibounded. Instead of variational principles we use analyticity and monotonicity properties. We apply our abstract result to operators factorised as

$$T = GJG^*$$

where $G^{-1}, J = J^*, J^{-1}$ are ewerywhere defined and bounded and G is perturbed into $\tilde{G} = G + \delta G$ with

$$\|\delta G^* \psi\| \leq \beta \|G^* \psi\|, \beta < 1.$$

It turns up that the perturbation bounds depend on the regularity of the positive J-selfadjoint operator $S = JG^*G$. Some apparently new and calculable criteria for the regularity are derived. Applications include various matrix operators defined as quadratic forms.