Kato Decompositions for Quasi-Fredholm Relations

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A closed linear operator A in a Hilbert space $\mathfrak{H}$ is said to be semi-Fredholm if $\operatorname{ran} A$ is closed and $\ker A$ or $\mathfrak{H}/\operatorname{ran} A$ is finite-dimensional. T. Kato has shown that these operators allow an algebraic decomposition. The more general class of quasi-Fredholm operators was investigated by J.-Ph. Labrousse. A range space relation A in a Hilbert space $\mathfrak{H}$ is said to be quasi-Fredholm of degree $d \in \mathbb{N} \cup \{0\}$ if

1. $\operatorname{ran} A^n \cap \ker A = \operatorname{ran} A^d \cap \ker A$ for all $n \geq d$;
2. $\ker A \cap \operatorname{ran} A^d$ is closed in $\mathfrak{H}$;
3. $\operatorname{ran} A + \ker A^d$ is closed in $\mathfrak{H}$.

Quasi-Fredholm relations of degree d are characterized by an Kato-like decomposition into a quasi-Fredholm relation of degree 0 and a nilpotent operator. The adjoint of a quasi-Fredholm relation of degree d is a quasi-Fredholm relation of degree d.