Admissible Majorants for de Branges Spaces of Entire Functions

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The notion of admissible majorants for shift-coinvariant subspaces of the Hardy space $H^2(\mathbb{C}^+)$ was recently introduced in a series of papers by V. Havin and J. Mashreghi. It applies in particular to de Branges spaces of entire functions.

Let $\mathcal{H}(E)$ be a de Branges space and let ω be a nonnegative function on $\mathbb{R}$. Then we investigate the subspace

$$\mathcal{R}_\omega(E) = \operatorname{clos}_{\mathcal{H}(E)}\big\{F \in \mathcal{H}(E) : \exists C > 0 : |E^{-1}F| \le C\omega \text{ on } \mathbb{R}\big\}$$

of $\mathcal{H}(E)$.

We show that $\mathcal{R}_\omega(E)$ is a de Branges subspace and describe all subspaces of $\mathcal{H}(E)$ which can be represented in this form. We study those majorants ω such that $\mathcal{R}_\omega(E) = \mathcal{H}(E)$ and give a criterion for the existence of positive minimal majorants. Note that the condition $\mathcal{R}_\omega(E) = \mathcal{H}(E)$ means that all elements of $\mathcal{H}(E)$ can be approximated by functions which are in a sense small on the real line. To illustrate the general situation we give some examples, which are obtained from the Beurling-Malliavin Theorem and from the theory of canonical systems of differential equations.